35-21. The maxima of a two-slit interference pattern are at angles θ given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be replaced by θ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of m is $\Delta\theta = (m/d)(\lambda_2 - \lambda_1)$, and their separation on a screen a distance D away is

$$\begin{aligned}\Delta y &= D\tan\Delta\theta \approx D\,\Delta\theta = \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1) \\ &= \left[\frac{3(1.0\,\mathrm{m})}{5.0\times 10^{-3}\,\mathrm{m}}\right](600\times 10^{-9}\,\mathrm{m} - 480\times 10^{-9}\,\mathrm{m}) = 7.2\times 10^{-5}\ \mathrm{m}\ .\end{aligned}$$

The small angle approximation $\tan\Delta\theta \approx \Delta\theta$ (in radians) is made. Without the small angle approximation:

$$D\tan(\arcsin(m\lambda_1/d)) - D\tan(\arcsin(m\lambda_2/d)) = 7.20\times 10^{-5}\ \mathrm{m}$$

35-27. Consider the two waves, one from each slit, that produce the seventh bright fringe in the absence of the mica. They are in phase at the slits and travel different distances to the seventh bright fringe, where they have a phase difference of $2\pi m = 14\pi$. Now a piece of mica with thickness x is placed in front of one of the slits, and an additional phase difference between the waves develops. Specifically, their phases at the slits differ by

$$\frac{2\pi x}{\lambda_m} - \frac{2\pi x}{\lambda} = \frac{2\pi x}{\lambda}(n-1)$$

where λ_m is the wavelength in the mica and n is the index of refraction of the mica. The relationship $\lambda_m = \lambda/n$ is used to substitute for λ_m. Since the waves are now in phase at the screen,

$$\frac{2\pi x}{\lambda}(n-1) = 14\pi$$

or

$$x = \frac{7\lambda}{n-1} = \frac{7(550\times 10^{-9}\,\mathrm{m})}{1.58-1} = 6.64\times 10^{-6}\ \mathrm{m}\ .$$

35-31. Adding the complex amplitudes: $10e^{0i} + 15e^{i\pi/6} + 5e^{-i\pi/4} = (26.5, 3.96) = 26.8\angle 8.50^\circ$
Now:

$$\sum_{k=1}^{N} a_k \sin(\omega t + \delta_k) = \mathrm{Im}\left[\left(\sum_{k=1}^{N} a_k e^{i\delta_k}\right)\ e^{i\omega t}\right] = A\sin(\omega t + \phi)$$

where: $\sum_{k=1}^{N} a_k e^{i\delta_k} = A\ e^{i\phi}$, so here:

$$\sum_{k=1}^{N} a_k \sin(\omega t + \delta_k) = 26.8\sin(\omega t + 8.50^\circ)$$

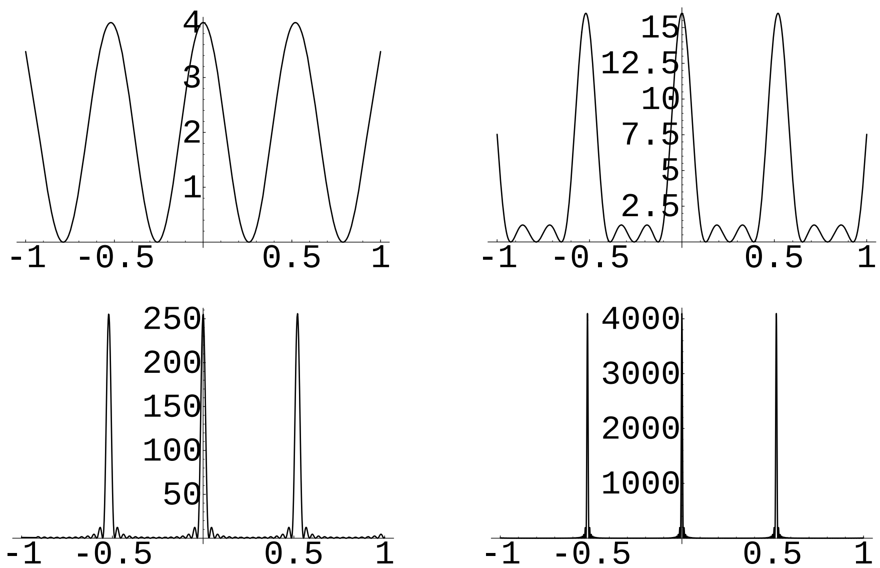
Plots:

```
Plot[4 Cos[ Pi (4.5/.580) (y/Sqrt[4^2+y^2]) ]^2,{y,-1,1}]

Plot[(Sin[4 Pi (4.5/.580) (y/Sqrt[4^2+y^2]) ]/Sin[ Pi (4.5/.580) (y/Sqrt[4^2+y^2]) ])^2,{y,-1,1},
PlotRange->All]

Plot[(Sin[16 Pi (4.5/.580) (y/Sqrt[4^2+y^2]) ]/Sin[ Pi (4.5/.580) (y/Sqrt[4^2+y^2]) ])^2,{y,-1,1},
PlotRange->All]

Plot[(Sin[64 Pi (4.5/.580) (y/Sqrt[4^2+y^2]) ]/Sin[ Pi (4.5/.580) (y/Sqrt[4^2+y^2]) ])^2,{y,-1,1},
PlotRange->All,PlotPoints->1000]
```



