The phase difference between V & I in L/C makes Kirchhoffs Laws difficult to apply eg

$$V = RI + L\frac{dI}{dt} \longrightarrow V_0\cos(\omega t) = \underbrace{RI_0\cos(\omega t) - L\omega I_0\sin(\omega t)}_{\text{must now add cos \& sin}}$$

If we go to complex numbers phase difference becomes just multiplication & allows us to factor out time.

$$V = RI + L\frac{dI}{dt} \rightarrow V_0 e^{i\omega t} = RI_0 e^{i\omega t} + i\omega L I_0 e^{i\omega t}$$

$$V_0 = RI_0 + \underbrace{i\omega L}_{\text{exactly like } R = i\omega L} I_0$$

Result: treat C as if "R" $= \frac{1}{i\omega C}$ (because $V = \frac{Q}{C}$ & $Q = \int I\,dt = I_0\int e^{i\omega t} = \frac{I_0}{i\omega} e^{i\omega t}$)

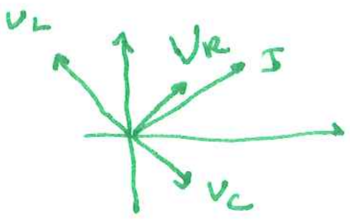
And L as if "R" $= i\omega L$

Note: EE's need symbol "i" for current hence they !HH use $\sqrt{-1} = j$

Note: $I = I_0 e^{i\omega t}$ is the rotating phasor of 200

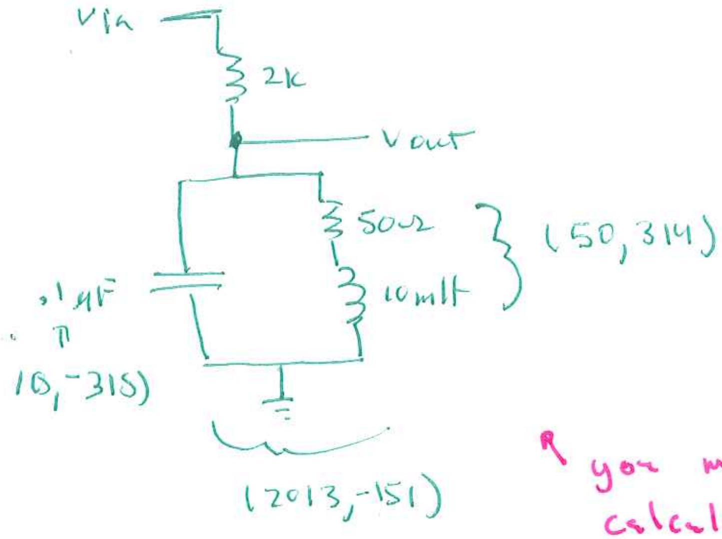
Then $V_L = i\omega L I$ is 90° ahead as $i = e^{i\pi/2}$ and $V_C = \frac{-i}{\omega C} I$ is 90° behind as $-i = e^{-i\pi/2}$

V_R is in phase with I



Note: All the rules from 200 which were derived via Kirchhoffs Laws now apply with these complex "resistors" eg $Z_1 \| Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$

