Time	Topics	Assignments
12:30 - 1:20	Introduction, units	Quiz #1
1:30 - 2:20	Displacement, velocity, acceleration	
2:30 - 3:30	Equations of motion	Homework #1

Day 1, Hour 1: Introduction

1.1 Physics

1. Science: study of nature, in which ideas are tested by experiment. Some examples:

Astronomy	meteors	planets	stars	galaxies
Biology	bugs	people	starfish	flowers
Chemistry	elements	molecules	organic molecules	orbitals
Geology	rocks	mountains	volcanoes	continents
...	...	...	...	...

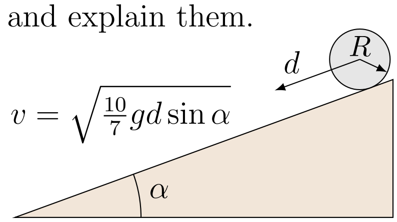
2. What is physics about? The laws of the universe.
   (a) As few laws as possible to cover everything.
   (b) Physics is the most basic science for that reason.
   (c) In each of the sciences, there is a connection with physics:
   - AstroPhysics; how laws of nature apply to stars, planets, etc.
   - BioPhysics; how laws of nature apply to living things
   - Chemical Physics, or Physical Chemistry; how laws of nature apply to atoms and molecules.
   - GeoPhysics; how laws of nature apply to rocks and volcanoes

   (d) So physicists are interested in pretty much everything scientific.
   - The law of gravity has to work for stars and also starfish.
   - If a biologist discovers an anti-gravity beetle, then the physicists would want to know about it, and would need to correct the law of gravity.

   (e) Physics doesn't explain everything.
   - Physicists don't know all the laws of the universe. It's "an expanding frontier of ignorance" according to one Nobel-prize-winning physicist.
   - Lots of things are much too difficult to try to explain in terms of physics.
     – Why do armadillos roll up in a ball on the highways of Texas? Don't ask a physicist, ask a bio-psychologist perhaps.

 – A physicist could estimate how fast the armadillo would roll down a hill.

(f) In physics, we usually oversimplify things so we can understand and explain them.
  - Approximate the armadillo as a smooth solid sphere.
  - Approximate the hill as a constant slope.
  - Disregard friction and air drag.



$$v = \sqrt{\frac{10}{7} g d \sin\alpha}$$

3. Physics is usually divided into a few subtopics as if they stood alone. For example,
   (a) Mechanics = study of motion (PHYS105)
   (b) Thermodynamics = study of heat and temperature effects (PHYS105)
   (c) Electromagnetism (includes light/optics) (PHYS106)
   (d) Quantum mechanics: atoms, nuclei (PHYS106)

   Other general areas can be focused on, such as Relativity or Waves; these could be considered part of Mechanics, but they are dealt with separately in PHYS106 where they fit best.

4. Any of these topics is much too big to cover in depth in a semester. In this course, we will only deal with two major topics, mechanics and thermodynamics, in a broad, shallow way.
   - Everything is simplified to emphasize basic big ideas and how they are applied, such as:
     – Unbalanced forces cause a mass to accelerate;
     – Energy and momentum are conserved;
     – Heat is a form of energy;
     – Substances are made of atoms that move faster at higher temperature.
   - The level of mathematics used will only require algebra and trigonometry, no calculus.

5. Plan for this course:
   (a) Book: OpenStax - can get free online, or buy for reasonable price at bookstore.
     - Worth bringing to class I think. But it is big and heavy.
   (b) Canvas - should see PHYS105 when you log in
   (c) Files ⇒ Syllabus
   (d) Book chapters 1 - 16 = about half of the book.
     i. Mechanics = Chs. 1-11, 16
     ii. Thermodynamics = Chs. 12-15
   (e) Read ahead, come with questions
   (f) 3 Exams on the dates
   (g) Labs: 7 experiments (usually more like 10 + a lab exam at the end).
     i. You will need 2 notebooks, one Lab manual (Pick one up today before you leave)
     ii. Come prepared, Do the 2-hour lab, Hand it in, You're done.
     iii. First lab is online: see the links, start ASAP, get done this week
   (h) Homework
     i. Canvas Home screen should show you a WebAssign link

ii. You create an account using your WebAssign code
iii. Then you can login and do your homework.
iv. An assignment for each chapter, due (before midnight) the day we start a new chapter.
v. So you can ask questions, and I can use them as examples.
vi. First assignment is due Wednesday (midnight). On math aspects of Ch. 1.

(i) Quizzes

i. When there is time, just a few minutes at the end of an hour to work out something related to what the class was about.

1.2 Physical quantities and units

To do physics, we need to be able to measure things. To describe how things move, we need to be able to measure

- Time: the standard unit is the second (s).
- Distance: standard unit is the meter (m).

And to explain why things move, we need a third quantity

- Mass: how much stuff is moving, usually measured in kilograms (kg).

Next semester you will have to deal with electricity and magnetism, and a fourth quantity is needed: Electric current, measured in a unit called the Ampere.

Derived units

By combining the fundamental units, you can make other units needed for studying motion. Examples:

Quantity	Unit
Velocity	m/s
Acceleration	$\mathrm{m/s/s = m/s^2}$
Force	$\mathrm{kg \cdot m/s^2 = N}$ = newton
Energy	$\mathrm{kg \cdot m^2/s^2 = N \cdot m = J}$ = joule
Momentum	$\mathrm{kg \cdot m/s}$

Metric prefixes

Often it is convenient to abbreviate powers of 10 applied to the units when measuring a quantity. Here are some examples:

Prefix	Symbol	Value	Example
mega	M	10^6	MJ = MegaJoule = 10^6 J
kilo	k	10^3	km = kilometer = 10^3 m
centi	c	10^{-2}	cm = centimeter = 10^{-2} m
milli	m	10^{-3}	mJ = millijoule = 10^{-3} J
micro	μ	10^{-6}	μs = microsecond = 10^{-6} s

1.4 Unit conversions and estimation

Units can be converted into other units by multiplying and dividing with conversion factors:

- Suppose this room is 10 m wide, and 1 foot is 0.3048 m. What is the width of the room in feet?
- $10\,\text{m} \times \frac{1\ \text{foot}}{0.3048\,\text{m}} = \boxed{32.8\ \text{feet}}$. Notice the cancellation of units: meters are gone, feet remain.

Units can be converted into completely different kinds of things as well.

- Suppose you will be driving 1000 miles: Can you convert 1000 miles into Dollars?
- $1000\ \text{mi} \times \frac{\text{gal}}{30\ \text{mi}} \times \frac{\$2}{\text{gal}} = \$67.$

For purposes of estimation, you don't really need a precise conversion factor — a guess is often good enough. An order-of-magnitude estimate is one in which you try to get the right power of ten, but don't worry about the rest.

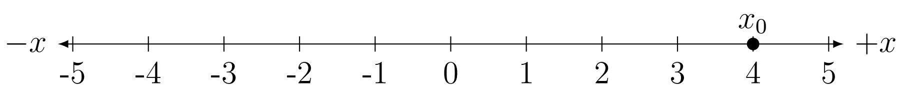
- The price of gasoline may not be exactly \$2/gal, but that's OK. We can be pretty sure it is not a power-of-ten lower (\$0.2/gal) or a power-of-ten higher (\$20/gal), so the estimate we get is probably the correct order-of-magnitude, or power-of-ten. Likewise, the conversion of gallons to miles is probably the right order-of-magnitude.

Quiz #1

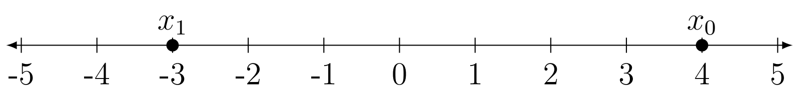
Day 1, Hour 2: One-dimensional motion

2.1 Displacement Δx

1. Our book, like most, uses x to represent the position of something along a line.
   - Imagine a line in both directions from an origin, with units. For instance, if the classroom really is 10 m wide, I could put $x = 0$ at the center of the blackboard, and mark off meters out to ± 5 to the left and right, and call this the x axis:



   - Now that the line exists, x could be anything along that line, and it would be represented by a number.
   - Suppose a kid we know named Boris is at $x_0 = 4\,\text{m}$. That is his position.
     - That subscript 0 is used to indicate that this is his position at a time we will call $t = 0$.
2. Things get more interesting if he moves to a different position: call it x_1, and let's say $x_1 = -3\,\text{m}$.



3. Displacement is the change in position: defined as

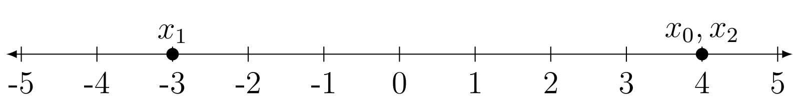
$$\Delta x = x_f - x_0 = \text{Displacement}$$

where the Greek letter Δ ("Delta" is a capital D in Greek) means the change in something, defined as the final value minus the initial value:

$$\Delta x = \underline{\text{final value}} \text{ of } x - \underline{\text{intitial value}} \text{ of } x.$$

$$\Delta x_1 = x_1 - x_0 = -3 - 4 = \boxed{-7\,\text{m.}}$$

   - Does the negative sign here make sense?
     - Change of position was in the direction defined as negative.
4. Suppose now Boris goes back to where he started: $x_2 = 4\,\text{m}$ again. What was his second displacement?

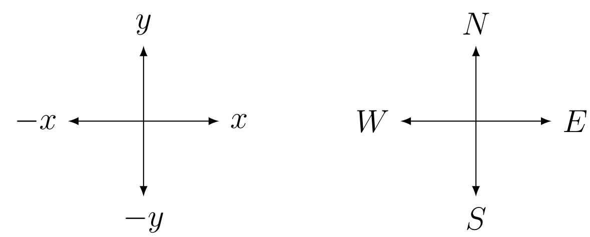


$$\Delta x_2 = x_2 - x_1 = 4 - (-3) = \boxed{+7\,\text{m.}}$$

5. What is his total displacement since the time 0? Well, $\Delta x = x_2 - x_0 = 4 - 4 = \boxed{0.}$
6. Displacement is different from Distance. Distance doesn't have a direction.
   - Total Distance Boris travelled $= 7\,\text{m} + 7\,\text{m} = \boxed{14\,\text{m.}}$
7. To emphasize this concept, the book has a section 2.2 on quantities with both a size and a direction.

2.2 Scalars and Vectors

1. Some things are measured by a single number (with units). These things are called scalars.
   - The number that defines a scalar is often called its magnitude.
   - Distance is a scalar. The distance Boris travelled had a magnitude of 14 m.
   - Lots of things are scalars: temperature (70°F), volume (5 gal), speed (70 mph), etc.
2. But other things have a direction along with a magnitude. These things are vectors.
   - Displacement is a vector, because the direction matters.
     - Boris's first displacement was $\Delta x_1 = -7\,\text{m}$. The magnitude was 7 m, and its direction was negative.
   - As we study motion, some other vectors we will meet include force, momentum, and torque.
   - When we get to two-dimensional motion, then we have to define two directions, with labels such as x and y, or North/South and East/West, or something like that.

