Hubble Trouble

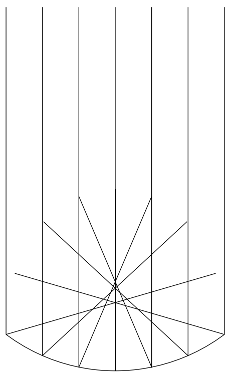
1 Discussion

In the cartoon version the incredibly expensive Hubble Space Telescope (HST) was so flawed that it needed to wear glasses. What happened?

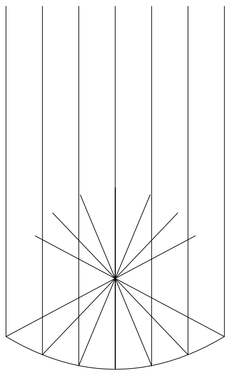
They say that history doesn't repeat itself, rather it rhymes. Jump back century: when George Ritchey was born (1864) the largest telescope in America was a $18\frac{1}{2}$-inch refractor built by Alvan Clark & Sons. When he died (1945) the great 200-inch reflecting telescope financed through the efforts of George Hale (1869–1938) was nearing completion atop Mt. Palomar. That change in emphasis: from builder to fund-raiser, is part of the odd history of how the discoveries of the most famous telescope designer and builder (Ritchey) were not incorporated in the "telescope of the century": the Palomar 200-inch.

The simplest telescope mirror to make has a spherical curvature. The problem with such a mirror is that it has "spherical aberration:" rays near the axis come to a focus further from the mirror than rays near the edge of the tube:

Spherical Aberration:

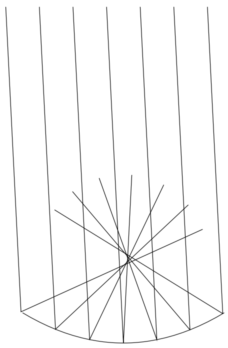


Solution: Parabolic Mirror



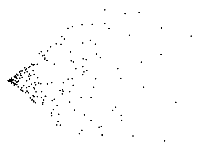
This problem of different focal points for rays going through different rings of the aperture, is solved by grinding the mirror into a parabola of revolution: a paraboloid. However the paraboloid mirror has another problem: coma. The image of objects not exactly in the center of the field of view are distorted. An off-axis star looks like a comet:

Off-axis Star Images Suffer from Coma:



Coma: Image of Star is Comet-like:

Each dot in this picture represents a ray's intersection with the focal plane. The rays far from the center of the aperture and near the scattering plane end up in a comet-like envelope of a cusp-like focus.

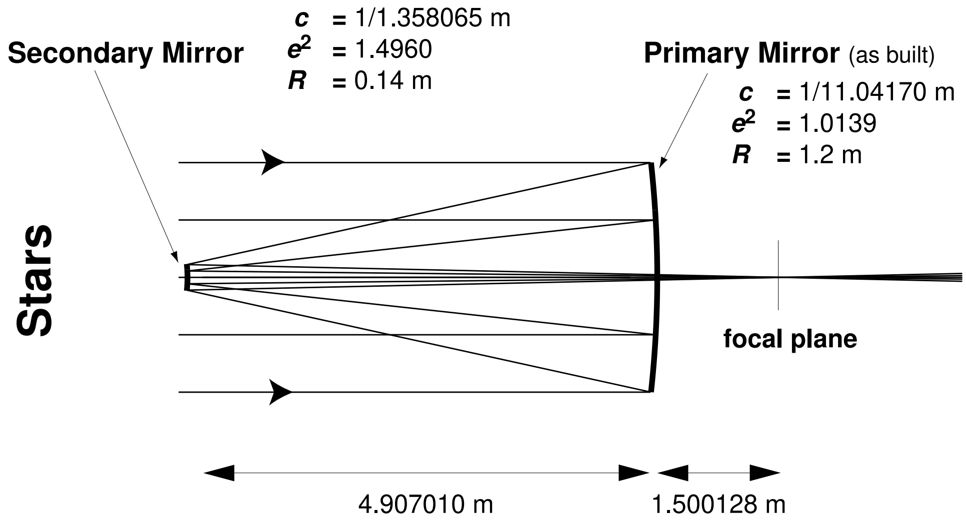


It turns out that it is impossible to remove coma using just one mirror. But Ritchey and Henri Chrétien[1]

