Answer 4 of the following 5 questions
'extra' answered problems will not contribute to your grade

Physical Constants

$\sigma = 5.6705 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$
$R = 8.3145\ \mathrm{J/(K \cdot mol)}$
$N_A = 6.0221 \times 10^{23}$
$k_B = 1.3807 \times 10^{-23}\ \mathrm{J/K}$
$\quad = 8.6173 \times 10^{-5}\ \mathrm{eV/K}$
$1\ \mathrm{eV} = 1.6022 \times 10^{-19}\ \mathrm{J}$

Properties of H_2O

$L_V = 2256\ \mathrm{J/g}$
$c_w = 4.19\ \mathrm{J/(g \cdot K)}$
$L_f = 333\ \mathrm{J/g}$
$c_i = 2.22\ \mathrm{J/(g \cdot K)}$
$\rho_w = 1000\ \mathrm{kg/m^3}$

1. In an insulated container, 250 g of ice at a temperature of $-40°\mathrm{C}$ is mixed with 50 g of water at 20°C). The final state is all solid ice. What will be the equilibrium temperature of this system?

2. The entropy of 1 g of liquid water at $T = 25°\mathrm{C}$ and $P = 100$ kPa is 3.89 J/K.

   (a) Find the number of microstates Ω.

   (b) A small amount of heat dQ is added to the water and the number of microstates increases to a billion (10^9) times the previous number. Find the change in entropy, dS. Assuming that the temperature remains constant (see following) find the amount of heat added, dQ.

   (c) Assuming a constant specific heat, with that added heat, the temperature of the water *will* change. Find that change in temperature, dT.

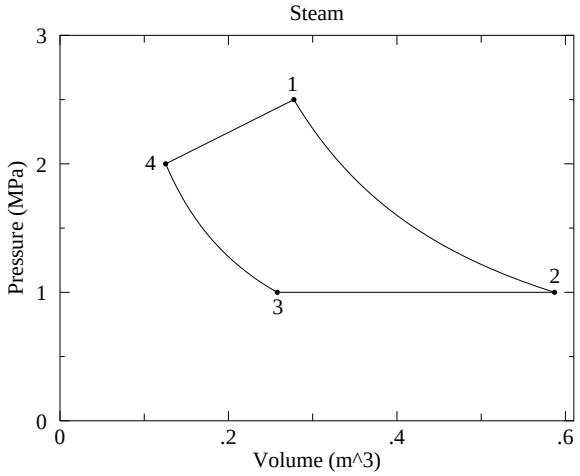
   Do you think these changes are measurable?

3. Carbon dioxide (CO_2, that's ^{12}C and ^{16}O) is a linear molecule. Report/guess the contributions (cause and value) you expect to the number of degrees of freedom f at room temperature. Report the per-mole and per-gram constant-pressure specific heats (c_p) you expect for this f. CO_2's lowest vibrationally excited states are at 0.083 eV and 2×0.083 eV above the ground state. Approximate the sum over all states by just these two excited states and the ground state. Find, at 300 K, the value of the partition function, Z, and the probability CO_2 is in the 0.083 eV state.

4. Consider the following cycle using 1 kg of water vapor (which is <u>not</u> an ideal gas). Starting at a pressure of 2.5 MPa, volume 0.2778 m^3, and temperature 1231°C:

   - The steam expands adiabatically to a volume of 0.5871 m^3; pressure of 1 MPa.
   - In a constant-pressure (a.k.a., isobaric) process, the volume is compressed to 0.2579 m^3.
   - An isothermal compression (at 300°C) reduces the volume to 0.1255 m^3
   - A straightline process returns to the initial state.

   The below graph displays this cycle. The below table reports state variables at the labeled points.



point	Volume (m^3)	Pressure (MPa)	Temperature (°C)	E_{int} (kJ)	Entropy (kJ/K)
1	0.2778	2.5	1231	4531	8.916
2	0.5871	1.0	1000	4051	8.916
3	0.2579	1.0	300	2793	7.124
4	0.1255	2.0	300	2773	6.768

   (a) Use the 1^{st} Law of Thermodynamics to calculate how much heat was removed in the isobaric compression $2 \to 3$?

   (b) Use ΔS to find the heat removed in the isothermal compression $3 \to 4$

   (c) How much heat was added in the straightline expansion $4 \to 1$?

   (d) Use all your heats to find the *net work* performed by this cycle.

5. A selection of rows from a run of `StatMech` with $N_A = 150$ (atoms), $N_B = 50$, and total energy $U = 2000\varepsilon$ can be found as the final page of this exam. This Einstein solid has $\varepsilon = .005$ eV.