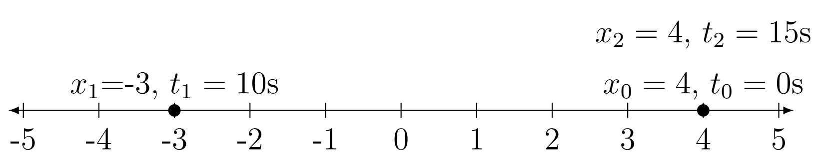


   - Vectors always have a magnitude and direction. But how we express these two properties is a matter of choosing what is most convenient.
     - A 2-dimensional displacement will still be given with two numbers, such as $(\Delta x, \Delta y)$, or (3 m East, 4 m North), or as a distance and an angle (5 m at 53.1° North of East).
   - For now, in dealing with 1-dimensional motion, using $\pm$ for direction is very convenient.

2.3 Time t, Velocity v, and Speed

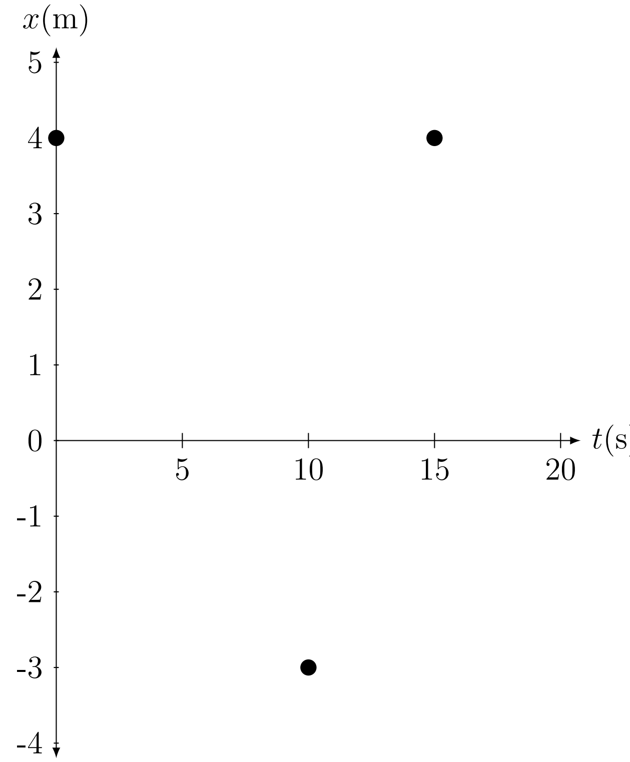
When describing motion, we will care not only about position, but also about time.

- Suppose I put time information into the picture of Boris's travels:
  - Say he started at position x_0 at a time $t_0 = 0$, then reached $x_1 = -3$ at time $t_1 = 10\,\text{s}$, and returned to his starting place at $t_2 = 15\,\text{s}$.



- These are important details, but the picture is just getting crowded without really doing much to help us understand the motion.

- A different way to handle the time and displacement information is to put them on a graph of x versus t.
  - Each dot with coordinates (t, x) shows the time t and corresponding position x.
  - The dots tell us a story:
    - Once upon a time, Boris was at 4 m from the origin on the x axis.
    - Ten seconds later, he was at -3 m.
    - In another 5 seconds, when the clock said 15, Boris was back at 4 m.
    - The End
  - You could probably write a better story, about how Boris rode his skateboard, or whatever you like.



- This graphical picture helps us see something about his velocity, defined by:

$$\boxed{\text{Average velocity} = \overline{v} = \frac{\Delta x}{\Delta t}} \text{ with units} = \frac{x \text{ units}}{t \text{ units}} = \frac{\text{m}}{\text{s}}$$

Velocity is a vector — it has a direction along with a magnitude. Speed only has magnitude, so speed is a scalar.

- For the first leg of his trip, Boris had an average velocity that was negative

$$\overline{v}_1 = \frac{-7\,\text{m}}{10\,\text{s}} = -0.7\,\text{m/s}$$

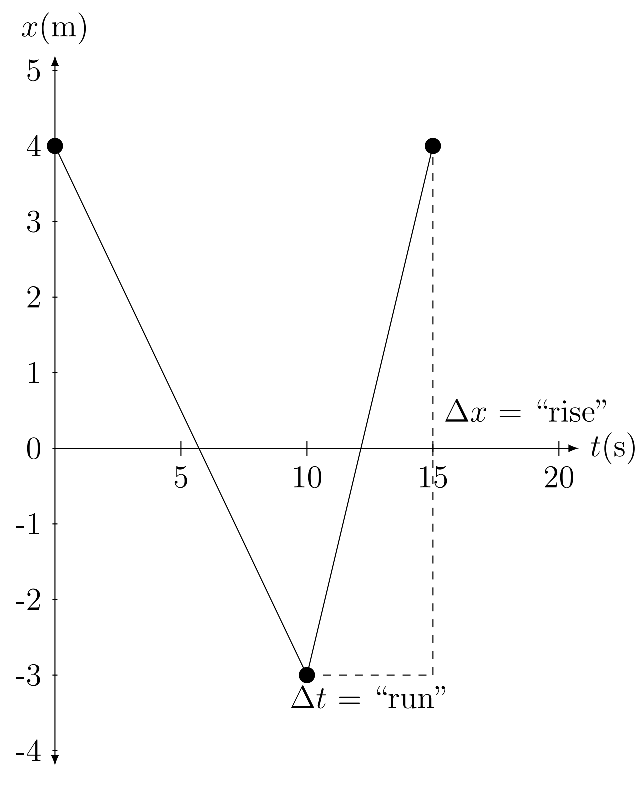
- For the second leg of his trip, Boris had an average velocity that was positive

$$\overline{v}_2 = \frac{7\,\text{m}}{5\,\text{s}} = 1.4\,\text{m/s}$$

- These average velocities are the slopes of the lines connecting the dots:

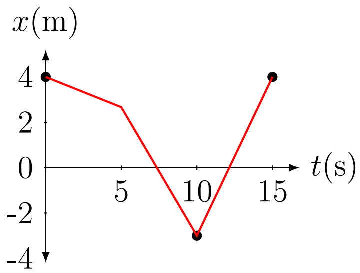
$$\overline{v} = \text{Average velocity} = \frac{\Delta x}{\Delta t} = \frac{\text{rise}}{\text{run}} = \text{ slope}$$

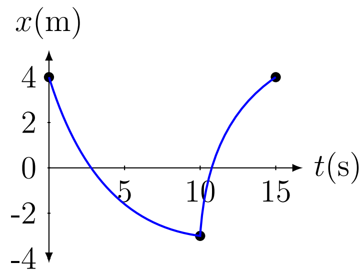
as suggested by the dashed lines from one point to the next.

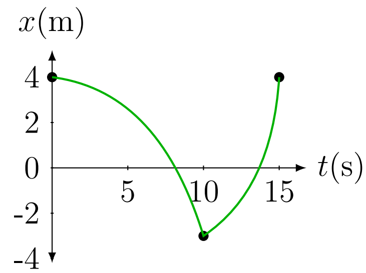


- So the x versus t graph can help you understand the motion better by showing velocities.

- But you have to be careful, because you may not actually have any information except the dots.
- The lines that connect them show how to calculate average velocities, but they don't tell you whether Boris really moved at these constant velocities.
- Suppose Boris's story actually went like one of these curves. What does each one tell you?







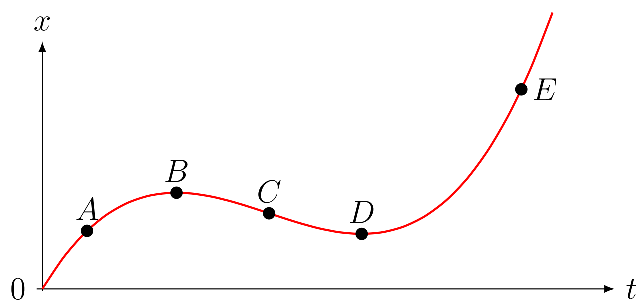
When we have a continuous curve, not just a few isolated points, we can see more than average velocities; we can see the "instantaneous velocity," defined as the slope at each point:

$$\boxed{\text{Instantaneous velocity } = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}.}$$

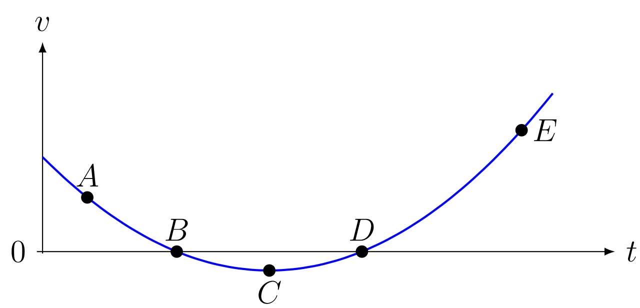
This means find the average velocity using two points, but move the two points infinitely close together.

The slope of an $x(t)$ curve at a single point is the instantaneous velocity v there. For a curving function, this is best done using calculus, but it won't be necessary in this class.

- You should be able to recognize points with positive velocity, such as points A and E, negative velocity, such as C, and points with zero velocity such as B and D.



When velocity is changing with time, then it is interesting to plot a graph of $v(t)$. This is not always easy, but here is an example. The blue $v(t)$ graph is the one that goes with the red $x(t)$ graph above. Notice the correspondence of points A, B, C, D, E in the two graphs.



- Points B and D are where $v = 0$.
- Between 0 and B, the velocity is positive, but decreasing.
- Between B and D the velocity is negative, and it is most negative at C.

It is important to be able to go one step further, and look at how the slope of the $v(t)$ graph changes.

- In this particular case, the slope of this velocity graph is always increasing.
- The slope of the $v(t)$ graph is known as the acceleration.
- The reason this is important for us is that whenever there is acceleration, then something interesting is happening to change the motion. A force causes acceleration.
- We will get to the topic of forces later, but for today, we will look at the concept of acceleration.

