TT → algebra : minterm (SOP), maxterm (POS), Karnaugh Map, look

Eg: half-adder

A	B	C	Σ
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

simple look should show: $\Sigma = A \oplus B$

$C = A \cdot B$

minterm: look at 1's — write as product

eg $C = A\cdot B$ $\quad \Sigma = \bar{A}B + A\bar{B}$

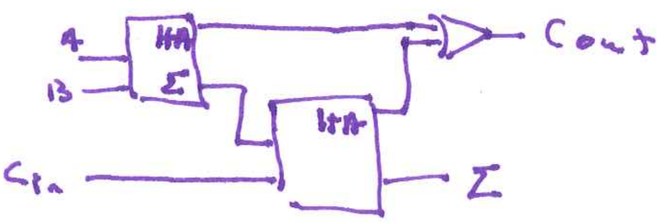
maxterm — look at 0's (ie $\bar{1}$)

write $\bar{\Sigma} = \bar{A}\cdot\bar{B} + A\cdot B \Rightarrow \Sigma = \bar{\bar{\Sigma}} = \overline{(\bar{A}\cdot\bar{B} + A\cdot B)}$

$= (A+B)\cdot(\bar{A}+\bar{B})$ "Factored"

full-adder (includes input of carry in = C_{in})

Note can build full adder from 2 half-adders (HA)



Note: building circuits from existing blocks (rather than going back to gates) is a good idea!

A	B	C_{in}	C_{out}	Σ
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

maxterm:

$\bar{\Sigma} = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C}$

$\Sigma = (A+B+C)\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+\bar{C})\cdot(\bar{A}+\bar{B}+C)$

minterm

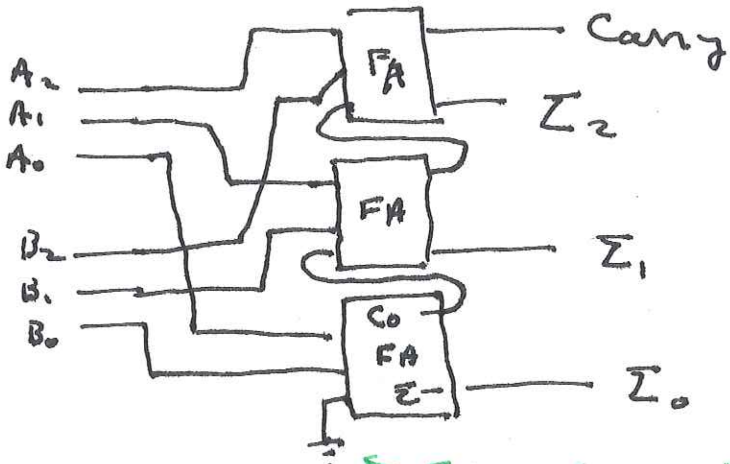
$\Sigma = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

minterm = $\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

maxterm: $\overline{C_{out}} = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$

$C_{out} = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+C)\cdot(\bar{A}+B+C)$

Note: can string together several full adders (FA) to add multi-bit parallel binary numbers



→ zero carry in for first bit

Again: building complex circuits using existing blocks is a good idea

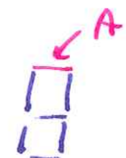
Karnaugh Maps: A 2d array of 0, 1, X (don't care)

Rows & Columns label via Gray Code (change just one bit at a time: 00, 01, 11, 10)

Circle blocks of 2^N 1 & Xs until every 1 covered.

The bigger the blocks the better.

Eg: A segment of seven segment display



A	B	C	D	a
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1 ← sometimes 0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
above				X

Gray Code order

CD \ AB	00	01	11	10
00	1	0	X	1
01	0	1	X	1
11	1	1	X	X
10	1	1	X	X

$\overline{B}\,\overline{D} + A + C + DB$

Remark: $\overline{B}\,\overline{D} + BD = \overline{B \oplus D}$ for which an IC exists

$\overline{a} = \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}B\overline{C}\,\overline{D}$

$a = (A+B+C+\overline{D})(A+\overline{B}+C+D)$ ← max term

K-map for Full Adder Cout

Cin \ AB	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$C_{in}A + AB + C_{in}B$

$DCBA_2$

Are the number of letters in the words 0-9 even?

True for 0, 4, 5, 9 ; X for 11-15

DC \ BA	00	01	11	10
00	1	0	0	0
01	1	*	0	0
11	X	X	X	X
10	0	1	X	X

$DA + C\overline{B} + \overline{A}\,\overline{B}\,\overline{D}$

Eg segment b of seven segment

always ON except for 5 & 6

ABCD

CD \ AB	00	01	11	10
00	1	1	X	1
01	1	0	X	1
11	1	1	X	X
10	1	0	X	X

$\overline{B} + CD + \overline{C}\,\overline{D}$