Solving a System of Linear Equations Using Matrices
With the TI-83 or TI-84 Graphing Calculator

To solve a system of equations using a TI-83 or TI-84 graphing calculator, the system of equations needs to be placed into an augmented matrix. The augmented matrix can be input into the calculator which will convert it to reduced row-echelon form. From this form, we can interpret the solution to the system of equations.

To put a system of equations into an augmented matrix, the coefficients associated with each variable and the constants are placed into their corresponding locations as shown in the following example:

Given the system $\begin{cases} x + 3y + 5z = 7 \\ 2x + 4y + 6z = 8 \\ 3x + 9y + 7z = 12 \end{cases}$,the augmented matrix is $\begin{bmatrix} 1 & 3 & 5 & | & 7 \\ 2 & 4 & 6 & | & 8 \\ 3 & 9 & 7 & | & 12 \end{bmatrix}$.

Each equation must be in standard form. For any missing variable, a zero must be entered in the matrix as this would be the coefficient of that variable.

Now the system of equations can be solved by entering the matrix into the calculator.
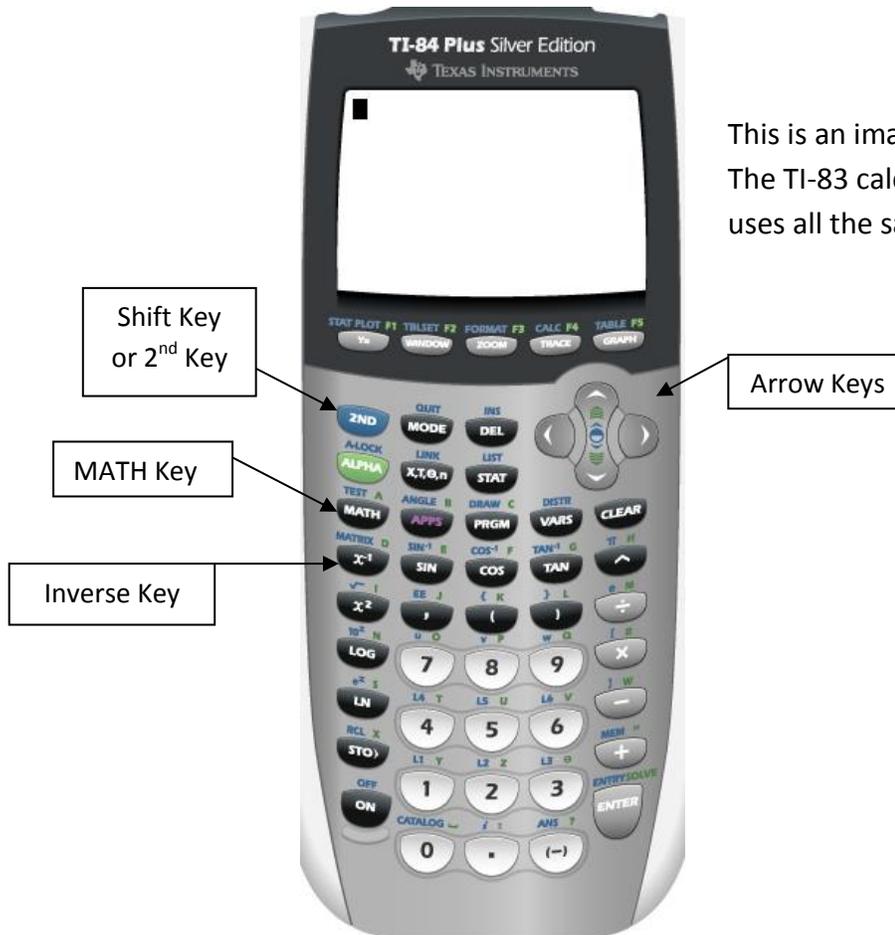
This is an image of a TI-84 calculator. The TI-83 calculator is similar in look and uses all the same key processes.

Shift Key or 2nd Key

Arrow Keys

MATH Key

Inverse Key

Step 1: To access the matrix features of your calculator, you need to press the shift key and then the inverse key. This will show the matrix menu of the calculator, which looks like:
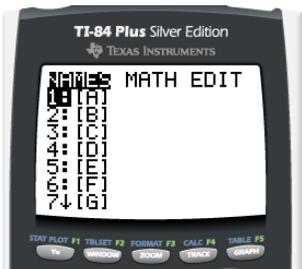
*If this feature has been used before, then the dimensions of the matrices entered will appear next to the matrix letter name.

Step 2: Press the right arrow key twice so that EDIT is highlighted.