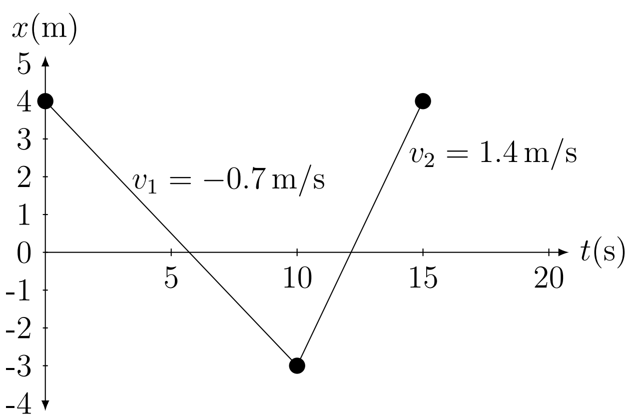
2.4 Acceleration a

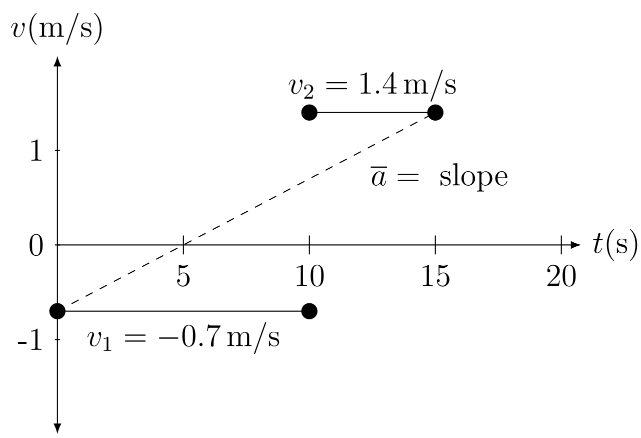
When velocity changes, then there is acceleration, which is defined just as you might expect from what we have seen about velocity. First,

$$\boxed{\text{Average acceleration} = \overline{a} = \frac{\Delta v}{\Delta t}} \text{ with units} = \frac{v \text{ units}}{t \text{ units}} = \frac{\text{m/s}}{\text{s}} = \text{m/s}^2.$$

$$\boxed{\text{Instantaneous acceleration } = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}.}$$

For example, if Boris's travels happened at two different constant velocities, as we found earlier, then we can calculate the average acceleration over some time interval. Let's use the entire interval from $t = 0$ to $t = 15\,\text{s}$. Here are the $x(t)$ and $v(t)$ graphs:



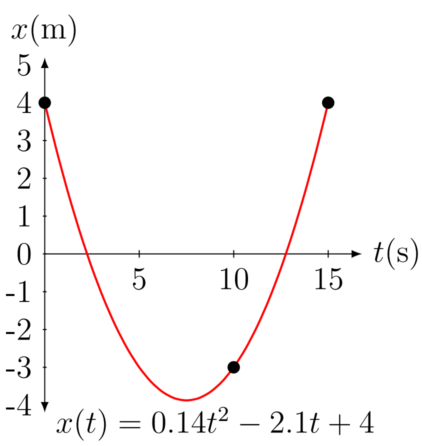


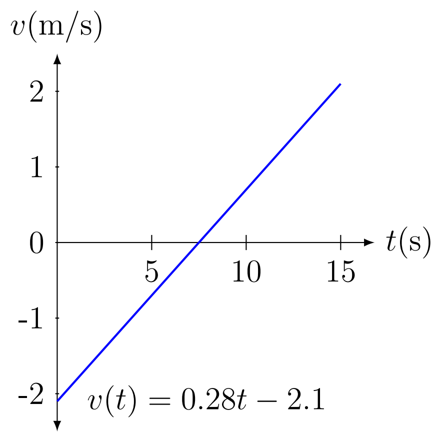
and we find the average acceleration over this time interval:

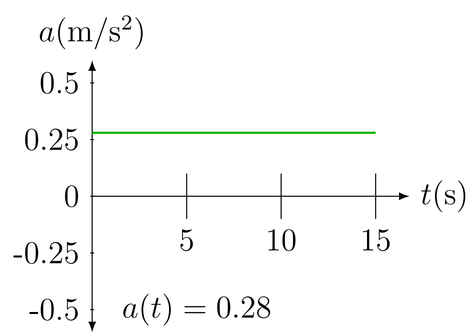
$$\overline{a} = \frac{v_2 - v_1}{t_2 - t_0} = \frac{1.4 - (-0.7)}{15 - 0} = \frac{2.1}{15} = \boxed{0.14\,\text{m/s}^2.}$$

But this analysis assumes there was no change in velocity anywhere except at the instant $t = 10\,\text{s}$, and there is no way to find the slope of $v(t)$ at that instant because the function is discontinuous there.

To calculate instantaneous acceleration, we need a continuous function for $v(t)$. Boris's actual $x(t)$ trip could have been something different, such as the red curve below. From a continuous $x(t)$ we can find $v(t)$ everywhere; the $v(t)$ graph in blue corresponds to the $x(t)$ graph in red. This $v(t)$ is a straight line from (0,-2.1) to (15,2.1), so its slope is a constant everywhere. Calculating the value of a is simple:







$$a = \frac{v_2 - v_0}{t_2 - t_0} = \frac{2.1 - (-2.1)}{15 - 0} = \boxed{0.28\,\text{m/s}^2.}$$

Day 1, Hour 3: Equations of motion

We have begun to describe it in terms of these quantities:

Symbol	Definition	Name
Δt	$t_f - t_i$	Time interval
Δx	$x_f - x_i$	Displacement
v	$\Delta x/\Delta t$	Velocity
a	$\Delta v/\Delta t$	Acceleration

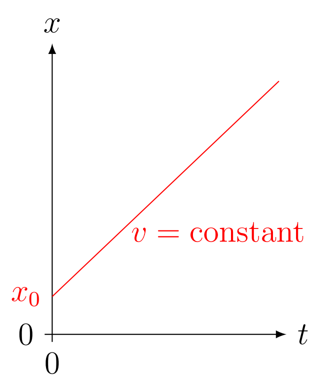
But motion in the real world can be very complicated. Some engineers need to keep going and define

Symbol	Definition	Name
j	$\Delta a/\Delta t$	Jerk
s	$\Delta j/\Delta t$	Snap
c	$\Delta s/\Delta t$	Crackle
p	$\Delta c/\Delta t$	Pop

For this course (until we get to chapter 16), we will assume a is constant, so $\Delta a = 0$, and all these other fun quantities don't matter.

2.5, 2.8 Motion Equations for Constant Acceleration in One Dimension

Now that we know the meaning of displacement, velocity, and acceleration, and how these look on graphs, we can cook up some equations that describe motion. In some cases, we can simply look at a graph, and write an equation for what we see there.



This graph shows a straight line with a slope. You probably remember from some math class that the equation of a line is something like

$$y = b + mx$$

where m is the slope, and b is the y-intercept. But our graph has x on the vertical axis instead of y, and t on the horizontal axis instead of x. The intercept is called x_0, the initial position, and the slope, as we have learned, is the velocity v. So the equation for this straight line is

$$\boxed{x = x_0 + vt.} \tag{1}$$

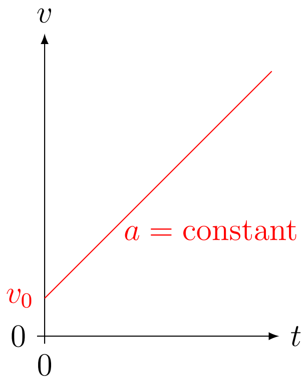
The line has a constant slope, which means velocity v = constant, and the acceleration is $a = 0$.

Before going on, it is also worth pointing out that when we know the average velocity, we can use it to determine displacement as in Eq. (1). If you travel at an average velocity of 50 miles per hour north for 2 hours, then you will go 100 miles north; this is true whether your velocity is constant or not — you only need the average velocity. So we could also say, regardless of acceleration

$$\boxed{x = x_0 + \overline{v}t.} \tag{2}$$

Example: Boris starts at $x_0 = 3\,\text{m}$, and runs in the x direction at a constant velocity $v = 2\,\text{m/s}$. What is his location at the end of 5 seconds?

Solution: $x = x_0 + vt = 3 + (2)(5) = 3 + 10 = 13\,\text{m}$.
And if his velocity was not constant, but his average velocity was $2\,\text{m/s}$, he would reach the same place in 5 seconds.



This graph also shows a straight line with a slope. But notice that it is a graph of velocity versus time. So the intercept is the initial velocity v_0, and the slope is the acceleration a, which is not zero in this example. Thus the equation for this straight line is

$$\boxed{v = v_0 + at.} \tag{3}$$

Example: Boris is riding his bike with an initial velocity $v_0 = 2\,\text{m/s}$, and he accelerates at $a = 1\,\text{m/s}^2$. What will be his velocity at the end of 5 seconds?

Solution: $v = v_0 + at = 2 + (1)(5) = 2 + 5 = 7\,\text{m/s}$.

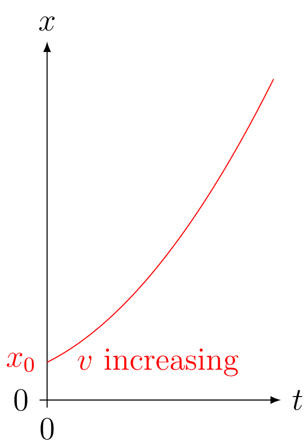
We can also see another useful equation from this graph: when a is constant, the average velocity is just the average of the beginning and ending velocites. That is,

$$\boxed{\overline{v} = \frac{1}{2}\left(v_0 + v\right).} \tag{4}$$

Example: In the previous example, what was Boris's average velocity during the 5-second interval?

Solution: $\overline{v} = \frac{1}{2}\left(v_0 + v\right) = \frac{1}{2}\left(2 + 7\right) = 4.5\,\text{m/s}$.

Suppose we need to calculate x instead of v when there is acceleration happening. The $x(t)$ graph is no longer a straight line, because v is changing, and we probably don't recall from that math class how to write the equation of a parabolic curve.



What we can do is use Eq. (2), but with the average velocity given by Eq. (4).

$$x = x_0 + \overline{v}t \text{ with } \overline{v} = \frac{1}{2}\left(v_0 + v\right) \Rightarrow x = x_0 + \frac{1}{2}\left(v_0 + v\right)t$$

and then, using Eq. (3), replace v with $v_0 + at$, so that

$$x = x_0 + \frac{1}{2}\left(v_0 + v_0 + at\right)t$$

which finally simplifies to

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2.} \tag{5}$$

This equation starts out like Eq. (1), but because there is acceleration, it adds the extra displacement due to acceleration.

Example: A car is travelling in the x direction at $40\,\mathrm{m/s}$ initially, but for 15 seconds it accelerates at $3\,\mathrm{m/s^2}$. What is its displacement during those 15 seconds?

Solution: $x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + (40)(15) + \frac{1}{2}(3)\left(15^2\right) = 600 + 337.5 = 937.5\,\mathrm{m}$.

One more useful item would be an equation that relates x, v_0, v and a, without having to specify the time t. This takes some algebra, rearranging Eq. (3) to get

$$t = (v - v_0)/a$$

and then using it in place of the t in that large Eq. (5) as follows.

