Please note the connection between problem 4.8 and the material we've already covered. Eq. (1.47) relates acceleration in polar coordinates:

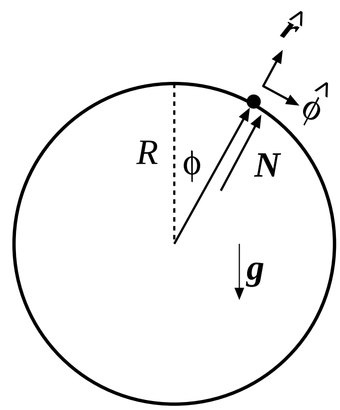
$$\ddot{\mathbf{r}} = \mathbf{a} = (\ddot{r} - r\dot{\phi}^2)\,\hat{\mathbf{r}} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\,\hat{\boldsymbol{\phi}}$$

In the context of this problem $r$ is the constant $R$ (at least while still in contact with the sphere) so:

$$\ddot{\mathbf{r}} = \mathbf{a} = (-R\dot{\phi}^2)\,\hat{\mathbf{r}} + (R\ddot{\phi})\,\hat{\boldsymbol{\phi}}$$

and while a sphere is named in the problem, the motion will be 'straight down' i.e., on a circle (so we can use polar coordinates).

Example 1.2 describes a skateboard oscillating around the bottom of a pipe. This is essentially the opposite of our problem. Example 1.2 defines $\phi$ from the bottom of the pipe; in problem 4.8 you'll want to define $\phi$ from the top of the sphere.




The location of the particle on the sphere is defined by $\phi$:

$$\begin{aligned}
\mathbf{r} &= R\sin\phi\,\hat{\mathbf{i}} + R\cos\phi\,\hat{\mathbf{k}} = R\,\hat{\mathbf{r}} \\
\mathbf{v} &= R\cos\phi\,\dot{\phi}\,\hat{\mathbf{i}} - R\sin\phi\,\dot{\phi}\,\hat{\mathbf{k}} = R\dot{\phi}\,\hat{\boldsymbol{\phi}} \\
\mathbf{a} &= (-R\dot{\phi}^2)\,\hat{\mathbf{r}} + (R\ddot{\phi})\,\hat{\boldsymbol{\phi}}
\end{aligned}$$

The total force on the particle is:

$$\begin{aligned}
\mathbf{F} &= m\mathbf{g} + \mathbf{N} \\
&= mg(-\cos\phi\,\hat{\mathbf{r}} + \sin\phi\,\hat{\boldsymbol{\phi}}) + N\hat{\mathbf{r}}
\end{aligned}$$

If $N > 0$ we have the usual situation of the sphere pushing the particle out from the surface. If $N < 0$ we have the impossible situation of the sphere sucking the particle into the surface. Evidently the moment when $N = 0$ is the moment that the particle leaves the surface of the sphere.

As stated in the problem, by using conservation of energy you should be able to calculate $v$ for any angle $\phi$, and from that calculate $\dot{\phi}$ for any angle $\phi$.

By looking at the radial component for the equation $\mathbf{F} = m\mathbf{a}$ you should then be able to find the angle at which $N = 0$.