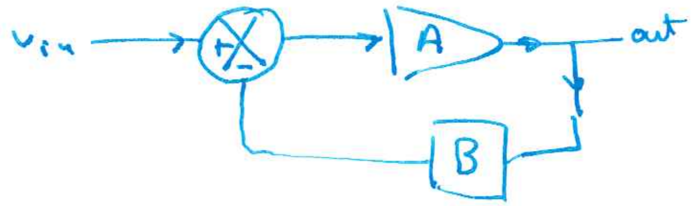
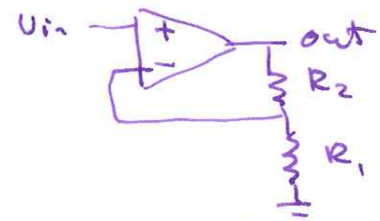
Feedback — some notation & algebra to show that "things get better if you 'throw away' gain"

Notation:





$A(V_{in} - B V_{out}) = V_{out}$

$B = \frac{R_1}{R_1 + R_2}$

Remark: not related to transistor β

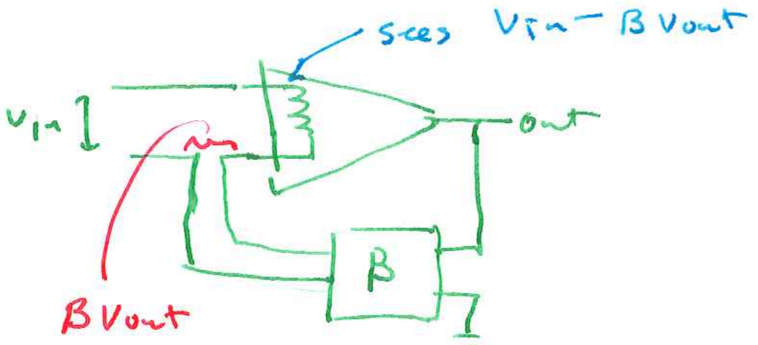
$V_{in} = \left(B + \frac{1}{A}\right) V_{out}$

$\frac{1}{\left(B + \frac{1}{A}\right)} = \frac{V_{out}}{V_{in}}$ ← circuit gain } ← in contrast to op amps gain

$= \frac{1/B}{\left(1 + \frac{1}{AB}\right)}$ ← $\frac{R_1 + R_2}{R_1} = 1 + R_2/R_1$

AB ← sometimes called the "loop gain" A_L

alternate form

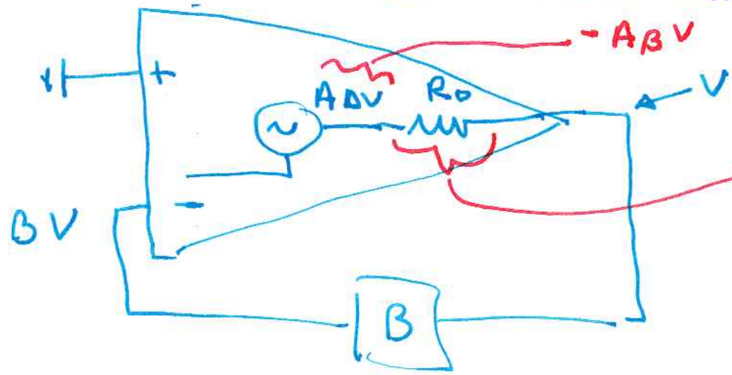


Note: the above is negative feedback; positive feedback → expo growth and is generally a bad idea.

The invention of negative feedback amps is attributed to Harold Black — EE working at Bell Labs — 1927

Output Impedance – a trick to mathematically calculate a circuit output impedance. Consider $V_{in}=0$ ("ground the input") this (in theory) would produce $V_{out}=0$ but we (mathematically) apply an external voltage V to the output & calculate the resulting current, I. the circuits output impedance is the V/I.

(In an experimental situation we'd more likely remove current from amp – by applying a load – and measure the resulting voltage droop – the circuits Z_{out} would then be $\frac{\Delta V}{I}$)



voltage drop $= V - {}^{-}A\beta V$

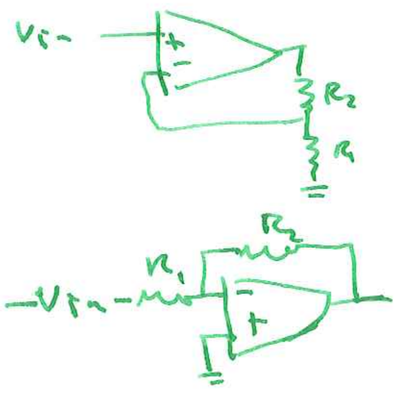
$= (1+A\beta)V$

current $= \frac{\text{voltage drop}}{R_o}$

$$Z_o = \frac{V}{I} = \frac{R_o}{(1+A\beta)}$$ ← note reduced from what was probably a small R_o to begin with.