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = x_0 + v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

This gets worse, but then better when we do the algebra, canceling and combining terms:

$$x = x_0 + \cancel{\frac{v_0 v}{a}} - \frac{v_0^2}{a} + \frac{1}{2}\cdot\frac{v^2}{a} - \cancel{\frac{1}{2}\cdot\frac{2v_0 v}{a}} + \frac{1}{2}\cdot\frac{v_0^2}{a}$$

$$\Rightarrow x = x_0 + \frac{v^2 - v_0^2}{2a}$$

or, a more popular version is this arrangement:

$$\boxed{v^2 = v_0^2 + 2a\left(x - x_0\right).} \tag{6}$$

Example: A car is travelling in the x direction at $40\,\mathrm{m/s}$ initially, but then it accelerates at $3\,\mathrm{m/s^2}$. What is its velocity at the end of $300\,\mathrm{m}$ with this acceleration?

Solution: $v^2 = v_0^2 + 2a\left(x - x_0\right) = 40^2 + 2(3)(300) = 1600 + 1800 = 3400 \Rightarrow v = \sqrt{3400} = 58.3\,\mathrm{m/s}$.

Collecting these equations together in one place, we have

Equation	Conditions on a
$x = x_0 + vt$	$a = 0$
$x = x_0 + \overline{v}t$	a can be anything
$\overline{v} = \frac{1}{2}(v_0 + v)$ $x = x_0 + v_0 t + \frac{1}{2}at^2$ $v = v_0 + at$ $v^2 = v_0^2 + 2a(x - x_0)$	$a = \text{constant}$

DAY 1 Homework Assignment: Practice a variety of unit-conversion and 1-dimensional motion problems. See Canvas / WebAssign. Due Tuesday 9/1 @ midnight, but don't wait to get started. Email questions or ask in class Tuesday.

Time	Topics	Assignments
12:30 - 1:20	Free fall, g, Graphs	
1:30 - 2:20	2D motion and vectors	Quiz #2
2:30 - 3:30	Projectile motion	Homework #2

Day 2, Hour 1: Free fall, g, graphs

Last time, we looked at some basics about motion, some definitions $(\Delta x, v, a)$ and equations.
Today we will start by looking at a special case, free fall, when the acceleration is due only to gravity.

2.7 Falling objects

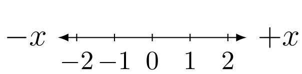
- The term "free fall" means there are no other forces but gravity — no air drag or friction.

It may seem surprising that heavy objects and light ones accelerate alike due to gravity.

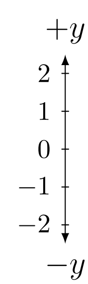
- Find a couple of objects to drop, one heavier than the other, and see what you think.
  - If you can, try objects about the same size and shape, but different weights.
- Also try racing a flat sheet of paper against a small object.
  - Then try crumpling the paper into a ball and compare again.
- Air drag can be important, but there are ways to reduce it.
- For now, to keep things simple, we will just ignore air drag effects.

In that case, near the earth, things fall with an acceleration of about $9.8\,\mathrm{m/s^2}$.

- In equations, the symbol g is used for the magnitude of free-fall acceleration, with $\boxed{g = 9.8\,\mathrm{m/s^2}.}$
- Also, for vertical motion, we will use y instead of x, with the y direction upward.
- Because gravity is downward, the direction of free-fall acceleration is the negative y direction.
- So in our equations of motion, a is replaced by $-g$ for describing free fall:



Constant a equations	Free-fall equations
$x = x_0 + v_0 t + \frac{1}{2} a t^2$	$y = y_0 + v_0 t - \frac{1}{2} g t^2$
$v = v_0 + at$	$v = v_0 - gt$
$v^2 = v_0^2 + 2a\,(x - x_0)$	$v^2 = v_0^2 - 2g\,(y - y_0)$



- Remember that up is positive, down is negative, then these equations are even easier to use, because you already know the acceleration is $-g$ in any free-fall situation.

Here are some examples in which these equations are used in various ways, and some graphs of the free-fall motion.

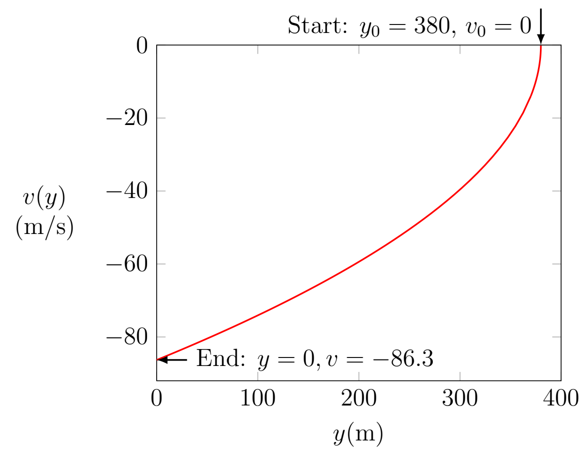
Example: King Kong drops a brick from the roof of the Empire State Building, a height of 380 m above the ground. Assuming free fall, what is the speed of the brick when it reaches the ground?

Solution: The initial velocity $v_0 = 0$ when the brick is dropped, and $y = 0$ at the ground. So, using the $v(y)$ equation,

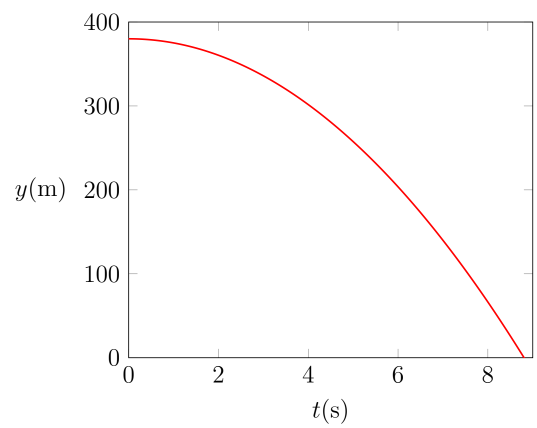
$$v^2 = v_0^2 - 2g\,(y - y_0) = 0 - 2(9.8)\,(0 - 380)$$

$$v^2 = 7448\,\mathrm{m^2/s^2} \Rightarrow v = -\sqrt{(2)(9.8)(380)} = -86.3\,\mathrm{m/s}.$$

Note that this is the velocity of the brick, and it is negative because it is downward. But the speed of the brick is 86.3 m/s.



There is usually more than one way to solve a problem. We were not given the time, only the starting height. Could we find the time for the brick to reach the ground, and then use that time to find v? Here is that calculation

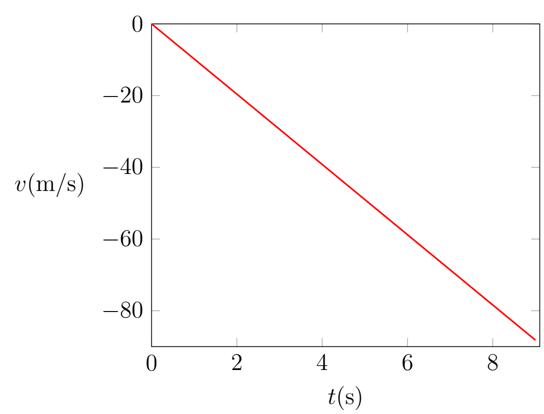


Solution:

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \Rightarrow 0 = 380 - \frac{1}{2}(9.8)t^2$$

$$\Rightarrow t = \sqrt{(380)(2)/(9.8)} = 8.81\,\mathrm{s}.$$

Next, we can use this time in the $v(t)$ equation to calculate v:



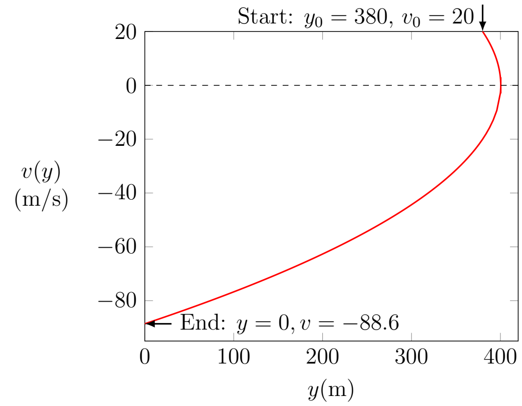
Solution:

$$v = v_0 - gt \Rightarrow v = 0 - (9.8)(8.81) = -86.3\,\mathrm{m/s},$$

and the speed is 86.3 m/s again.

Notice that this simple v versus t graph shows a constant, negative slope, as it should, because the acceleration is a constant downward $g = 9.8\,\mathrm{m/s^2}$.

Suppose there is also some initial velocity given to the brick:



Example: King Kong throws a brick upward at 20 m/s from the roof of the Empire State Building, a height of 380 m above the ground. Assuming free fall, what is the velocity of the brick when it reaches the ground?

Solution: $v^2 = v_0^2 - 2g(y - y_0) = (20^2) - 2(9.8)(0 - 380) = 7848\,\text{m}^2/\text{s}^2 \Rightarrow v = -88.6\,\text{m/s}$. Note that this is the negative square root, because downward is the negative direction.

What is the brick's maximum height above the ground?

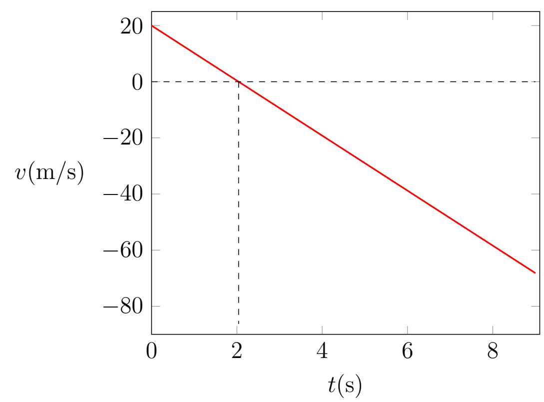
- You can see in the previous graph that the largest y is somewhere beyond 380 m, of course, and you may be able to see that this is where $v = 0$.

Solution: At the highest point, the brick stops rising, so its upward velocity there is $v = 0$, and we can solve for y:

$$v^2 = v_0^2 - 2g(y - y_0)$$

$$0 = 20^2 - 2(9.8)(y - 380) \Rightarrow y = 380 + \frac{20^2}{2(9.8)} = 400.4\,\text{m}.$$

At what time does the brick reach its maximum height?



