Answer 5 of the following 6 questions

Physical Constants

$\sigma = 5.6705 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$
$R = 8.3145\ \mathrm{J/(K \cdot mol)}$
$N_A = 6.0221 \times 10^{23}$
$k_B = 1.3807 \times 10^{-23}\ \mathrm{J/K}$
$1\ \mathrm{eV} = 1.6022 \times 10^{-19}\ \mathrm{J}$
$1\ \mathrm{atm} = 1.0133 \times 10^{5}\ \mathrm{Pa}$

Properties of H_2O

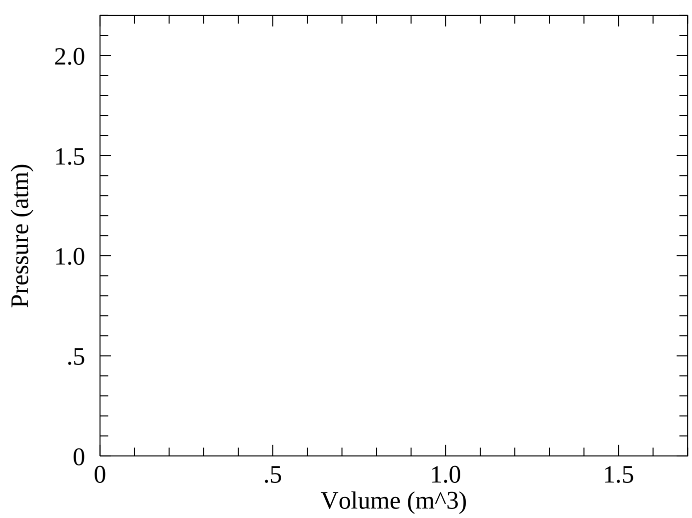
$L_V = 2256\ \mathrm{J/g}$
$c_w = 4.19\ \mathrm{J/(g \cdot K)}$
$L_f = 333\ \mathrm{J/g}$
$c_i = 2.22\ \mathrm{J/(g \cdot K)}$
$\rho_w = 1000\ \mathrm{kg/m^3}$

1. In an insulated container, 250 g of water at 35°C is mixed with 75 g of ice at −20°C. The final state consists of 100% liquid water. What will be the equilibrium temperature of this system?

2. Consider the following cycle starting with 1 $\mathrm{m^3}$ of a diatomic ideal gas at a pressure of 0.5 atm and a temperature of 300 K.

   (a) The gas is adiabatically compressed until the temperature reaches 400 K.
   (b) With the volume held constant, the temperature is increased to 450 K.
   (c) The gas is then isothermally expanded until the pressure reaches 0.5 atm.
   (d) In a constant-pressure (a.k.a., isobaric) process, the volume is returned to 1 $\mathrm{m^3}$.

   On the below graph, accurately plot and label each leg of this cycle. This will require calculating various pVT values at the end of some cycles. Fill in the below table giving the sign $(+, -, 0)$ of the quantity for each leg of the cycle.



path:	a	b	c	d
ΔT				
ΔE_{int}				
Q				
W				
ΔS				

3. A selection of rows from a run of `StatMech` with $N_A = 200$ (atoms), $N_B = 50$, and total energy $U = 2000\varepsilon$ can be found as the final page of this exam. This Einstein solid has $\varepsilon = .005$ eV.

   (a) Of the rows shown: (i) the entropy of A is largest in which row? (ii) the entropy of B is largest in which row? (iii) the combined entropy of A and B together is largest in which row? Including rows not recorded here, where (approximately) would you expect the combined entropy of A and B together to be largest? Why?

   (b) Approximate $\dfrac{\partial S}{\partial U}$ as a (small) finite difference $\dfrac{\Delta S}{\Delta U}$. Use this result to find an equation for the temperature of this Einstein solid. Simplify your result using the properties of logarithms. Calculate T_A (the temperature of system A) using $\Delta U = 1\varepsilon$ for $U_A = 100\varepsilon$, $U_A = 1600\varepsilon$, $U_A = 2000\varepsilon$. Calculate T_B (the temperature of system B) using $\Delta U = 1\varepsilon$ for $U_B = 1900\varepsilon$, $U_B = 400\varepsilon$.

4. A particular atom has three states: a ground state (at energy $E_0 = -1$ eV), and two excited states (at energies $E_1 = -.995$ eV, $E_2 = -.994$ eV). What is the probability that the atom is in the state E_1 (a) if the temperature is 100 K? (b) if the temperature is 10^9 K? What is the relative probability the atom is in the state E_2 compared to state E_1 (c) if the temperature is 100 K? (d) if the temperature is 10^9 K?. Would your answer to these questions change if the energies instead were: $E_0 = 0$ eV, $E_1 = .005$ eV, $E_2 = .006$ eV?