f = 5000 Hz



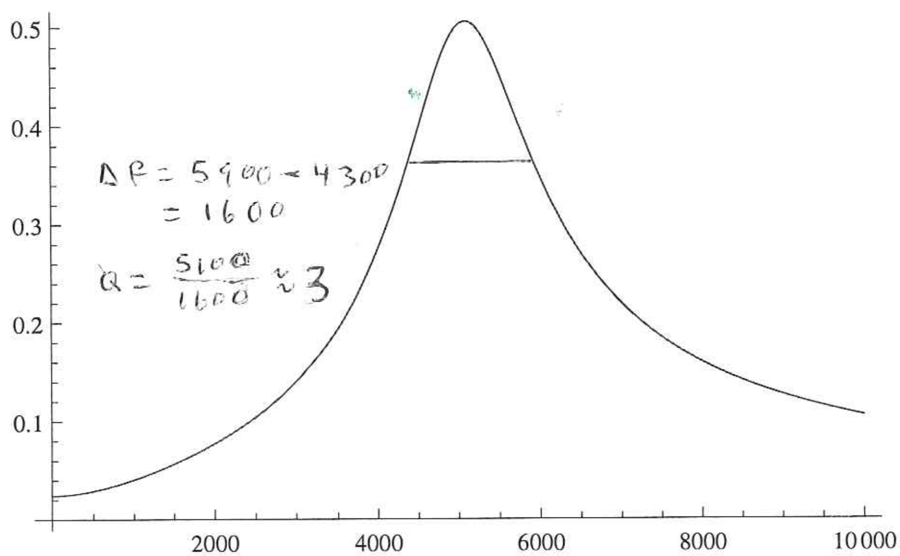
$$\frac{V_{out}}{V_{in}} = \frac{(2013, -151)}{2000 + (2013, -151)}$$

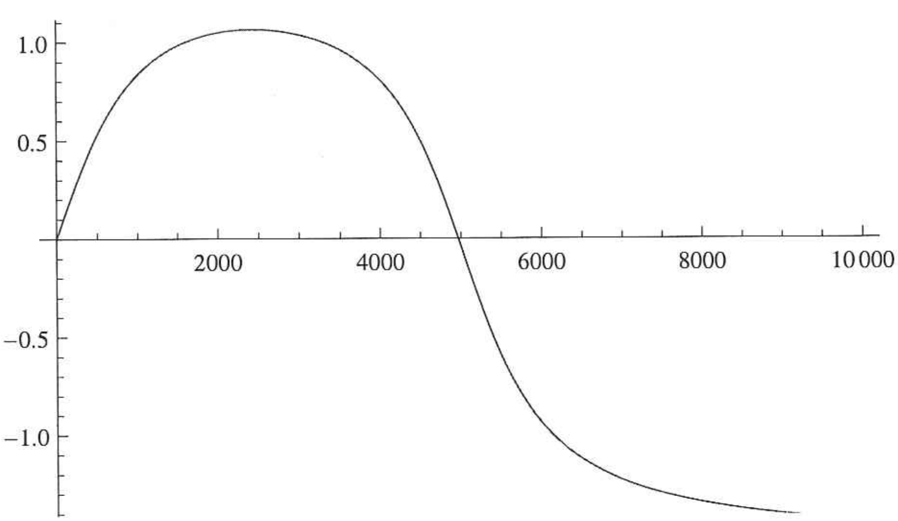
$$= (.5, -1.88E{-2})$$

$$= .5 \angle -2.1^\circ$$

magnitude phase

↑ you must be able to do complex calculations like this using your calculator!





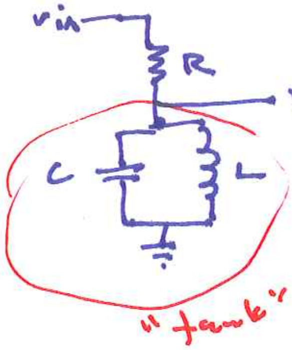
03/18/14

```
L=.01
c=10^(-7)
r=2000
f=5000
zC=1/(I 2 Pi f c)
zL=I 2 Pi f L + 50
z=zC zL/(zC+zL)
Out[7]= 2012.56 - 151.243 I

z/(2000+z)
Out[8]= 0.502272 - 0.0187606 I

{Abs[%],Arg[%]*180/Pi}
Out[9]= {0.502622, -2.13909}

ClearAll[f]
zC=1/(I 2 Pi f c)
zL=I 2 Pi f L + 50
z=zC zL/(zC+zL)
z/(2000+z)
Plot[Abs[%],{f,1,10000}]
Plot[Arg[%%],{f,1,10000}]
GraphicsGrid[{{%15},{%16}}]
Export["tank.eps",%]
```



$$V_{out} = \frac{\frac{1}{i\omega C} \,\|\, i\omega L}{\underbrace{\frac{1}{i\omega C} \,\|\, i\omega L}_{\frac{L/C}{i\left(\omega L - \frac{1}{\omega C}\right)}} + R}\, V_{in} = \frac{L/CR^2}{\underbrace{\frac{L}{CR^2}}_{\tau_L/\tau_R} + i\underbrace{\left(\omega\frac{L}{R} - \frac{1}{\omega RC}\right)}_{\tau_L \quad \tau_R}}\, V_{in}$$