- A different graph, this time v versus t, shows that the velocity decreases from 20 m/s to 0 at a time slightly over 2 s, and it then increases in the negative direction as it falls.
- To solve for t, we again set $v = 0$:

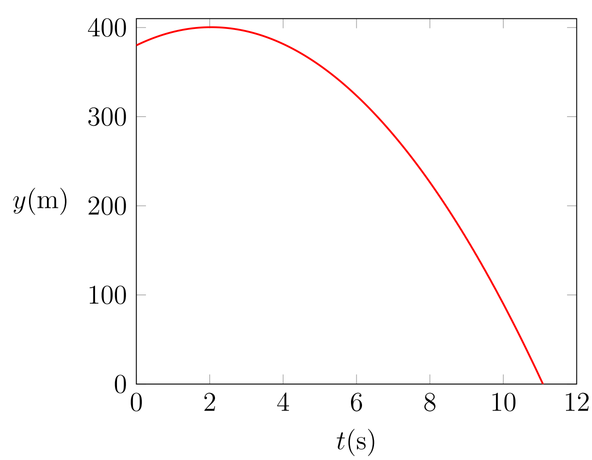
Solution: $v = v_0 - gt \Rightarrow 0 = 20 - 9.8t \Rightarrow t = \frac{20}{9.8} = 2.04\,\text{s}$.

At what time does the brick reach the ground?

Solution:

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

$$0 = 380 + 20t - 4.9t^2$$



This is a quadratic equation. You might recall that

$$\text{if } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Writing our equation in that form, we can say

$$4.9t^2 - 20t - 380 = 0 \text{ so}$$

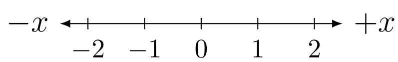
$$t = \frac{20 \pm \sqrt{20^2 - 4(4.9)(-380)}}{2(4.9)} = \frac{20}{9.8} \pm \frac{\sqrt{7848}}{9.8} = 2.04 \pm 9.04 = 11.08\,\text{s}$$

since only the + sign makes sense, giving a positive amount of time.

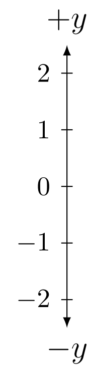
Day 2, Hour 2: Two-dimensional motion and vectors

3.1 Kinematics in two dimensions: introduction

In chapter 2 we looked at one-dimensional motion in different situations. The table below summarizes what we needed for motion with no acceleration, and and for free-fall motion:

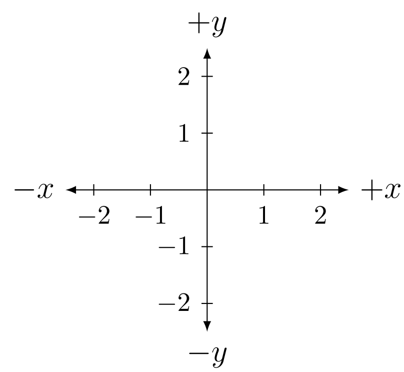


One-dimensional motion	
No acceleration	Free fall
$x = x_0 + v_0 t$	$y = y_0 + v_0 t - \frac{1}{2} g t^2$
$v = v_0 = \text{constant}$	$v = v_0 - gt$
	$v^2 = v_0^2 - 2g\,(y - y_0)$



In chapter 3 we combine these separate sets of equations, and use them both at once. Two-dimensional motion is a combination of unaccelerated horizontal motion and free-fall vertical motion, happening simultaneously. So these two sets of equationst are connected by using the same time t in both.

Two-dimensional motion	
Horizontal	Vertical
$x = x_0 + v_{0x} t$	$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$
$v_x = v_{0x} = \text{constant}$	$v_y = v_{0y} - gt$
	$v_y^2 = v_{0y}^2 - 2g\,(y - y_0)$



- The only change made in the equations is that subscripts x and y are attached to the velocities. This is because the velocity vector can be thought of as having two separate parts, or components: a horizontal x component, and a vertical y component. The two components together tell us all about the velocity.
- Likewise, the x and y values are components of the displacement vector, so together they tell us the displacement.

This component approach to describing vectors is explained in the next section. Regarding notation:

- The book uses boldface type to indicate that something is a vector: for example, $\boldsymbol{A}$. In these notes I can do likewise, but I can't do this very well on the blackboard, so I might show it as $\vec{A}$.
- The magnitude of a vector $\boldsymbol{A}$ is indicated by just the letter A, not in boldface. So $\boldsymbol{A}$ is a vector, but A is a scalar.
- The direction of a vector is commonly shown by an angle θ ("theta" is the Greek letter for a "th" sound) measured counterclockwise from the x or horizontal axis of a coordinate system.

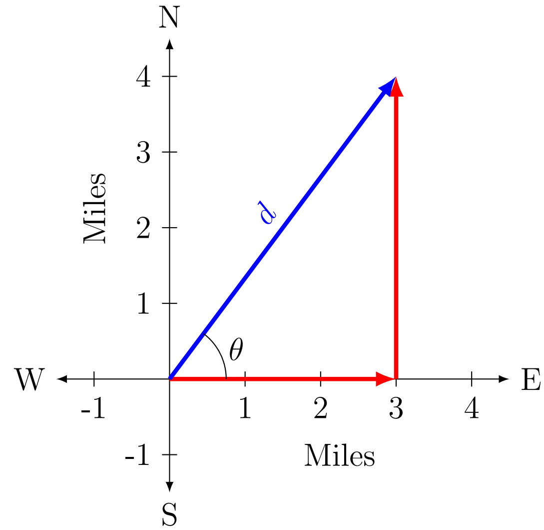
3.2 – 3.3 Vector addition and subtraction

Because displacement is a vector, it has both magnitude and direction. But describing the vector in terms of magnitude and direction is not the only way to be specific about it.

- Instead of specifying magnitude and direction, it is often easier to specify its components — the amount of displacement in two perpendicular directions, such as East and North, or the x and y directions.

Examples:

1. Suppose you ride your bike 3 miles East, then you turn left and ride another 4 miles North. What is your displacement from the place you started?
   The two parts of the trip are shown in the graph at the right as red vectors, one with a magnitude of 3 and direction East, the other with a magnitude 4 in the direction marked North. These two displacement vectors add up to one vector shown in blue. How can we specify this displacement vector?



   (a) It has a magnitude that is easy to calculate because it is the hypotenuse of a right triangle:
   $$d = \sqrt{3^2 + 4^2} = 5 \text{ miles.}$$

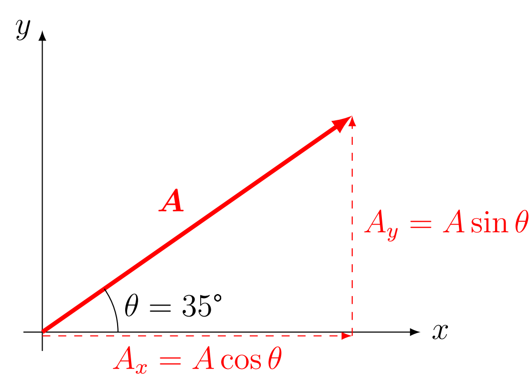
   (b) For a direction, we might say it is sort of northeast, but that is not specific enough. Instead, we can say it goes at an angle θ:
   $$\theta = \arctan\left(\frac{4}{3}\right) = 53.1° \text{ North of East.}$$
   (Note: "arctan $\left(\frac{4}{3}\right)$" means "the angle whose tangent is 4/3." Make sure you know how your calculator does this kind of calculation.)

   So the displacement vector could be described in either of two ways:

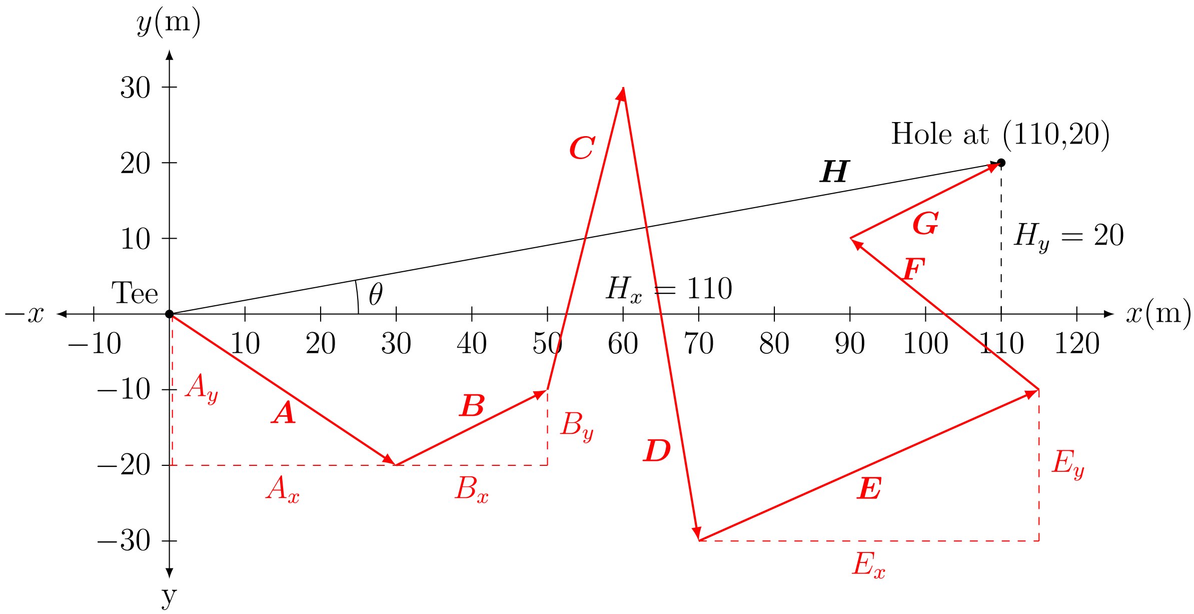
   - 5 miles at 53.1° North of East (magnitude and direction).
   - 3 miles East, 4 miles North (perpendicular components).

   Most of the time we will use x-y coordinates, and you should be able to find the components of a vector, or its magnitude and direction, using trigonometry.

2. Find the components of this vector $\boldsymbol{A}$ that has a magnitude of 5 units at an angle of 35° from the x axis as shown.
   Solution:
   $$A_x = A\cos\theta = 5\cos(35°) = 4.096$$
   $$A_y = A\sin\theta = 5\sin(35°) = 2.868$$
   If this trigonometry is unfamiliar, you should review the basic functions (sine, cosine, and tangent) so you can use them effectively. You will get some practice in this course..



3. Suppose Boris tries golfing for the frst time, and doesn't quite understand how to do it. The tee is at the origin, and the hole is marked at 100 m away along the x axis and 20 m in the y direction. A displacement vector $\boldsymbol{H}$ that goes from the tee to the hole would be a good shot — a hole-in-one.



- Instead, Boris hits the ball from the tee to the end of vector $\boldsymbol{A}$.
- From there he hits again, and vector $\boldsymbol{B}$ shows the next displacement of the ball.
- Vectors $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{E}$, and $\boldsymbol{F}$ are his next few attempts.
- He finally gets the ball into the hole with vector $\boldsymbol{G}$.

