36-6. (a) In combination with the small angle approximation ($\sin\theta \approx \tan\theta \approx \theta$), we use Eq. 36-3 to calculate the separation between the first ($m_1 = 1$) and fifth ($m_2 = 5$) minima:

$$\Delta y = D\Delta\tan\theta \approx D\Delta\sin\theta == D\Delta\left(\frac{m\lambda}{a}\right) = \frac{D\lambda}{a}\Delta m = \frac{D\lambda}{a}(m_2 - m_1) \ .$$

Solving for the slit width, we obtain

$$a = \frac{D\lambda(m_2 - m_1)}{\Delta y} = \frac{(400\,\text{mm})(550\times 10^{-6}\,\text{mm})(5-1)}{0.35\,\text{mm}} = 2.5143\ \text{mm} \ .$$

Without the small angle approximation (i.e., using $\tan\theta = \sin\theta/\sqrt{1-\sin^2\theta}$), we have:

$$\Delta y = D\left(\frac{m_2\lambda/a}{\sqrt{1-(m_2\lambda/a)^2}} - \frac{m_1\lambda/a}{\sqrt{1-(m_1\lambda/a)^2}}\right)$$

or

$$\frac{\Delta y}{D} = \left(\frac{m_2}{\sqrt{\alpha^2 - m_2^2}} - \frac{m_1}{\sqrt{\alpha^2 - m_1^2}}\right)$$

where $\alpha = a/\lambda$. Using *Mathematica*:

```
FindRoot[.35/400==5/Sqrt[alf^2-5^5]-1/Sqrt[alf^2-1^2],{alf,4545}]
Out[1]= {alf -> 4571.86}
```

So $a = 4571.86\times\lambda = 2.5145$ mm, i.e., much the same result.

(b) For $m = 1$,

$$\sin\theta = \frac{m\lambda}{a} = \frac{(1)(.550\times 10^{-3}\,\text{mm})}{2.51\,\text{mm}} = 2.19\times 10^{-4} \ .$$

The angle is $\theta = \sin^{-1}(2.19\times 10^{-4}) = 2.19\times 10^{-4}$ rad.

36-18. (a) We use Eq. 36-14:

$$\theta_\text{R} = 1.22\,\frac{\lambda}{d} = \frac{(1.22)(540\times 10^{-6}\,\text{mm})}{5.0\,\text{mm}} = 1.32\times 10^{-4}\ \text{rad} \ .$$

(b) The linear separation is $D = L\theta_\text{R} = (400\times 10^3\,\text{m})(1.32\times 10^{-4}\,\text{rad}) = 52.7\,\text{m}$.

36-35. (a) The first minimum of the diffraction pattern is at 5.00°, so

$$a = \frac{\lambda}{\sin\theta} = \frac{0.440\,\mu\text{m}}{\sin 5.00^\circ} = 5.05\,\mu\text{m} \ .$$

(b) Since the fourth bright fringe is missing, $d = 4a = 4(5.05\,\mu\text{m}) = 20.2\,\mu\text{m}$.

(c) For the $m = 1$ bright fringe,

$$\alpha = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(5.05\,\mu\text{m})\sin 1.25^\circ}{0.440\,\mu\text{m}} = 0.787\ \text{rad} \ .$$

Consequently, the intensity of the $m = 1$ fringe is

$$I = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 = (7.0\,\text{mW/cm}^2)\left(\frac{\sin 0.787\,\text{rad}}{0.787}\right)^2 = 5.70\ \text{mW/cm}^2 \ ,$$

which agrees with Fig. 36-43. Similarly for $m = 2$, the intensity is $I = 2.9\,\text{mW/cm}^2$, also in agreement with Fig. 36-43.

36-102. We use Eq. 36-14:

$$\theta_\text{R} = 1.22\,\frac{\lambda}{d} = \frac{D}{L}.$$

Solve for d given: $\lambda = 500\times 10^{-9}$ m, $d = 5\times 10^{-3}$ m, and $L = .25$ m. The result is: $D = 30.5\,\mu$m.

36-43. The angular positions of the first-order diffraction lines are given by $d\sin\theta = \lambda$. Let λ_1 be the shorter wavelength (430 nm) and θ be the angular position of the line associated with it. Let λ_2 be the longer wavelength (680 nm), and let $\theta+\Delta\theta$ be the angular position of the line associated with it. Here $\Delta\theta = 20°$. Then, $d\sin\theta = \lambda_1$ and $d\sin(\theta+\Delta\theta) = \lambda_2$. We write $\sin(\theta+\Delta\theta)$ as $\sin\theta\cos\Delta\theta+\cos\theta\sin\Delta\theta$, then use the equation for the first line to replace $\sin\theta$ with λ_1/d, and $\cos\theta$ with $\sqrt{1-\lambda_1^2/d^2}$. After multiplying by d, we obtain

$$\lambda_1\cos\Delta\theta + \sqrt{d^2-\lambda_1^2}\sin\Delta\theta = \lambda_2 \ .$$