[1] 1879–1956, French astronomer who first met Ritchey while working at Mt. Wilson in 1910

discovered that slight changes in the shapes of the two mirrors that make up the usual astronomical telescope can cancel both spherical aberration and coma. Excited by his discovery, Ritchey tried to convince his boss Hale to allow him to incorporate the "new curve" in the mirrors of the Mt. Wilson 100-inch telescope then under construction. Hale found Ritchey guilty of taking his plan to Hale's patron for the project (John Hooker), and then grousing that the old curve mirror he was constructing for Hale was inferior to what might have been. As soon as Ritchey finished the Mt. Wilson mirror, Hale fired him and proceeded to use his great influence to make Ritchey an "un-person" in American astronomy. (E.g., he convinced historians to delete mention of Ritchey's quite significant contributions to astronomy [e.g., his discovery of novas in distant galaxies] and blackballed nomination of Ritchey for several prestigious awards.) Ritchey never saw his great idea executed in a big telescope. It took nearly fifty years for the effects of Hale's suppression to be undone. In 1958 Aden Meinel, first directory of the Kitt Peak national Observatory, modified the original plan and decided to build the 84-inch as a Ritchey-Chrétien. Since that time essentially all big telescopes (e.g., the 10 m Keck) have been built to the "new curves" calculated by Ritchey and Chrétien.

Return to the recent past: The Hubble Space Telescope (a Ritchey-Chrétien, of course), the world's most expensive telescope, is found to be perfectly ground to the wrong shape. The classic instruction: "measure twice, cut once" had been violated. With faulty measuring rod in hand, the folks at Perkin-Elmer, ground the mirror until the faulty measuring rod said "stop!". As a result the primary mirror was about a factor of five more hyperbolic than the curve specified by Ritchey and Chrétien.



The above is a diagram of HST. The light moves from the left, bounces off the primary mirror and heads back towards a focus just beyond the secondary mirror. It never reaches that focus; instead it bounces off the secondary which reduces the convergence of the beam, and sends the light back through a hole in the primary to a focus about 1.5 m behind the primary. According to Ritchey and Chrétien the mirrors should be constructed of hyperbolas of revolution—hyperboloids:

$$-r^2 + (e^2 - 1)z^2 + \frac{2}{c} z = 0 = \phi$$

which can be solved as:

$$z(r) = \frac{cr^2}{1 + \sqrt{1 + c^2 r^2 (e^2 - 1)}}$$

`z1[x_,e_,c_]=c x^2/(1+Sqrt[1+c^2 x^2 (e^2-1)])` ... we'll be using x as our radial variable

`z2[x_,e_,c_,d_]=c x^2/(1+Sqrt[1+c^2 x^2 (e^2-1)])+d` ... The secondary hyperboloid is displaced a distance `d` above the first

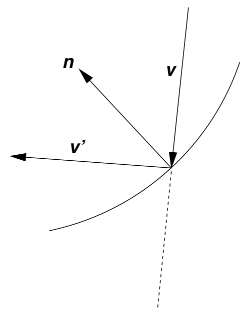
We seek to ray-trace the light from the stars. These rays are lines, and hence:

$$\mathbf{r} = (x, z) = \mathbf{r}_0 + \mathbf{v}t = (x_0 + v_x t, z_0 + v_z t)$$

where the velocity $\mathbf{v}$ should be the speed of light (but speed is of no account in what follows—we seek only the path of the light), and $\mathbf{r}_0$ is the initial position. We can find the location where the light ray hits the primary mirror:

```
Solve[(vx t+ x0)^2==(vz t+z0)^2(e^2-1)+2 (vz t+z0)/c,t]
mirrort1[x0_,z0_,vx_,vz_,e_,c_]=t /. Last[%]
normal[x_,z_,e_,c_]={-x,z(e^2-1)+1/c}/Sqrt[x^2+(z(e^2-1)+1/c)^2]
```

The function `mirrort1` now gives us the "time" the ray hits the mirror. (The `Last` is needed as there are two intersection points between a hyperboloid and line; `Last` picks out the correct one.) In order to reflect our ray, we need a vector perpendicular (`normal`) to the surface. $\nabla\phi$ is normal to constant ϕ surfaces.



We seek a vector description for reflection rather than the usual "angle of incidence equals angle of reflection". We note that the component of $\mathbf{v}$ perpendicular to the normal is unchanged whereas the component of $\mathbf{v}$ parallel to the normal is reversed. Thus:

$$\mathbf{v}' = \mathbf{v} - 2\mathbf{n}(\mathbf{n}\cdot\mathbf{v})$$

```
{vxp,vzp}={vx,vz}-2 normal[x,z,e,c]( normal[x,z,e,c].{vx,vz} )
```

We take as our $\mathbf{r}_0$ for the new line the bounce point, i.e., the point of intersection between the line and the mirror.

```
tp=mirrort1[x0,z0,vx,vz,e,c]
x0p=vx tp+ x0
z0p=vz tp+ z0
```

Thus `x0p, z0p, vxp, vzp` describe our outgoing line.