Each of the vectors $\boldsymbol{A}$ through $\boldsymbol{H}$ has a magnitude and a direction shown by an arrow. But each one is also completely described by its two components in the x and y directions. For a few of these vectors, the x and y components are also shown as dashed lines. They are not all shown, to keep the diagram from getting too cluttered, but you can imagine the others.

What is the ball's displacement vector ($\boldsymbol{H}$) when it arrives at the hole?

- The magnitude and direction of vector $\boldsymbol{H}$ are

$$H = \sqrt{(110)^2 + (20)^2} = 111.8\,\text{m}, \text{ and } \theta = \arctan(20/110) = 10.3°.$$

- Alternatively, we can specify the vector $\boldsymbol{H}$ by giving its x and y components:

$$H_x = 110\,\text{m}, \text{ and } H_y = 20\,\text{m}.$$

Another way to describe what happened is to say that all the displacement vectors were added:

$$\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} + \boldsymbol{D} + \boldsymbol{E} + \boldsymbol{F} + \boldsymbol{G} = \boldsymbol{H}.$$

But this means that all the x components were added and the y compnents were added:

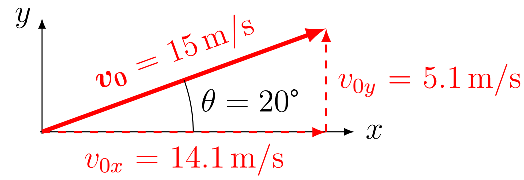
$$A_x + B_x + C_x + D_x + E_x + F_x + G_x = H_x$$

$$A_y + B_y + C_y + D_y + E_y + F_y + G_y = H_y$$

The text shows in section 3.2 how you can use a ruler and protracter to add vectors graphically, but for us, knowing trigonometry makes the use of x and y components much easier.

4. We can do the same things with vectors that happen to be velocity instead of displacement:

Example: A soccer ball is kicked so that its initial velocity vector $\boldsymbol{v_0}$ is 15 m/s at an angle of 20° upward from horizontal. What are the x and y components of this initial velocity?



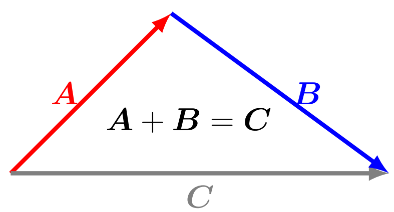
Solution: $v_{0x} = v_0 \cos\theta = 15\cos(20°) = 14.1\,\text{m/s}$

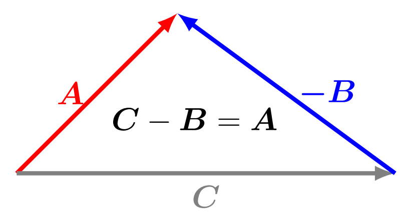
$v_{0y} = v_0 \sin\theta = 15\sin(20°) = 5.1\,\text{m/s}$

Of course, these are only the initial components of the ball's velocity.

- Because there is gravity downward, the vertical velocity component will change with time.
- Because there is not horizontal gravity, the horizontal velocity component stays constant.

5. Subtracting vectors. If $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{C}$ then we can also say $\boldsymbol{A} = \boldsymbol{C} - \boldsymbol{B} = \boldsymbol{C} + (-\boldsymbol{B})$, as shown.





Subtracting a vector means adding a vector of the same magnitude but with the opposite direction.

5-minute Quiz #2

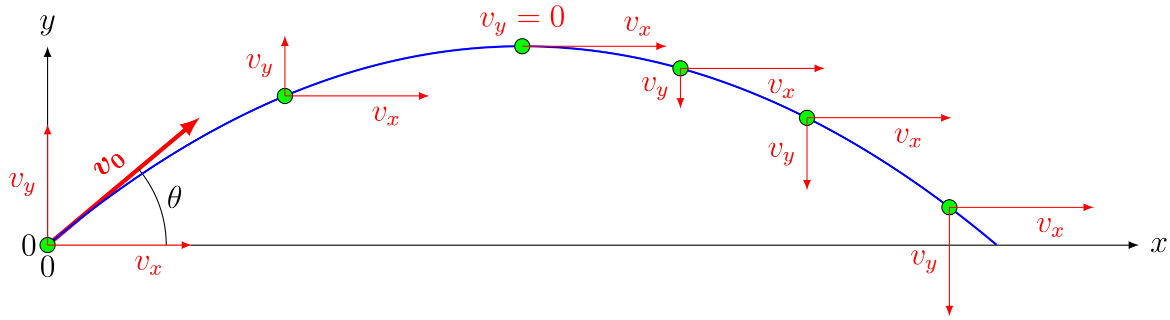
Day 2, Hour 3: Projectile motion

3.4 Projectile motion

Now that we know how to break up a displacement or velocity vector into x and y components, we can use this set of equations for two-dimensional motion:

Two-dimensional motion	
Horizontal	Vertical
$x = x_0 + v_{0x}t$	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$
$v_x = v_{0x} = \text{constant}$	$v_y = v_{0y} - gt$
	$v_y^2 = v_{0y}^2 - 2g\,(y - y_0)$

Suppose a green jelly bean is launched at a speed v_0 at an angle θ from horizontal. Its initial velocity vector is pictured in red, but the jelly bean does not continue upward at this angle. It travels along a curve like the one sketched in blue. The points (x, y) along this curve are given by the equations for $x(t)$ and $y(t)$. A few are highlighted in as green dots. At each point, the velocity vector has components that are given by $v_x(t)$ and $v_y(t)$. While v_x stays constant, v_y changes from upward to downward because of gravity. At the top of the flight path, $v_y = 0$. The path is symmetric which makes the analysis easier.



Now, there are a number of questions we might want to ask about the flight of the jelly bean, such as the following ones:

1. At what time will it return to the ground?

   - Set $y = 0$ at the ground, and $v_{y0} = v_0 \sin\theta$, and solve the $y(t)$ equation for t. You should be able to show that
   $$t = 2\frac{v_0 \sin\theta}{g}$$

2. What maximum height will it reach? There are several approaches:

   - Knowing $v_y = 0$ at the top of the flight, solve for y with the $v(y)$ equation.
   - Calculate the time at which $v_y = 0$, then calculate $y(t)$.
   - Use the fact that at the top, t would be half the total flight time, and solve for $y(t)$.

3. What horizontal distance will it travel before reaching the ground?

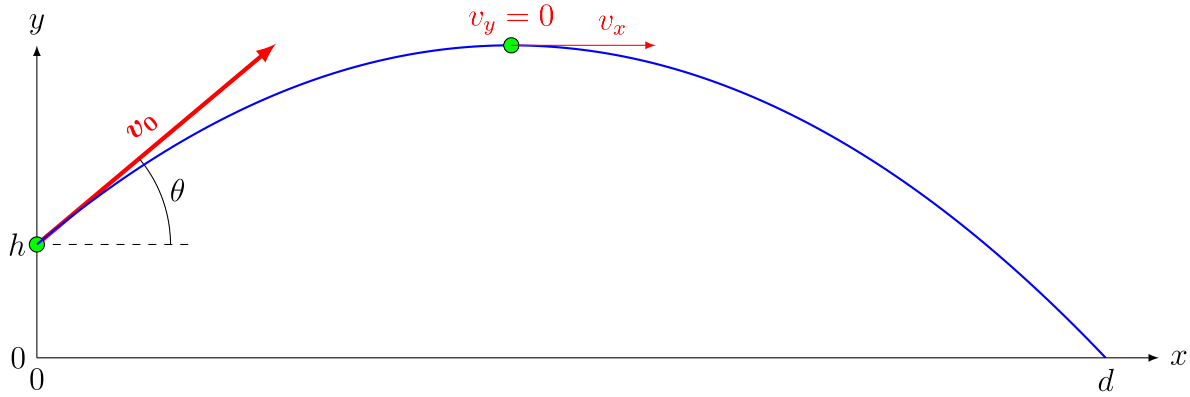
   - Again, if $y = 0$ at the ground, you can find the time t for the flight, and use it in the $x(t)$ equation. This distance is known as the range: you should be able to show that the range is
   $$R = \frac{2v_0^2 \sin\theta\cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \quad [\text{because } 2\sin\theta\cos\theta = \sin(2\theta)]$$
   - The largest value of $\sin(2\theta)$ is 1, when $2\theta = 90°$, so R is maximum when $\theta = 45°$. For larger or smaller launch angles the projectile would not go as far; this is not true if air resistance is present however.

4. How fast will it be going when it hits the ground?

   - In free-fall, with no air drag, the flight is symmetric. For example, it takes the same time to go up and down, and it has similar speeds (but opposite directions) when falling as it had when rising.

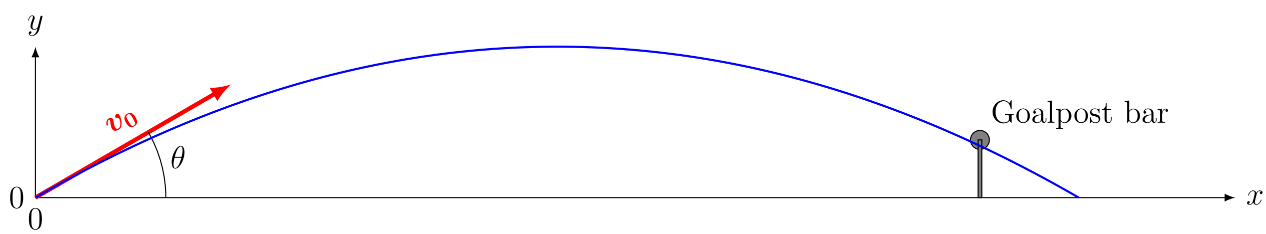
Things get a bit more complicated if the flight is not a symmetric one from the ground to the ground:

- A projectile is launched from a height h, with speed v_0 at angle θ. The flight time and range are no longer the same as when the projectile was launched from the ground. We encounter quadratic equations in these cases. For instance, the flight time turns out to be $\frac{v_{0y}+\sqrt{v_{0y}^2+2gh}}{g}$; from this the distance d is easy to get because $x = v_{0x} \cdot t$.



In some cases, you have to find out something you don't really want to know, in order to find out what you do want to know. Here is an example:

1. A football is kicked from the ground with a speed of 25 m/s at an angle of 30°. Will it go over, or under a goalpost bar that is 3.048 m high (that's 10 feet) at a distance of 50 m away?