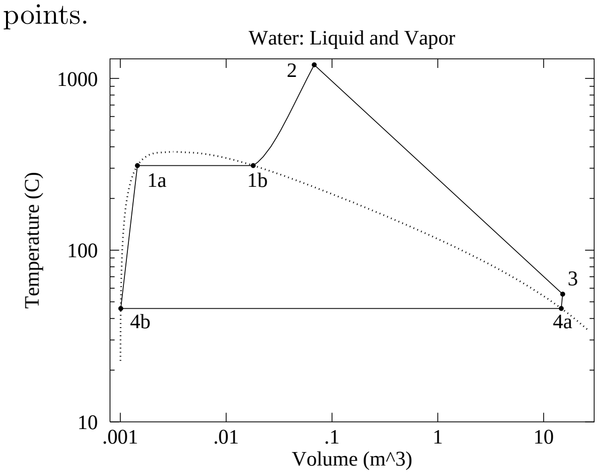
5. Define five of the following terms:

   (a) carnot cycle
   (b) degree of freedom
   (c) entropy
   (d) equipartition theorem
   (e) heat
   (f) multiplicity
   (g) second law of thermodynamics
   (h) temperature

6. The following problem is based on "steam tables"—tables of V, T, E_{int}, S etc. which substitute for the simple equations like $pV = nRT$, $\Delta S = nC_p \ln(T_f/T_i)$ etc. that apply only to the mythical ideal gas. Again steam is a non-ideal gas; you must use the tabulated V, T, E_{int}, S etc. not formulas based on $pV = nRT$.

   - 1 kg of liquid water (1a) at a temperature of 311°C and pressure of 10 MPa has been pumped into a boiler. In an isobaric process the water is totally evaporated (point 1b) and then the resulting vapor is heated to 1200°C (point 2).
   - The high pressure steam is piped to a turbine where it expands adiabatically to pressure of 0.01 MPa in the process of doing work (point 3)
   - In an isobaric process, the steam is cooled until it starts to condense (point 4a) and finally all the vapor is converted to liquid (point 4b).
   - A pump is then used to re-inject this low pressure, low temperature water back into the boiler. Approximate this process as a straight-line pV process.

   This cycle is displayed below on a log-log T-V diagram. The region below the dotted curve consists of a mixed phase: part liquid and part vapor. In an isobaric boiling process the system moves horizontally from the left boundary to the right as 100% liquid is converted to a much larger volume of 100% vapor at a constant temperature. The following table reports state variables at the labeled points.



point	Volume (m^3)	Pressure (MPa)	Temperature (°C)	E_{int} (kJ)	Entropy (kJ/K)
1a	0.00145	10.	311	1405	3.3605
1b	0.01803	10.	311	2544	5.6141
2	0.06789	10.	1200	5124	8.2055
3	15.122	0.01	55	2793	8.2055
4a	14.674	0.01	46	2558	8.1511
4b	0.00101	0.01	46	192	0.6493

   (a) Find the heat required to evaporate the water at a pressure of 10 MPa (i.e., the process 1a → 1b) from ΔS.

   (b) Find the work done in the turbine during 2 → 3.

   (c) Find the heat released when the vapor is condensed to liquid at a pressure of 0.01 MPa (i.e., the process 4a → 4b) from the first law of thermodynamics.

   (d) Find the heat added in the straightline expansion $4b \to 1$?

Number of atoms in System A = 200; Number of atoms in System B = 50
Total number of microstates = 3.0004E+697; Total system energy = 2000 units