$$I = \frac{(1+A\beta)V}{R_o}$$

Slight practical consideration – in real circuit feedback network β itself would draw some current



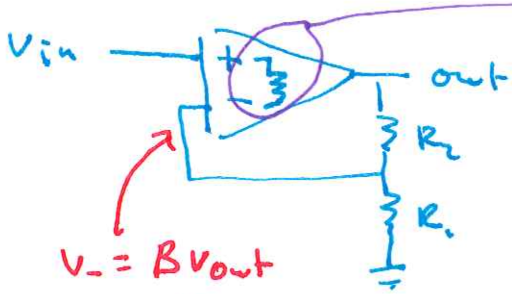
in both cases an addition current flow of V/R_1+R_2 happens – ie must add this current to above

result: $$Z_{out} = \frac{R_o}{(1+A\beta)} \,||\, R_1+R_2$$

this should be much smaller than R_1+R_2 so

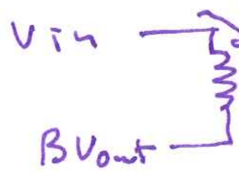
$$Z_o \approx R_o/(1+A\beta)$$

Input Impedance (Note: for inverting amp circuit this will be R which is not huge — say kΩ)



$V_- = BV_{out}$

$B = \dfrac{R_1}{R_1 + R_2}$



$$I = \frac{V_{in} - BV_{out}}{R_{in}} = \frac{V_{in}}{R_{in}}(1 - BG)$$

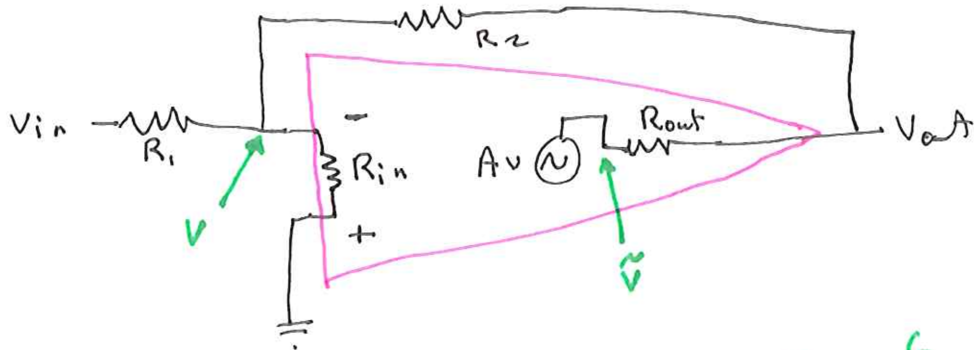
$$G = \frac{V_{out}}{V_{in}} = \frac{A}{1 + BA}$$

$$\frac{V_{in}}{I} = \frac{R_{in}}{(1 - BG)} = \frac{R_{in}}{\left(1 - \frac{AB}{1 + BA}\right)}$$

$$= R_{in}(1 + BA)$$

an initially large value increased even more.

Inverting Amp: Input/Output Impedance



open loop gain A

closed loop gain $G = \frac{V_o}{V_i}$

loop gain AB

$$B = \frac{R_1}{R_1 + R_2}$$

$$V = \frac{\frac{V_i}{R_1} + \frac{V_o}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}}} = -\frac{1}{A} V_o \Rightarrow \frac{\frac{1}{R_1} + \frac{G}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}}} = \frac{-G}{A}$$

math $\Rightarrow$ $$G = -\left(\frac{A R_2 R_{in}}{R_1 R_2 + R_{in} R_1 (A+1) + R_{in} R_2} \right)$$

$$\xrightarrow{R_{in} \to \infty} \frac{-A R_2}{R_1 (A+1) + R_2} = \frac{-A(1-B)}{AB+1} \xrightarrow{A \to \infty} \frac{-(1-B)}{B} = \frac{-R_2}{R_1}$$

$$I_{in} = \frac{(V_{in} - V)}{R_1} = \frac{(V_{in} + \frac{1}{A} V_{out})}{R_1} = \frac{V_{in}(1 + \frac{G}{A})}{R_1}$$

$$\frac{R_1}{(1 + G/A)} = \frac{V_{in}}{I_{in}} = R_1 + R_{in} \parallel \frac{R_2}{(1+A)}$$ (math)