- We need to know the height y for the ball when it has travelled a distance $x = 40\,\text{m}$.
- So we must first find that time; we can use $x = 40\,\text{m}$ to find the time t when the ball has travelled that far:

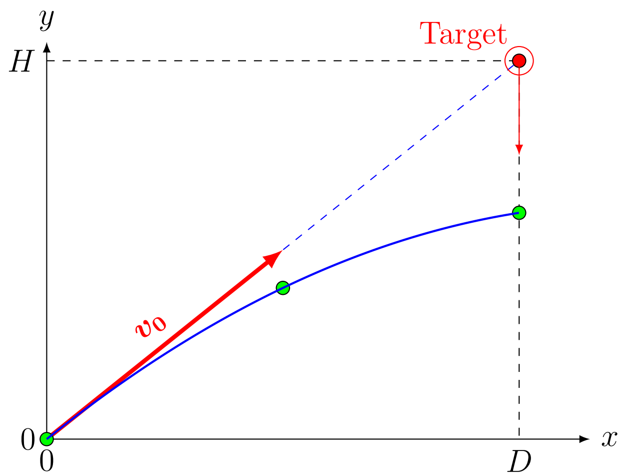
$$x = v_{0x}t \Rightarrow t = x/v_{0x} = x/(v_0 \cos\theta) = 40/(35 \cos 28°) = 1.294\,\text{s}$$

- Now that we know the time, we can solve for the height of the ball:

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2 \\ &= 0 + (35)(\sin 28°)(1.294) - (0.5)(9.8)(1.294^2) \\ &= 2.73\,\text{m} \;\; \text{(it goes under the bar.)} \end{aligned}$$

- If the ball was kicked slightly faster (25.2 m/s) or at a slightly larger angle (30.5°) it would get over the bar.

2. Suppose a marble is aimed directly at a target that is at a height H, and a horizontal distance D away, as shown. But at the same instant the marble is launched, the target falls. Will the marble hit or miss the target?



Just like in the previous example, we can find

- the time for the marble to go distance D horizontally is $t = D/(v_0 \cos\theta)$.
- the height of the marble at that time:

$$y_m = (v_0 \sin\theta)\left(\frac{D}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{D}{v_0\cos\theta}\right)^2$$
$$= D\tan\theta - \frac{g}{2}\left(\frac{D}{v_0\cos\theta}\right)^2$$
$$y_m = H - \frac{g}{2}\left(\frac{D}{v_0\cos\theta}\right)^2 \quad \text{since } \tan\theta = D/H$$

- But the height of the target at that time, since it fell from rest from height H is also:

$$y_t = H - \frac{1}{2}gt^2 = H - \frac{g}{2}\left(\frac{D}{v_0\cos\theta}\right)^2 = y_m$$

So we should expect the marble to hit the target; they are at the same place at the same time.

Homework Assignment 2:

Several problems on Canvas/WebAssign for practice with free-fall motion, vectors and projectiles.
Due Thursday 9/3 @ midnight. Email questions or ask in class.

Time	Topics	Assignments
12:30 - 1:20	Newton's laws	Quiz #3
1:30 - 2:20	Applying Newton's laws	
2:30 - 3:30	Friciton and fluid drag	

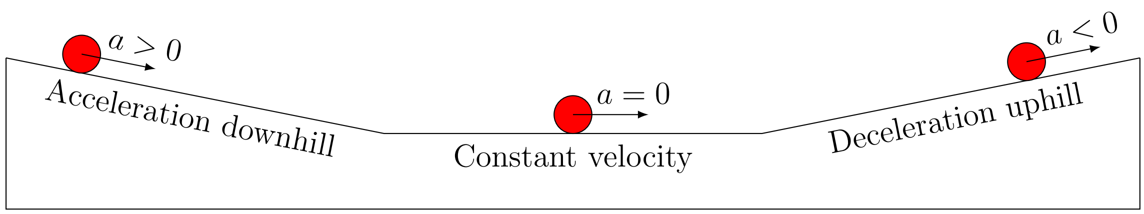
Day 3, Hour 1: Newton's laws

Now that we know quite a bit about how things move, in terms of displacement, velocity, and acceleration, next, in Chapter 4 we begin looking at why things move. This is a subject that has interested people for a long time. For example, Aristotle, the Greek philosopher who lived in the 300s B.C. as part of his philosophy, divided motion into two categories:

- Natural motion, for things like planets in orbit, or smoke rising. Nothing is needed to cause their motion, because it is natural for them to move.
- Unnatural motion, for everything else, requires a force to maintain it. It is unnatural for a cart to move, so a horse is needed to pull it.
- Aristotle taught that heavier objects would fall faster than lighter ones of the same shape.

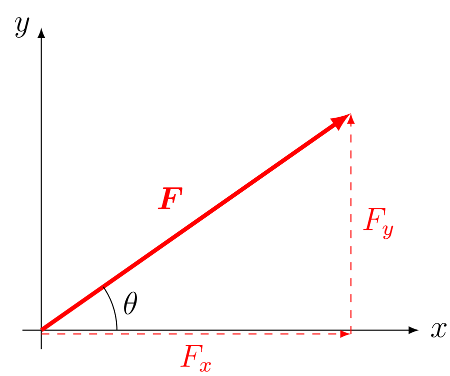
Of course, Aristotle had plenty more to say about these things, and his opinions were generally accepted for about 2000 years. In the 1600s, Galileo experimented on motion, and differed with Aristotle.

- He understood that, apart from air resistance, all masses accelerate alike due to gravity.
- He found that a force is needed to change how something moves, not to maintain motion.
  - Experiments with objects rolling on inclined planes convinced him that, if friction is eliminated, a particle moving on a smooth horizontal surface continues moving with constant velocity.



4.1 Force

A force is simply a push or a pull, and we will look at various ways that forces occur. Force is a vector quantity — that is, a force has both a magnitude and a direction. As with other vectors, we will often need to consider the components of a force in perpendicular directions such as x and y, or East and North, and to relate these to the magnitude and direction of the force.



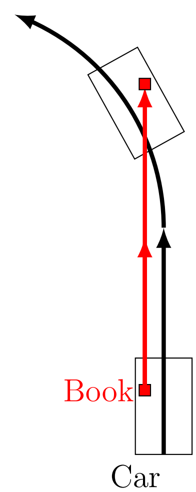
$$F = \sqrt{F_x^2 + F_y^2}, \quad F_x = F\cos\theta, \quad F_y = F\sin\theta, \quad \tan\theta = \frac{F_y}{F_x}$$

4.2 Newton's first law

Isaac Newton (1642 – 1727) was taught Aristotle's theories of motion as a college student at Cambridge, but was also familiar with the writings of Galileo. Newton's first law of motion is the one Galileo found:

$$\boldsymbol{v} = \text{ constant unless a force changes it.}$$

- This seems obvious in the case of an object at rest — it stays at rest until a force moves it.
- It is less obvious for an object in motion; our experience may suggest what Aristotle believed, that a force is needed to maintain motion, not to change it.
  - But if we take steps to reduce forces such as friction and air resistance, we see that it is these kinds of forces that are acting to change the motion of the object. When they are eliminated, Newton's first law turns out to work fine.
- Newton's first law is often called the law of inertia, where the word inertia means this property that objects have of maintaining a constant velocity.
  - Another word for inertia is mass. The larger the mass of an object, the more force is needed to accelerate it.
- The first law does not seem to work from the point of view of an observer who is accelerating:
  1. For instance, imagine driving a car with your physics book sitting on the slippery dashboard.
     - As long as you go in a straight line at constant speed, the book stays in place, having a constant speed of zero relative to the car.
     - But if you turn a corner, the book may go sliding across the dash toward the outside of the turn. To account for this strange behavior, one might suppose there is a force that only happens on turns — this gets the name "centifugal force." But it only seems to be present if one's frame of reference is the car, which is no longer moving at constant velocity. The car is a non-inertial reference frame when it is turning, and that is where fictitious forces appear to act.



     - From outside the car (say, looking down at the car from a tall tree), one could see the book travelling with constant velocity until the side of the car causes $\boldsymbol{v}$ to change by exerting an inward force.
  2. Similarly, if the car begins to accelerate forward in a straight line, the book might slip off the dash toward the back of the car, and the heads of passangers might tilt back.
     - In the non-inertial reference frame of the car, these motions would require an explanation involving some backward forces on the book and the passengers.
     - But an observer moving at constant velocity would see the book moving at constant velocity until a force (such as the car seat it landed on) causes it to accelerate, and peoples' heads accelerated forward only when their neck muscles exert a force that way.
- Newton's first law is obeyed when the motion is seen from the point of view of a non-accelerating observer, otherwise known as an inertial reference frame. But in a non-inertial reference frame, additional forces seem to be at work without any physical explanation for their origin.

4.3 Newton's second law

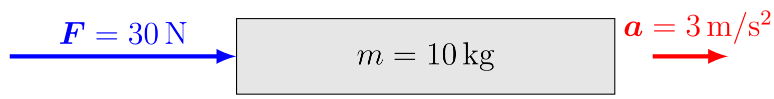
The second law is one will use a great deal in this course. While the first law affirms that a force causes a change in vleocity, the second law tells how this works. As the result of a force $\boldsymbol{F}$ on an object, there is an acceleration $\boldsymbol{a}$ that is proportional to $\boldsymbol{F}$, and the mass m of the object determines how large $\boldsymbol{a}$ is.

$$\boldsymbol{F} = m\boldsymbol{a}.$$

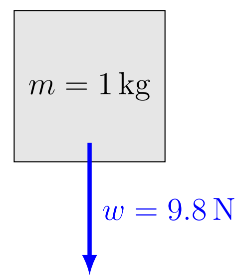
- The larger the mass, the smaller the acceleration when the same force is applied.
- The standard unit of mass is the kilogram (kg), and the standard units for acceleration are $\mathrm{m/s^2}$, so the unit of force is defined by the product of these, and called the newton, abbreviated as N.

  $$1\,\mathrm{kg}\cdot\mathrm{m/s^2} = 1 \text{ newton} = 1\,\mathrm{N}$$

  So a force that causes 10 kg of mass to accelerate at $3\,\mathrm{m/s^2}$ would have a magnitude of 30 N.

- Another common force unit is the pound (lb): their relationship is that $1\,\mathrm{N} = 0.225$ lb.
- The force exerted on an object by gravity is, of course, its weight, so, using the acceleration g due to gravity at the earth's surface, the weight of an object with mass m is

  $$w = mg.$$

  Thus, one kilogram on earth has a weight of 9.8 N.

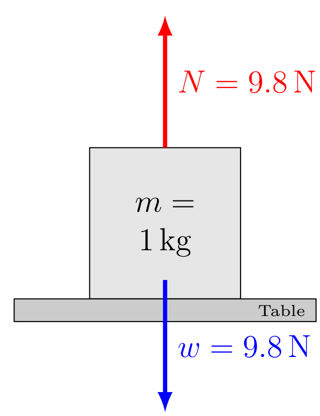
  – The mass of the object would be the same anyplace, but g would be smaller on the moon ($1.67\,\mathrm{m/s^2}$), larger on Jupiter ($24.8\,\mathrm{m/s^2}$), because g depends on the size and mass of the world.