$\omega \to \infty$ $+i\infty$

$\omega \to 0$ $-i\infty$

$\omega = \frac{1}{\sqrt{LC}}$ 0

$$= \frac{A}{A + i\Delta}\, v_{in}$$

$$\left|\frac{V_{out}}{v_{in}}\right| = \frac{A}{\sqrt{A^2 + \Delta^2}} \quad \begin{array}{ll} \to 0 & \omega \to \infty \\ \to 1 & @\ \omega_0 \\ \to 0 & \omega \to 0 \end{array}$$

Taylor Δ near ω_0: $\quad \Delta \approx 0 + \Delta'\big|_{\omega_0} \Delta\omega$

$$\frac{L}{R} + \frac{1}{\omega^2}\frac{1}{RC} = 2\frac{L}{R}$$

$$\left|\frac{V_{out}}{v_{in}}\right| = \frac{A}{\sqrt{A^2 + \left(\frac{2L}{R}\Delta\omega\right)^2}}$$

find $\Delta\omega$ for $\frac{1}{\sqrt{2}}$ $\quad A = \frac{2L}{R}\Delta\omega = \frac{L}{CR^2}$

$$\Delta\omega = \frac{1}{2}\frac{1}{RC}$$

full width $= \frac{1}{RC}$

$$Q = \frac{\omega_0}{\text{full}} = \frac{\frac{1}{\sqrt{LC}}}{1/RC} = \sqrt{\frac{R^2C}{L}} \qquad \left(\frac{1}{\alpha} \text{ of series}\right)$$

$\to$ LRC series: $\quad Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$

$$I_0 = \frac{V_0}{Z} \;;\quad |I_0| = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

peak at $\omega L = \frac{1}{\omega C}$

$\omega = \frac{1}{\sqrt{LC}}$

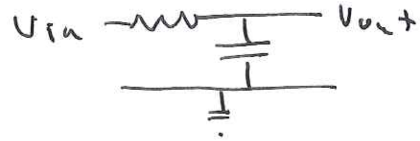
Taylor expand Δ @ $\omega = \frac{1}{\sqrt{LC}}$: $\quad \Delta \approx 0 + \left(L + \frac{1}{\omega^2 C}\right)\Delta\omega$

$$= 2L\,\Delta\omega$$

half-width $\Delta\omega$ for $\frac{1}{\sqrt{2}} \to 2L\Delta\omega = R \to \Delta\omega = \frac{R}{2L}$

$$Q = \cancel{\frac{2L\Delta\omega}{\omega_0}}\ \cancel{2LR}\ \frac{\omega_0}{2\Delta\omega} = \sqrt{\frac{L}{R^2C}}$$

Low pass RC circuit:



$$\frac{V_{out}}{V_{in}} = \frac{-i/\omega C}{R + \frac{-i}{\omega C}} = \frac{1}{i\omega RC + 1}$$