Ignore R_{in}, Ground V_{in}, Apply voltage V_o to output, measure inflowing current I_o

$$V = \frac{\frac{V_o}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} \quad ; \quad -Av = \tilde{V}$$

$$I_{out} = \frac{(\tilde{V} - V_o)}{R_{out}} = \frac{-V_o}{R_{out}} \left\{ \frac{\frac{A}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} + 1 \right\}$$

$$\frac{R_{out}}{\left\{ \frac{A/R_2}{1/R_1 + 1/R_2} + 1 \right\}} = \frac{-V_o}{I_{out}} = \frac{R_{out}}{AB+1}$$

$$\frac{A R_1}{R_2 + R_1} + 1 = AB + 1$$

```
Solve[(1/R1+G/R2)/(1/R1+1/R2+1/Ri)==-G/A,G]

                                    A R2 Ri
Out[1]= {{G -> -(------------------------------)}}
                 R1 R2 + R1 Ri + A R1 Ri + R2 Ri

R1/(1+G/A) /. First[%]

Simplify[%]

        R2 Ri + R1 (R2 + Ri + A Ri)
Out[3]= ---------------------------
              R2 + Ri + A Ri

R1+Ri R2/(1+A)/(Ri+R2/(1+A))

Simplify[%]

        R2 Ri + R1 (R2 + Ri + A Ri)
Out[5]= ---------------------------
              R2 + Ri + A Ri
```

Vocabulary

AC or DC amp

bandwidth (f_{-3dB}) ; "mid band" gain

inverting / non inverting

clipping

differential amp ($out = A_d(V_+ - V_-)$)

- common mode gain (when $V_+ = V_- = V_{in}$) A_c
- differential gain (when $V_+ - V_- = V_{in}$) A_d
- common mode rejection ratio = $\frac{A_d}{A_c}$ (often in dB)

noise - note odd unit $\mu V/\sqrt{Hz}$ or $pA/\sqrt{Hz}$

For op-Amps

Gain-Bandwidth product [note Compensated op-Amps]

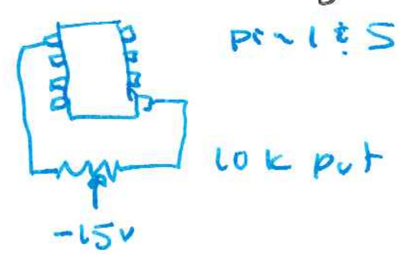
Slew rate

"rail-to-rail" or limited output voltage swing

input offset voltage - "as if"

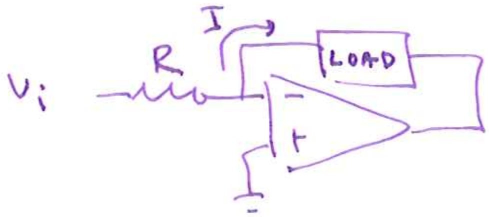
input bias current

open loop gain

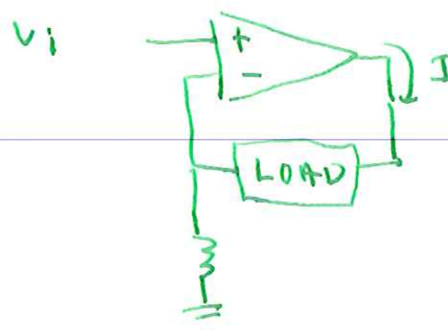


Current sources — supplies a fixed current — if R is big a big voltage results; if R small → small voltage. Every current source has its limits — if R is too big will not be able to produce required voltage.

[Contrast: voltage sources supply fixed voltage — if R is big a small current results; if R small → large current. Every voltage source has its limits — if R is too small will not be able to produce required current]

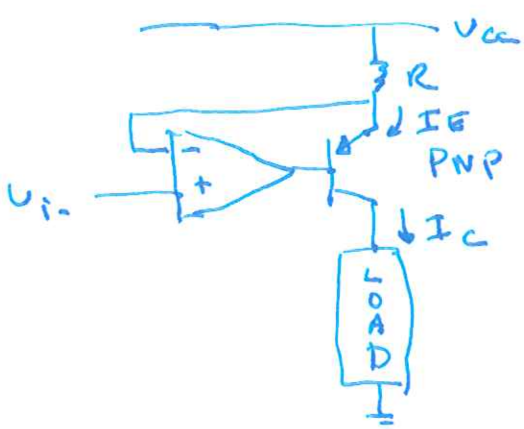


$I = \frac{V_i}{R}$



$I = \frac{V_i}{R}$

Above require a "floating" load — often want load connected to ground. Also above limited by current output of op-amp → a few mA



Since $I_E \approx I_c$

$I_c \approx \frac{V_{cc} - V_{in}}{R}$