35-36. Constructive:

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \qquad \text{where} \quad m = 0, 1, 2, \ldots$$

or

$$\frac{2Ln}{m + \frac{1}{2}} = \lambda \qquad \text{where} \quad m = 0, 1, 2, \ldots$$

For $m = 2, 3, 4, 5$, $\lambda = 672, 480, 373, 305$ nm

Destructive:

$$2L = (m)\frac{\lambda}{n} \qquad \text{where} \quad m = 1, 2, 3, \ldots$$

or

$$\frac{2Ln}{m} = \lambda \qquad \text{where} \quad m = 1, 2, 3, \ldots$$

For $m = 3, 4, 5$, $\lambda = 560, 420, 336$ nm

35-37. Light reflected from the front surface of the coating suffers a phase change of π rad while light reflected from the back surface does not change phase. If L is the thickness of the coating, light reflected from the back surface travels a distance $2L$ farther than light reflected from the front surface. The difference in phase of the two waves is $2L(2\pi/\lambda_c) - \pi$, where λ_c is the wavelength in the coating. If λ is the wavelength in vacuum, then $\lambda_c = \lambda/n$, where n is the index of refraction of the coating. Thus, the phase difference is $2nL(2\pi/\lambda) - \pi$. For fully constructive interference, this should be a multiple of 2π. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) - \pi = 2m\pi$$

for L. Here m is an integer. The solution is

$$L = \frac{(2m+1)\lambda}{4n} \; .$$

To find the smallest coating thickness, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{560 \times 10^{-9}\,\text{m}}{4(2.00)} = 7.00 \times 10^{-8}\ \text{m} \; .$$

35-55. The situation is analogous to that treated in Sample Problem 35-6, in the sense that the incident light is in a low index medium, the thin film has somewhat higher $n = n_2$, and the last layer has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \qquad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. The value of L which corresponds to no reflection corresponds, reasonably enough, to the value which gives maximum transmission of light (into the highest index medium – which in this problem is the water).

(a) If $2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2}$ (Eq. 35-36) gives zero reflection in this type of system, then we might reasonably expect that its counterpart, Eq. 35-37, gives maximum reflection here. A more careful analysis such as that given in §35-7 bears this out. We disregard the $m = 0$ value (corresponding to $L = 0$) since there is *some* oil on the water. Thus, for $m = 1, 2, \ldots$ maximum reflection occurs for wavelengths

$$\lambda = \frac{2n_2L}{m} = \frac{2(1.20)(460\,\text{nm})}{m} = 1104\,\text{nm}\,,\ 552\,\text{nm}\,,\ 368\,\text{nm}\ \ldots$$

We note that only the 552 nm wavelength falls within the visible light range.

(b) As remarked above, maximum transmission into the water occurs for wavelengths given by

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \implies \lambda = \frac{4n_2L}{2m+1}$$

which yields $\lambda = 2208\,\text{nm}\,,\ 736\,\text{nm}\,,\ 442\,\text{nm}\ \ldots$ for the different values of m. We note that only the 442 nm wavelength (blue) is in the visible range, though we might expect some red contribution since the 736 nm is very close to the visible range.

35-81. Let ϕ_1 be the phase difference of the waves in the two arms when the tube has air in it, and let ϕ_2 be the phase difference when the tube is evacuated. These are different because the wavelength in air is different from the wavelength in vacuum. If λ is the wavelength in vacuum, then the wavelength in air is λ/n, where n is the index of refraction of air. This means

$$\phi_1 - \phi_2 = 2L\left[\frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda}\right] = \frac{4\pi(n-1)L}{\lambda}$$

where L is the length of the tube. The factor 2 arises because the light traverses the tube twice, once on the way to a mirror and once after reflection from the mirror. Each shift by one fringe corresponds to a change in phase of 2π rad, so if the interference pattern shifts by N fringes as the tube is evacuated,

$$\frac{4\pi(n-1)L}{\lambda} = 2N\pi$$

and

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60(500 \times 10^{-9}\,\text{m})}{2(5.0 \times 10^{-2}\,\text{m})} = 1.00030\,.$$