Solving for d, we find

$$\begin{aligned} d &= \sqrt{\frac{(\lambda_2-\lambda_1\cos\Delta\theta)^2+(\lambda_1\sin\Delta\theta)^2}{\sin^2\Delta\theta}} \\ &= \sqrt{\frac{[(680\,\text{nm})-(430\,\text{nm})\cos 20°]^2+[(430\,\text{nm})\sin 20°]^2}{\sin^2 20°}} \\ &= 914\,\text{nm} = .914\,\mu\text{m} \ . \end{aligned}$$

There are $1/d = 1/(.914\times10^{-3}\,\text{mm}) = 1090$ rulings per mm. With *Mathematica*:

```
Solve[{.43==d Sin[t],.68==d Sin[t+Pi/9]},{t,d}]
Out[1]= {{d -> -0.91421, t -> -2.6519}, {d -> 0.91421, t -> 0.489689}}
```

36-52. (a) We find $\Delta\lambda$ from $R = \lambda/\Delta\lambda = Nm$:

$$\Delta\lambda = \frac{\lambda}{Nm} = \frac{500\,\text{nm}}{(600/\text{mm})(5.0\,\text{mm})(3)} = 0.056\,\text{nm} = 56\ \text{pm} \ .$$

(b) Since $\sin\theta = m_{\text{max}}\lambda/d < 1$,

$$m_{\text{max}} < \frac{d}{\lambda} = \frac{1}{(600/\text{mm})(500\times10^{-6}\,\text{mm})} = 3.3 \ .$$

Therefore, $m_{\text{max}} = 3$. No higher orders of maxima can be seen.

36-57. We use Eq. 36-34. From the peak on the left at angle 0.75° (estimated from Fig. 36-44), we have

$$\lambda_1 = 2d\sin\theta_1 = 2(0.94\,\text{nm})\sin(0.75°) = 0.025\,\text{nm} = 25\ \text{pm} \ .$$

This estimation should be viewed as reliable to within ± 2 pm. We now consider the next peak:

$$\lambda_2 = 2d\sin\theta_2 = 2(0.94\,\text{nm})\sin 1.15° = 0.038\,\text{nm} = 38\ \text{pm} \ .$$

One can check that the third (fourth) peak from the left is the second-order one for λ_1 (λ_2).

`grating.txt:`
Looking at `gratingA1.pdf` I find the $m = +4$ maximum at $y = 2.8$ m.

$$\lambda = \frac{d\sin\theta}{m} = \frac{dy}{m\sqrt{D^2+y^2}} = \frac{4.5\times10^{-6}\cdot 2.8}{4\sqrt{4^2+2.8^2}} = 0.645\ \mu\text{m}$$

It appears to me that there is a diffraction zero at $y = 2$ m, so

$$a = \frac{\lambda}{\sin\theta} = \frac{\lambda\sqrt{D^2+y^2}}{y} = \frac{.645\times10^{-6}\cdot\sqrt{4^2+2^2}}{2} = 1.44\ \mu\text{m}$$

Looking at `gratingA2.pdf` I find 9 zeros between the $m = 0$ and $m = +1$ maximums; therefore $N = 10$

`old exam: 211t2_06.pdf #5`

Looking at the left plot, I find $m = 1$ peaks at $y = .58$ m and $y = .75$ m; $d = 1/528$ mm which is 1.894 μm.

$$\lambda_1 = \frac{d \sin\theta}{m} = \frac{dy}{m\sqrt{D^2 + y^2}} = \frac{1.894 \times 10^{-6} \cdot .58}{\sqrt{2^2 + .58^2}} = 0.528\ \mu\text{m}$$

$$\lambda_2 = \frac{d \sin\theta}{m} = \frac{dy}{m\sqrt{D^2 + y^2}} = \frac{1.894 \times 10^{-6} \cdot .75}{\sqrt{2^2 + .75^2}} = 0.665\ \mu\text{m}$$

For $m = 3$, λ_1 we have

$$\sin\theta = \frac{m\lambda_1}{d} = .8363 \Rightarrow y = D\tan\theta = 3.05 \text{ m}$$

whereas for $m = 3$, λ_2 we have

$$\sin\theta = \frac{m\lambda_2}{d} = 1.05 \text{ impossible!}$$

I estimate a diffraction zero for λ_1 at $y = 2.5$ m (with a rather large uncertainty, in fact I can't rule out $y > 4$ m).

$$a = \frac{\lambda_1}{\sin\theta} = 0.528\ \mu\text{m}\ \frac{\sqrt{D^2 + y^2}}{y} = 0.68\ \mu\text{m}$$