Step 3: Select a matrix by using the down arrow until the desired matrix is highlighted. (Here we will use matrix [A], so we do not need to use the arrows). Press ENTER to edit the matrix.

Step 4: Enter the number of rows of your matrix, then press ENTER. Enter the number of columns of your matrix, then press ENTER. (Here we are using a 3 x 4 matrix) These are the dimensions of the matrix.

Step 5: When the matrix values are entered, they are entered by rows. Using the previous example, enter the first row of values, and then the second row. Finally, enter the third row. Your screen should look like the following:
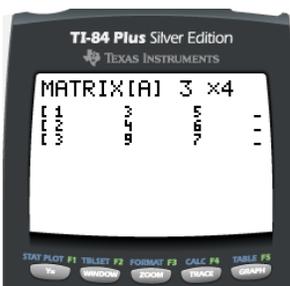
*The screen can only display 3 columns. To display the column that you cannot see, use the arrow keys to move from one end of the matrix to the other.

*The line between the 3rd and 4th columns of our augmented matrix will not appear on the calculator.

Step 6: Now that the matrix is entered into the calculator, the reduced row-echelon form of our matrix is needed. To have the calculator complete this operation, use the matrix menu by pressing shift then the inverse key. Press the right arrow key once until MATH is highlighted. Arrow down until rref(is highlighted. Your screen should look like:

Press ENTER. Your screen should look like:
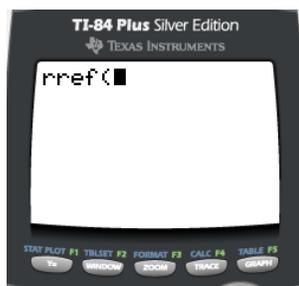
Now go back into the matrix feature by pressing shift and then the inverse key. Arrow down to highlight your matrix, if necessary, and press ENTER. Now you are back to the main screen. Close the parentheses around your matrix. Your screen should look like:
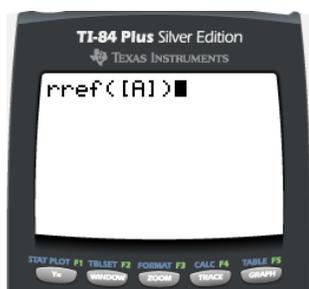
Press ENTER. This is the reduced row-echelon form for the matrix. The result should look like:
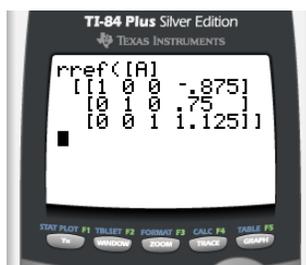
The calculator may express the answer as a decimal. We can either leave the answer in decimal form or convert it to fractions (if possible). Here we will convert our decimals into fractions.

Converting your results into fractions: Press the MATH key once (this MATH key is below the ALPHA key). The highlighted option converts decimals into fractions. Press ENTER once to get back to the home screen, then press ENTER a second time to carry out the operation. The result will look like:
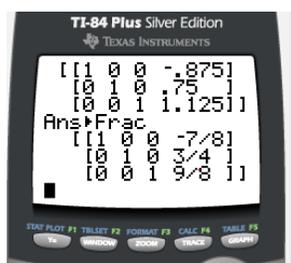
Interpreting Results:

For the given example, the reduced row-echelon form of the matrix is $\begin{bmatrix} 1 & 0 & 0 & -\frac{7}{8} \\ 0 & 1 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{9}{8} \end{bmatrix}$.

Converting each row in the matrix back to an equation in standard form, the first row shows that x = -7/8. The second row shows that y = 3/4, and the third row shows that z = 9/8.

When solving a system of equations with matrices, there are 3 possible results when reducing the matrix into Reduced Row Echelon Form.

1) **Independent**

> When a system is independent, there is exactly <u>one solution</u> to the system. The result will look like:
>
> $\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}$ with a, b, and c being real numbers
>
> The solution to the system would be (a, b, c).

2) **Dependent**

> When a system is dependent, there are <u>infinitely many solutions</u> to the system. The result will look similar to:
>
> $\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 0 & 0 \end{bmatrix}$
>
> The row of zeroes in the bottom row indicates that there are infinite solutions. *The final answer can be expressed in many ways. Refer to your notes or professor in order to be sure you are expressing the final answer in the correct format.

3) **Inconsistent**

> When a system is inconsistent, there are <u>no solutions</u> to the system.
>
> $\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 0 & 1 \end{bmatrix}$
>
> The bottom row indicates that there are no solutions. When the bottom row shows that zero is equal to anything other than zero, there are no solutions to the system.