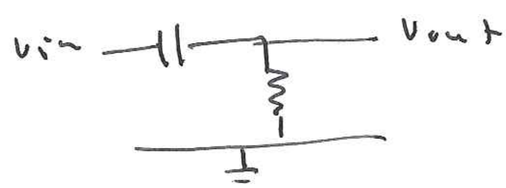
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{(\omega RC)^2 + 1}} = \begin{cases} \omega \ll \frac{1}{\tau_c} & \rightarrow 1 \\ \omega \gg \frac{1}{\tau_c} & \rightarrow \frac{1}{\omega RC} \end{cases}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}} \quad @ \quad \omega = \frac{1}{\tau_c} \quad \leftarrow \text{"-3dB freq"}$$

$$\text{phase } \frac{V_{out}}{V_{in}} = \begin{cases} \omega \ll \frac{1}{\tau_c} & \rightarrow 0 \\ \omega = \frac{1}{\tau_c} & \rightarrow -\pi/4 \\ \omega \gg \frac{1}{\tau_c} & \rightarrow -\pi/2 \end{cases}$$

Bode Plot: $\log\left(\frac{V_{out}}{V_{in}}\right)$ vs $\log(\text{freq})$

High Pass RC circuit:

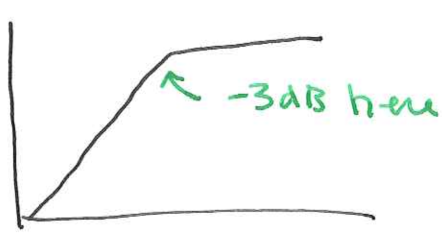


$$\frac{V_{out}}{V_{in}} = \frac{R}{R - \frac{i}{\omega C}} = \frac{1}{1 - \frac{i}{\omega RC}}$$

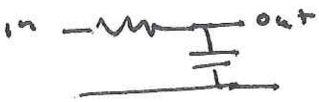
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} = \begin{cases} \omega \gg \frac{1}{\tau_c} & \rightarrow 1 \\ \omega = \frac{1}{\tau_c} & \rightarrow \frac{1}{\sqrt{2}} \\ \omega \ll \frac{1}{\tau_c} & \rightarrow \omega RC \end{cases}$$

phase
0
$+\pi/4$
$+\pi/2$

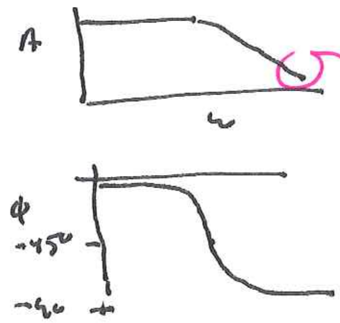
Bode Plot



Low-pass



$$\frac{V_{out}}{V_{in}} = \frac{1}{1+i\omega RC}$$



$$V_{out} \approx 0$$

$$I = \frac{(V_{in}-V_{out})}{R}$$

$$= \frac{dQ}{dt} = C\frac{dV_{out}}{dt}$$

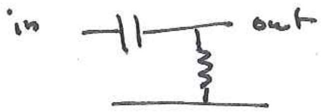
$$\Rightarrow \frac{dV_{out}}{dt} \approx \frac{1}{RC} V_{in}$$

so $V_{out} \approx \frac{1}{RC}\int V_{in}\,dt$

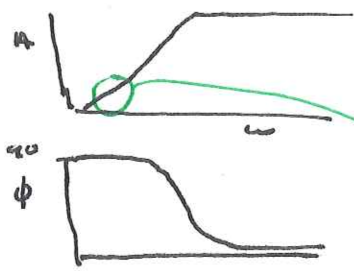
$$\frac{V_{out}}{V_{in}} \approx \frac{1}{i\omega RC}$$

$$V_{out} \approx \frac{1}{RC}\underbrace{\frac{1}{i\omega}V_{in}}_{\int V_{in}\,dt}$$

High-pass



$$\frac{V_{out}}{V_{in}} = \frac{1}{1 - i/\omega RC}$$



$$V_{out} \approx 0$$

$$I = \frac{dQ}{dt} = C\frac{d}{dt}(V_{in}-V_{out})$$

$$V_{out} = IR \approx RC\frac{d}{dt}V_{in}$$

$$\frac{V_{out}}{V_{in}} \approx i\omega RC$$

$$V_{out} \approx RC\underbrace{i\omega V_{in}}_{\frac{d}{dt}V_{in}}$$