   (a) Of the options listed, which value of U_A maximizes: (i) the entropy of A? (ii) the entropy of B?, (iii) the entropy of the combined system?

   (b) Approximate $\dfrac{\partial S}{\partial U}$ as a (small) finite difference $\dfrac{\Delta S}{\Delta U}$. Use this result to find an equation for the temperature of this Einstein solid. Simplify your result using the properties of logarithms. Calculate T_A (the temperature of system A) using $\Delta U_A = 1\varepsilon$ for the following four situations: $U_A = 300\varepsilon$, $U_A = 500\varepsilon$, $U_A = 1500\varepsilon$, and $U_A = 1700\varepsilon$. Calculate T_B (the temperature of system B) using $\Delta U_B = 1\varepsilon$ for $U_B = 100\varepsilon$, $U_B = 300\varepsilon$. System B at $U_B = 100\varepsilon$ and system A at $U_A = 300\varepsilon$ have the same energy per atom; are you surprised that, under those conditions, they have approximately the same temperature? In the situation that maximizes the total entropy, we expect the temperatures of A and B to be approximately equal. Is this the case?

   (c) Compare the change in temperature that results when 200ε of heat is added in the process: $U_A : 300\varepsilon \to 500\varepsilon$ to the change in temperature that results in the process: $U_A : 1500\varepsilon \to 1700\varepsilon$. Does the specific heat seem approximately constant? Compare the change in temperature that results when 200ε of heat is added in the process: $U_B : 100\varepsilon \to 300\varepsilon$ to the change in temperature that results in the process: $U_A : 300\varepsilon \to 500\varepsilon$. With three times fewer atoms you should expect that B's temperature change would be approximately three times that of A for the same Q (as $\Delta T = Q/mc$). Approximately so?

StatMech: Calculation Results

Number of atoms in System A = 150
Number of atoms in System B = 50
Total combined system energy = 2000 units
Total number of microstates = 4.0915E+607