U(A)	U(B)	Omega(A)	Omega(B)	Omega(AB)	Fraction of states
0	2000	1	4.3751E+233	4.3751E+233	1.46E-464
1	1999	600	4.0718E+233	2.4431E+236	8.14E-462
2	1998	180300	3.7893E+233	6.8321E+238	2.28E-459
3	1997	36180200	3.5263E+233	1.2758E+241	4.25E-457
4	1996	5454165150	3.2815E+233	1.7898E+243	5.97E-455
5	1995	6.58863E+11	3.0535E+233	2.0119E+245	6.71E-453
6	1994	6.64354E+13	2.8413E+233	1.8877E+247	6.29E-451
7	1993	5.75140E+15	2.6438E+233	1.5205E+249	5.07E-449
8	1992	4.36388E+17	2.4599E+233	1.0735E+251	3.58E-447
9	1991	2.94804E+19	2.2887E+233	6.7471E+252	2.25E-445
10	1990	1.79536E+21	2.1293E+233	3.8229E+254	1.27E-443
99	1901	2.5146E+122	2.9931E+230	7.5263E+352	2.51E-345
100	1900	1.7577E+123	2.7755E+230	4.8785E+353	1.63E-344
101	1899	1.2182E+124	2.5737E+230	3.1353E+354	1.04E-343
199	1801	1.4503E+193	1.2998E+227	1.8851E+420	6.28E-278
200	1800	5.7939E+193	1.2005E+227	6.9554E+420	2.32E-277
201	1799	2.3060E+194	1.1087E+227	2.5567E+421	8.52E-277
299	1701	3.8784E+246	3.6916E+223	1.4318E+470	4.77E-228
300	1700	1.1622E+247	3.3943E+223	3.9449E+470	1.31E-227
301	1699	3.4751E+247	3.1207E+223	1.0845E+471	3.61E-227
399	1601	1.1929E+290	6.5269E+219	7.7856E+509	2.59E-188
400	1600	2.9792E+290	5.9711E+219	1.7789E+510	5.93E-188
401	1599	7.4293E+290	5.4625E+219	4.0582E+510	1.35E-187
499	1501	8.5877E+326	6.7758E+215	5.8188E+542	1.94E-155
500	1500	1.8876E+327	6.1639E+215	1.1635E+543	3.88E-155
501	1499	4.1444E+327	5.6069E+215	2.3237E+543	7.74E-155
599	1401	9.9210E+358	3.8510E+211	3.8205E+570	1.27E-127
600	1400	1.9825E+359	3.4808E+211	6.9008E+570	2.30E-127
601	1399	3.9585E+359	3.1460E+211	1.2453E+571	4.15E-127
699	1301	2.5636E+387	1.1008E+207	2.8220E+594	9.41E-104
700	1300	4.7573E+387	9.8767E+206	4.6986E+594	1.57E-103
701	1299	8.8224E+387	8.8611E+206	7.8176E+594	2.61E-103
799	1201	8.6912E+412	1.4261E+202	1.2394E+615	4.131E-83
800	1200	1.5199E+413	1.2687E+202	1.9283E+615	6.427E-83
801	1199	2.6565E+413	1.1286E+202	2.9980E+615	9.992E-83
899	1101	1.3523E+436	7.3545E+196	9.9453E+632	3.315E-65
900	1100	2.2523E+436	6.4779E+196	1.4590E+633	4.863E-65
901	1099	3.7496E+436	5.7051E+196	2.1392E+633	7.130E-65
999	1001	2.4279E+457	1.2807E+191	3.1094E+648	1.036E-49
1000	1000	3.8822E+457	1.1148E+191	4.3277E+648	1.442E-49
1001	999	6.2053E+457	9.7022E+190	6.0204E+648	2.007E-49
1099	901	1.0131E+477	6.0852E+184	6.1649E+661	2.055E-36
1100	900	1.5648E+477	5.2217E+184	8.1708E+661	2.723E-36
1101	899	2.4161E+477	4.4800E+184	1.0824E+662	3.608E-36
1199	801	1.6947E+495	5.9448E+177	1.0075E+673	3.358E-25
1200	800	2.5407E+495	5.0124E+177	1.2735E+673	4.244E-25
1201	799	3.8079E+495	4.2254E+177	1.6090E+673	5.363E-25
1299	701	1.7528E+512	8.1081E+169	1.4211E+682	4.737E-16
1300	700	2.5604E+512	6.6868E+169	1.7121E+682	5.706E-16
1301	699	3.7392E+512	5.5132E+169	2.0615E+682	6.871E-16
1399	601	1.5909E+528	8.9082E+160	1.4172E+689	4.723E-09
1400	600	2.2716E+528	7.1385E+160	1.6216E+689	5.405E-09
1401	599	3.2428E+528	5.7184E+160	1.8544E+689	6.180E-09
1499	501	1.6894E+543	3.4739E+150	5.8687E+693	1.956E-04
1500	500	2.3640E+543	2.6776E+150	6.3298E+693	2.110E-04
1501	499	3.3074E+543	2.0629E+150	6.8227E+693	2.274E-04
1599	401	2.6651E+557	1.3076E+138	3.4850E+695	0.0116154
1600	400	3.6629E+557	9.5339E+137	3.4922E+695	0.0116391
1601	399	5.0333E+557	6.9464E+137	3.4963E+695	0.0116530
1699	301	7.6357E+570	4.9399E+122	3.7720E+693	1.257E-04
1700	300	1.0326E+571	3.3043E+122	3.4120E+693	1.137E-04
1701	299	1.3962E+571	2.2077E+122	3.0825E+693	1.027E-04
1799	201	4.7113E+583	2.0469E+102	9.6434E+685	3.214E-12
1800	200	6.2791E+583	1.1755E+102	7.3810E+685	2.460E-12
1801	199	8.3675E+583	6.7363E+101	5.6366E+685	1.879E-12
1899	101	7.2442E+595	9.00449E+71	6.5230E+667	2.174E-30
1900	100	9.5280E+595	3.63781E+71	3.4661E+667	1.155E-30
1901	99	1.2530E+596	1.46097E+71	1.8306E+667	6.101E-31
1990	10	2.9639E+606	2.13192E+15	6.3188E+621	2.106E-76
1991	9	3.8556E+606	1.34083E+14	5.1697E+620	1.723E-77
1992	8	5.0150E+606	7.63764E+12	3.8303E+619	1.277E-78
1993	7	6.5222E+606	3.89179E+11	2.5383E+618	8.460E-80
1994	6	8.4815E+606	17463172650	1.4811E+617	4.937E-81
1995	5	1.1028E+607	675993780	7.4549E+615	2.485E-82
1996	4	1.4338E+607	21947850	3.1468E+614	1.049E-83
1997	3	1.8638E+607	573800	1.0695E+613	3.564E-85
1998	2	2.4226E+607	11325	2.7436E+611	9.144E-87
1999	1	3.1485E+607	150	4.7228E+609	1.574E-88
2000	0	4.0915E+607	1	4.0915E+607	1.364E-90