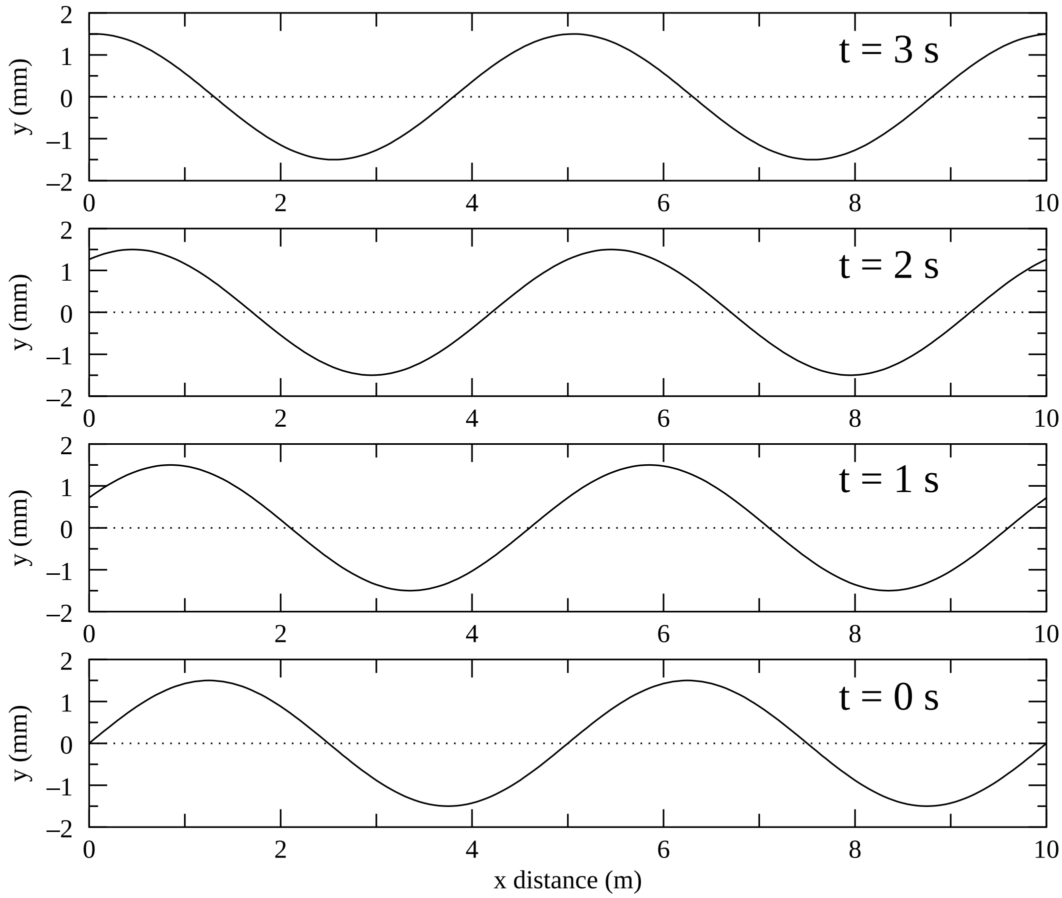
Answer five of the following seven problems

Properties of Sea Level Air:

sound speed: $v = 340.3$ m/s
density: $\rho = 1.225$ kg/m^3
pressure: $p = 1$ atm $= 1.0133 \times 10^5$ Pa
temperature: 288.2 K = 15.0°C

1. The below plots are frames from a movie showing the transverse displacement y of a sinusoidal wave on a string at times ranging from $t = 0$ s (bottom) to $t = 3$ s (top). Write down the function $y(x, t)$ that describes this wave.



2. Four cosine functions with amplitudes and offsets: $a_1 = 5.32$, $\delta_1 = 1.253$ rad; $a_2 = 1.92$, $\delta_2 = 4.532$ rad; $a_3 = 3.21$, $\delta_3 = 2.932$ rad; and $a_4 = 2.13$, $\delta_4 = .325$ rad are to be added together:

$$
\begin{aligned}
g(t) &= a_1 \cos(\omega t + \delta_1) + a_2 \cos(\omega t + \delta_2) + a_3 \cos(\omega t + \delta_3) + a_4 \cos(\omega t + \delta_4) \\
&= A \cos(\omega t + \phi)
\end{aligned}
$$

Find the resulting function (i.e., A and ϕ).

3. Consider a sound traveling wave with frequency 441 Hz and intensity 41.3 dB. Write down the displacement function (i.e., $s(x,t)$) describing this wave. What intensity (in dB) would be required to make a sound wave with ten times the displacement amplitude (s_m) as the given wave?

4. The sound wave described in #3 above excites an open-open pipe into a resonant oscillation at its fifth harmonic.

   (a) Sketch the resulting *pressure* standing wave; label nodes and anti-nodes.

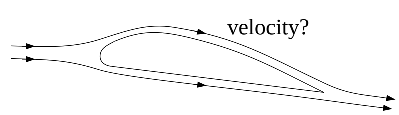
   (b) Write down the pressure function (i.e., $\Delta p(x,t)$) describing this standing wave assuming the displacement amplitudes at the ends of the pipe are 1 nm. (Set up your coordinate system so that $x = 0$ is one end of the pipe.)

   (c) How long is the pipe?

5. Consider the following situation: a Boeing 747 (i.e., a large airplane, with mass 271,000 kg) is cruising at Mach 0.543 at an altitude where the air pressure is 30.74 kPa, the air density is .466 kg/m^3, and the speed of sound is 303.5 m/s. The plane, in straight and level flight, is in equilibrium with the upward pressure force on the 541 m^2 wings balancing the force of gravity.

   (a) Making the 'spherical cow' approximation that the pressure difference between the top and bottom surface of the wings is uniform, what pressure difference is required to support the plane?

   (b) Assuming that (in the rest frame of the wing) the air speed on the bottom of the wing is Mach 0.543, what air speed on the top of the wing is required to achieve the pressure difference calculated in (a) above.



6. An F-15 approaches the 747 of problem #5 directly from the rear at a speed of Mach .763. The F-15's jet engine produces a high frequency whine at 17.1 kHz. What frequency is heard by the pilot of the 747?

7. The function, $f(x)$, (plotted below) is periodic with a period $L = 2$. On the interval $[-1, 1]$, $f(x) = x^2$. Find a general formula for the Fourier series coefficients a_m (Don't bother to simplify your formula.) Separately calculate the value of a_0. Write down the resulting formula for $f(x)$ expressed as a Fourier series. The following integral may be of use:

$$\int x^2 e^{ax}\,dx = \frac{e^{ax}(a^2x^2 - 2ax + 2)}{a^3}$$