U(A)	U(B)	Omega(A)	Omega(B)	Omega(AB)	Fraction of states
0	2000	1	4.3751E+233	4.3751E+233	1.07E-374
1	1999	450	4.0718E+233	1.8323E+236	4.48E-372
2	1998	101475	3.7893E+233	3.8452E+238	9.40E-370
3	1997	15288900	3.5263E+233	5.3914E+240	1.32E-367
4	1996	1731467925	3.2815E+233	5.6818E+242	1.39E-365
5	1995	1.57217E+11	3.0535E+233	4.8007E+244	1.17E-363
6	1994	1.19223E+13	2.8413E+233	3.3875E+246	8.28E-362
7	1993	7.76653E+14	2.6438E+233	2.0533E+248	5.02E-360
8	1992	4.43663E+16	2.4599E+233	1.0914E+250	2.67E-358
9	1991	2.25775E+18	2.2887E+233	5.1673E+251	1.26E-356
10	1990	1.03631E+20	2.1293E+233	2.2066E+253	5.39E-355
99	1901	1.1782E+111	2.9931E+230	3.5265E+341	8.62E-267
100	1900	6.4685E+111	2.7755E+230	1.7954E+342	4.39E-266
101	1899	3.5225E+112	2.5737E+230	9.0657E+342	2.22E-265
199	1801	1.2623E+172	1.2998E+227	1.6408E+399	4.01E-209
200	1800	4.0963E+172	1.2005E+227	4.9175E+399	1.20E-208
201	1799	1.3247E+173	1.1087E+227	1.4687E+400	3.59E-208
299	1701	1.1691E+217	3.6916E+223	4.3158E+440	1.05E-167
300	1700	2.9189E+217	3.3943E+223	9.9073E+440	2.42E-167
301	1699	7.2729E+217	3.1207E+223	2.2697E+441	5.55E-167
399	1601	1.1783E+253	6.5269E+219	7.6907E+472	1.88E-135
400	1600	2.5010E+253	5.9711E+219	1.4934E+473	3.65E-135
401	1599	5.3013E+253	5.4625E+219	2.8958E+473	7.08E-135
499	1501	1.6503E+283	6.7758E+215	1.1182E+499	2.73E-109
500	1500	3.1322E+283	6.1639E+215	1.9306E+499	4.72E-109
501	1499	5.9393E+283	5.6069E+215	3.3301E+499	8.14E-109
599	1401	1.5758E+309	3.8510E+211	6.0683E+520	1.483E-87
600	1400	2.7550E+309	3.4808E+211	9.5895E+520	2.344E-87
601	1399	4.8132E+309	3.1460E+211	1.5142E+521	3.701E-87
699	1301	1.1157E+332	1.1008E+207	1.2282E+539	3.002E-69
700	1300	1.8314E+332	9.8767E+206	1.8088E+539	4.421E-69
701	1299	3.0044E+332	8.8611E+206	2.6622E+539	6.507E-69
799	1201	2.8434E+352	1.4261E+202	4.0550E+554	9.911E-54
800	1200	4.4393E+352	1.2687E+202	5.6321E+554	1.377E-53
801	1199	6.9278E+352	1.1286E+202	7.8184E+554	1.911E-53
899	1101	7.8716E+370	7.3545E+196	5.7892E+567	1.415E-40
900	1100	1.1799E+371	6.4779E+196	7.6430E+567	1.868E-40
901	1099	1.7678E+371	5.7051E+196	1.0086E+568	2.465E-40
999	1001	5.2923E+387	1.2807E+191	6.7779E+578	1.657E-29
1000	1000	7.6685E+387	1.1148E+191	8.5487E+578	2.089E-29
1001	999	1.1108E+388	9.7022E+190	1.0777E+579	2.634E-29
1099	901	1.5829E+403	6.0852E+184	9.6322E+587	2.354E-20
1100	900	2.2290E+403	5.2217E+184	1.1639E+588	2.845E-20
1101	899	3.1380E+403	4.4800E+184	1.4058E+588	3.436E-20
1199	801	3.3608E+417	5.9448E+177	1.9979E+595	4.883E-13
1200	800	4.6182E+417	5.0124E+177	2.3149E+595	5.658E-13
1201	799	6.3448E+417	4.2254E+177	2.6809E+595	6.552E-13
1299	701	7.3230E+430	8.1081E+169	5.9375E+600	1.451E-07
1300	700	9.8522E+430	6.6868E+169	6.5880E+600	1.610E-07
1301	699	1.3252E+431	5.5132E+169	7.3064E+600	1.786E-07
1399	601	2.2018E+443	8.9082E+160	1.9615E+604	4.794E-04
1400	600	2.9080E+443	7.1385E+160	2.0759E+604	5.074E-04
1401	599	3.8400E+443	5.7184E+160	2.1959E+604	5.367E-04
1499	501	1.1631E+455	3.4739E+150	4.0406E+605	0.0098755
1500	500	1.5113E+455	2.6776E+150	4.0466E+605	0.0098902
1501	499	1.9633E+455	2.0629E+150	4.0501E+605	0.0098988
1599	401	1.3179E+466	1.3076E+138	1.7234E+604	4.212E-04
1600	400	1.6877E+466	9.5339E+137	1.6091E+604	3.933E-04
1601	399	2.1611E+466	6.9464E+137	1.5012E+604	3.669E-04
1699	301	3.7852E+476	4.9399E+122	1.8699E+599	4.570E-09
1700	300	4.7850E+476	3.3043E+122	1.5811E+599	3.864E-09
1701	299	6.0480E+476	2.2077E+122	1.3352E+599	3.263E-09
1799	201	3.1733E+486	2.0469E+102	6.4953E+588	1.588E-19
1800	200	3.9649E+486	1.1755E+102	4.6606E+588	1.139E-19
1801	199	4.9533E+486	6.7363E+101	3.3367E+588	8.155E-20
1899	101	8.7578E+495	9.00449E+71	7.8860E+567	1.927E-40
1900	100	1.0827E+496	3.63781E+71	3.9388E+567	9.627E-41
1901	99	1.3385E+496	1.46097E+71	1.9555E+567	4.779E-41
1990	10	1.4198E+504	2.13192E+15	3.0269E+519	7.398E-89
1991	9	1.7400E+504	1.34083E+14	2.3331E+518	5.702E-90
1992	8	2.1322E+504	7.63764E+12	1.6285E+517	3.980E-91
1993	7	2.6126E+504	3.89179E+11	1.0168E+516	2.485E-92
1994	6	3.2009E+504	17463172650	5.5897E+514	1.366E-93
1995	5	3.9213E+504	675993780	2.6508E+513	6.479E-95
1996	4	4.8034E+504	21947850	1.0542E+512	2.577E-96
1997	3	5.8833E+504	573800	3.3759E+510	8.251E-98
1998	2	7.2055E+504	11325	8.1602E+508	1.994E-99
1999	1	8.8239E+504	150	1.3236E+507	3.23E-101
2000	0	1.0805E+505	1	1.0805E+505	2.64E-103