Following this logic through the second mirror allows us to define a function which gives us the location of the ray when it passes through the focal plane.:

```
lineFP[x0_,z0_,vx_,vz_,e_,c_,e2_,c2_,d_]:=(tp1=mirrort1[x0,z0,vx,vz,e,c];
x0p=vx tp1+ x0; z0p=vz tp1+ z0;
{vxp,vzp}={vx,vz}-2 normal[x0p,z0p,e,c]( normal[x0p,z0p,e,c].{vx,vz});
tp2=mirrort2[x0p,z0p,vxp,vzp,e2,c2,d];
x0p2=vxp tp2+ x0p; z0p2=vzp tp2+ z0p;
{vxp2,vzp2}={vxp,vzp}-2 normal2[x0p2,z0p2,e2,c2,d]( normal2[x0p2,z0p2,e2,c2,d].{vxp,vzp});
tp3=(-1.5-z0p2)/vzp2;
Return[vxp2 tp3+x0p2] )
```

2 Homework

Copy the above function on a sheet of paper and explain what every line does.

The following command will insert the Hubble function into *Mathematica*:

```
<<Hubble.m
```

With the given parameters, Hubble is out of focus. You need to find new mirror parameters to get the rays to properly focus. With spherical aberration, rays far from the axis miss the focal plane.

```
lineFP[1.2,5,0,-1,e,c,e2,c2,d]^2 + lineFP[-.9,5,0,-1,e,c,e2,c2,d]^2 +
lineFP[.6,5,0,-1,e,c,e2,c2,d]^2 + lineFP[-.3,5,0,-1,e,c,e2,c2,d]^2
```

These rays are all going straight down the tube (vx=0) at various off-axis locations (x0 $= 1.2, -.9, .6, -.3$) and should come to a focus at the origin of the focal plane. A miss (lineFP$\neq 0$) is an aberration; we want to minimize the square of the miss distance.

```
(lineFP[0,5,0.001,-1,e,c,e2,c2,d]-lineFP[1.2,5,0.001,-1,e,c,e2,c2,d])^2+
(lineFP[0,5,0.001,-1,e,c,e2,c2,d]-lineFP[-.9,5,0.001,-1,e,c,e2,c2,d])^2+
(lineFP[0,5,0.001,-1,e,c,e2,c2,d]-lineFP[.6,5,0.001,-1,e,c,e2,c2,d])^2+
(lineFP[0,5,0.001,-1,e,c,e2,c2,d]-lineFP[-.3,5,0.001,-1,e,c,e2,c2,d])^2,
```

These rays are from a star near the edge of the field of view. The off-axis rays (with x0 $= 1.2, -.9, .6, -.3$)) should go the same place as the the rays that start at the center of the tube. We minimize the square of the deviation. Thus I suggest:

```
FindMinimum[lineFP[-1.2,5,0,-1,e,c,eH2,cH2,dH]^2+lineFP[-.9,5,0,-1,e,c,eH2,cH2,dH]^2+
lineFP[-.6,5,0,-1,e,c,eH2,cH2,dH]^2+lineFP[-.3,5,0,-1,e,c,eH2,cH2,dH]^2+
(lineFP[0,5,0.001,-1,e,c,eH2,cH2,dH]-lineFP[1.2,5,0.001,-1,e,c,eH2,cH2,dH])^2+
(lineFP[0,5,0.001,-1,e,c,eH2,cH2,dH]-lineFP[.9,5,0.001,-1,e,c,eH2,cH2,dH])^2+
(lineFP[0,5,0.001,-1,e,c,eH2,cH2,dH]-lineFP[.6,5,0.001,-1,e,c,eH2,cH2,dH])^2+
(lineFP[0,5,0.001,-1,e,c,eH2,cH2,dH]-lineFP[.3,5,0.001,-1,e,c,eH2,cH2,dH])^2,
{c,.99*cH,1.01*cH},{e,.99*eH,1.01*eH}]
```

should find the needed correction to the primary. eH, cH, eH2, cH2, dH are the as-built Hubble parameters. We seek a minimum close to the current values, which should give us the design parameters for the primary mirror.

3 Postscript

It is disheartening to know that the Hale "success" story—success in this very early effort at funding "big science"—has at its core authoritarian suppression of real discovery. Hale sought discovery after "his" telescopes went on-line, and suppressed inconvenient discovery. We see similar flaws in the expensive programs at N.A.S.A.

I'd like to close on a happier note, and luckily this story has a happy ending, or perhaps one might say a prequal. While Ritchey and Chrétien solved the problem of coma with mirrors, it was first solved with lenses by Ernst Abbe (1840–1905). The solution is generally called the Abbe sine rule. In 1866 Abbe, then a professor of physics at Jena (Germany), was approached by Carl Zeiss with various optical problems. The Carl Zeiss Foundation describes Abbe's work at this time as follows:

> One year after beginning the manufacture of the Carl Zeiss compound microscope, in 1873, Herr Abbe released a scientific paper describing the mathematics leading to the perfection of this wonderful invention. For the first time in optical design, aberration, diffraction and coma were described and understood... As a reward for his efforts Carl Zeiss made Abbe a partner in his burgeoning business in 1876.

Becoming wealthy through his optical work and a partnership with Zeiss, Abbe set up and endowed the Carl Zeiss Foundation for research in science and social improvement in 1891.