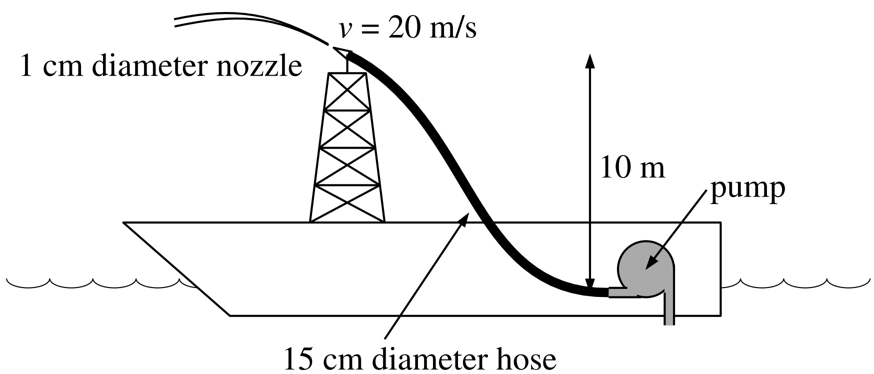
Answer five of the following six problems

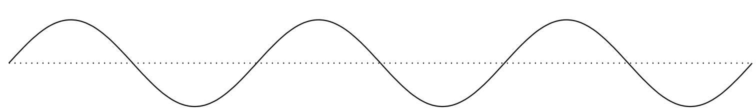
Properties of Air:
Use these values in the below problems unless otherwise stated.

sound speed: $v = 340.3$ m/s
density: $\rho = 1.225$ kg/m^3
pressure: $p = 1$ atm $= 1.0133 \times 10^5$ Pa
temperature: 288.2 K = 15.0°C

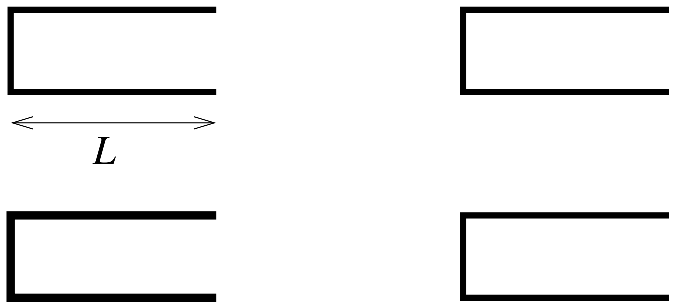
1. On a Lake Superior fire-fighting boat, a pump below decks provides the pressure to squirt water from the nozzle 10 m vertically above the pump. A 15 cm diameter hose connects the pump to the nozzle which has an end diameter of 1 cm. The velocity of the water as it leaves the nozzle is 20 m/s. The density of lake water is 1000 kg/m^3.
   (a) How much water must the pump suck from the lake (in m^3/s)?
   (b) What pressure does the pump produce, given that atmospheric pressure is 100 kPa?



2. A sinusoidal traveling wave is traveling in the positive x direction in a string (string tension: $T = 100$ N, mass: $m = .075$ kg, length: $L = 3$ m). The wave has a y amplitude of 3 mm and a frequency of 211 Hz. What is the wave speed? What is the wavelength? Write down the function for the transverse displacement $y(x,t)$. Directly on the below sinusoidal wave shape, label (A) amplitude (y_m), (B) wavelength (λ), (C) location of maximum kinetic energy density ($\mathcal{K}$), and (D) location of maximum potential energy density ($\mathcal{U}$) .



3. The below diagram shows the cross section of four identical organ pipes. Each pipe is open at one end and closed at the other. Directly on the below diagram, inside of each displayed pipe, sketch a *pressure* standing wave, starting with the fundamental and working sequentially through the higher frequency standing waves. Report the wavelength of each standing wave in terms of L (the length of the pipe). Next to each node label N. Next to each anti-node label A.



4. Four cosine functions with amplitudes and offsets: $a_1 = 4.32$, $\delta_1 = 2.253$ rad; $a_2 = 1.72$, $\delta_2 = 0.532$ rad; $a_3 = 3.51$, $\delta_3 = 1.932$ rad; and $a_4 = 2.63$, $\delta_4 = 3.325$ rad are to be added together:

$$\begin{aligned} g(t) &= a_1 \cos(\omega t + \delta_1) + a_2 \cos(\omega t + \delta_2) + a_3 \cos(\omega t + \delta_3) + a_4 \cos(\omega t + \delta_4) \\ &= A \cos(\omega t + \phi) \end{aligned}$$

Find the resulting function (i.e., A and ϕ).

5. Two trains are traveling toward each other. Train A is traveling east at 20 m/s and has a whistle that sounds at a frequency of 440 Hz. Train B is traveling west at 30 m/s and has a whistle that sounds at 500 Hz. If the air between them is still, when A's whistle blows, what frequency does B hear? If the air is moving 10 m/s towards the east when B's whistle blows, what frequency does A hear?

6. The function, $f(x)$, (plotted below) is periodic with a period $L = 2$. On the interval $[-1, 1]$, $f(x) = x^2$. Find a general formula for the Fourier series coefficients a_m. (Don't bother to simplify your formula.) Separately calculate the value of a_0. Write down the resulting formula for $f(x)$ expressed as a Fourier series. The following integral may be of use:

$$\int x^2 e^{ax}\, dx = \frac{e^{ax}(a^2x^2 - 2ax + 2)}{a^3}$$