- There may be more than one force acting on an object, and so it is best to write the law in terms of the net force, or the total of all the force vectors. The sum of the force vectors is also expressed as $\sum \boldsymbol{F}$, so the law states

  $$\boldsymbol{F}_{net} = m\boldsymbol{a} \qquad \text{or} \qquad \sum \boldsymbol{F} = m\boldsymbol{a}.$$

  Thus $\boldsymbol{a}$ is in the direction of the total force; we will often need to add force vectors, as you learned about in chapter 3.

- Here is a case where there are two forces acting: suppose the 1 kg mass is sitting on top of a table. The weight $w = 9.8\,\mathrm{N}$ is a downward force, but the object is not accelerating at all. Why not?

  – The net force on it must be zero.

  – The weight is downward, so there must be an upward force. This is due to the table; if the table is removed, the object will be in free fall.

  – The upward force exerted by the table is called the normal force $\boldsymbol{N}$.

  – The word "normal" is used in a mathematical sense, meaning "perpendicular"; the force is normal to the surfaces in contact. This will be explained a bit more later in connection with Newton's third law.

4.4 Newton's third law

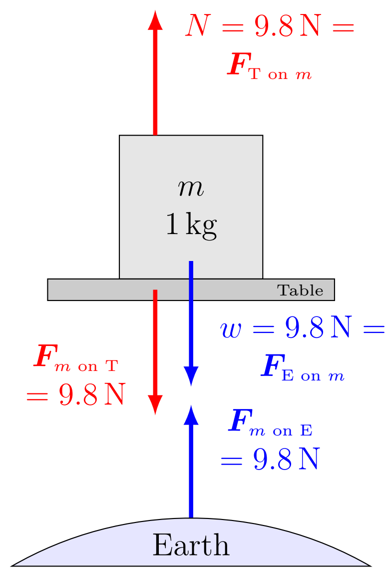
The third law is sort of a strange one, but necessary for understanding motion. It comes into play whenever we want to understand the interaction of two objects. The third law states that whenever one object exerts a force on another, it receives an equally large force going the opposite way.

- If object A pushes on object B, then object B also pushes on object A with a force of the same magnitude, in the opposite direction. We can write this as

$$\boldsymbol{F}_{A\,\text{on}\,B} = -\boldsymbol{F}_{B\,\text{on}\,A}$$

  - If you push on the wall, the wall pushes back with equal force on you. Try it on roller skates.
  - If you push on the floor, the floor pushes back on you just as hard. You can increase your force on the floor: think about how you jump, and the forces between you and the floor.
  - How do you climb a rope? How do you climb stairs?

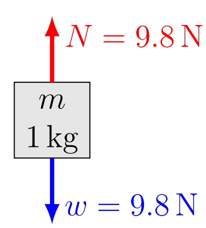
In a previous example, we saw two opposite but equal forces when an object rests on a table. It is easy to be confused about these forces — they are not what the third law is about. These are two forces on the same object. But now we can find two more forces to put into the picture.



- The weight $w = 9.8\,\text{N}$ is the force of the Earth pulling down on the 1 kg mass. The third law says the mass m therefore pulls up on the Earth with a force of 9.8 N.
- The normal force $\boldsymbol{N}$ is the table pushing up on the mass. This is because the mass is pushing down on the table.
- So the two red vectors show the third-law pair of forces involving the mass/table interaction, and the two blue vectors represent the mass/Earth interaction.

We could add more: the table needs legs, and maybe it stands on the floor of a house, etc. The picture could become very complicated with all the pairs of forces we could imagine. But, as we'll see, to understand how something moves, we only need the forces on that object.

Free-body diagrams



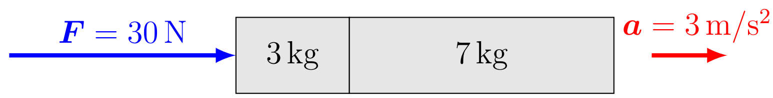
A free-body diagram, showing only the forces on the object of interest, is usually the first step in understanding the motion of the object. In the previous example, if we are interested in how mass m moves, there are only two forces on it: the weight $\boldsymbol{w}$ and normal force $\boldsymbol{N}$. All the forces extered by m on other objects (the earth, the table) are irrelevant.

After making a free-body diagram, then the usual procedure is to apply Newton's second law: sum the force vectors, and set that sum equal to $m \cdot \boldsymbol{a}$. Usually we will say up is the positive direction, down is negative. In this case

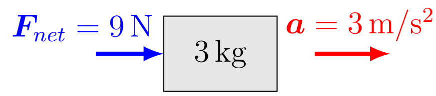
$$\sum \boldsymbol{F} = N - w = 9.8 - 9.8 = 0 = m\boldsymbol{a} \Rightarrow \boxed{\boldsymbol{a} = 0.}$$

Newton's third law is not always needed,especially if we are only interested in one object's motion. But it gives important insights when objects interact.

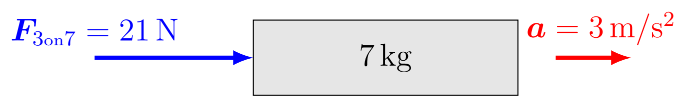
Earlier we looked at an example of a 30 N force on a 10 kg mass, causing $a = 3\,\mathrm{m/s^2}$. But suppose the 10 kg mass is made of two parts, with masses 3 kg and 7 kg, as shown, and we think about what each of these masses is doing using a free-body diagram for each.



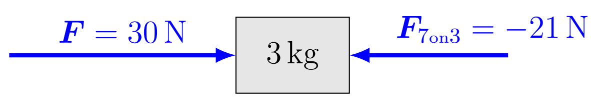
- When we see a 3 kg mass accelerated at $3\,\mathrm{m/s^2}$, the second law says there must be a net force of 9 N acting on it.



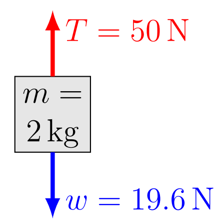
- The force that would cause the 7 kg mass to accelerate at $3\,\mathrm{m/s^2}$ is 21 N. But this must come from the 3 kg mass pushing on it, so we know $\boldsymbol{F}_{3\mathrm{on}7} = 21\,\mathrm{N}$.



- According to the third law, the 7 kg mass pushes back on the 3 kg mass with a force $\boldsymbol{F}_{7\mathrm{on}3} = -21\,\mathrm{N}$. So the net force on the 3 kg mass is indeed 9 N as expected.



As another example, suppose we tie a rope to a mass $m = 2\,\mathrm{kg}$ and pull upward on it with a force of 50 N. How will it accelerate? A free-body diagram is shown.



- The weight is $w = mg = 19.6\,\mathrm{N}$ downward, as always.
- The rope is pulling upward on the mass — this force is labeled as T, the tension in the rope.
  - Incidentally, tension goes both ways on a string, because of Newton's third law. If a hand pulls up on the rope, the rope pulls down on the hand. That is not part of the free-body diagram here however.

- Newton's second law gives the acceleration. Since all the forces are in the y direction, we can say

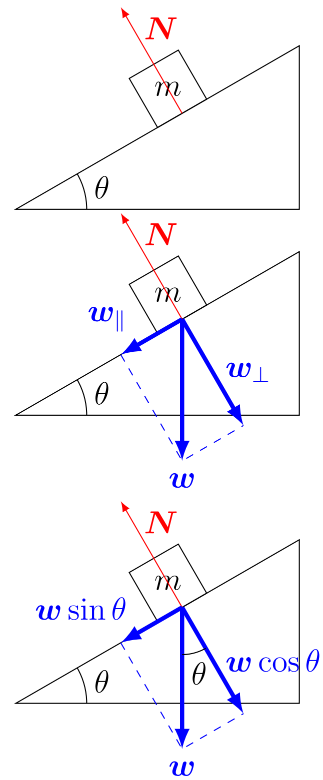
$$\sum \boldsymbol{F} = T - w = 50 - 19.6 = 30.4\,\mathrm{N} = ma = (2\,\mathrm{kg})a \Rightarrow \boxed{a = 15.2\,\mathrm{m/s^2}.}$$

5-minute Quiz 3

Day 3, Hour 2: Applying Newton's laws

Now that we have seen Newton's three laws, and how they connect with one another, we can get some practice applying them in various ways, and learn more about the nature of forces.

4.5 - 4.6 Normal force, tension, problem solving



One interesting type of force we have already seen is the normal force $\boldsymbol{N}$ that occurs when surfaces are in contact — the force is normal, or perpendicular, to the surfaces. We saw this applied to the case of a mass on a horizontal surface. Now consider an object placed on a hill that is tilted at an angle θ from horizontal.

- The normal force of the hill on the mass m is now directed as shown, perpendicular to the surfaces in contact.
- How large is $\boldsymbol{N}$? It is as large as the force that presses the object against the hill, and we can find that using a little geometry.
- The weight $\boldsymbol{w}$ must be vertically downward, as shown, but it is helpful to find its two components in directions parallel to the hill and perpendicular. The perpendicular one, $w_\perp$, gives N. The parallel one, $w_\parallel$ is also useful, as we shall see.
- Because the three angles in a triangle add to 180°, you can pretty easily see that θ is also the angle between $\boldsymbol{w}$ and its component perpendicular to the hill surface.
- Then, knowing a little trigonometry, you can see that the important components of $\boldsymbol{w}$ are

$$\boldsymbol{w}_\perp = \boldsymbol{w}\cos\theta \text{ and } \boldsymbol{w}_\parallel = \boldsymbol{w}\sin\theta.$$

  They are important because

  1. the normal force has the same magnitude as $w_\perp$: $N = w\cos\theta$; this will be needed when we look at the force of friction when one surface moves across another.
  2. the parallel component $w_\parallel$ is the force that would accelerate the object down the hill.

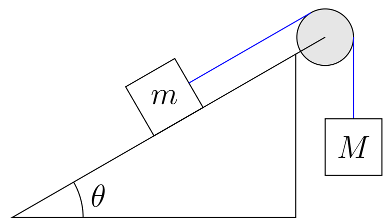
Example:

1. A mass m is placed on a slope of 30° where there is no friction. What will be its acceleration down the hill?

   - We can divide $\boldsymbol{a}$ into parallel and perpendicular components as we have done for the forces, so Newton's second law gives

$$ma_\perp = N - w\cos\theta = 0 \Rightarrow \boxed{a_\perp = 0.}$$
$$ma_\parallel = w\sin\theta = mg\sin\theta = m(9.8)30° \Rightarrow \boxed{a_\parallel = 4.9\,\mathrm{m/s^2}.}$$

2. A mass m is on a hill sloped at angle θ, and attached to it is a string that goes over a pulley, and another mass M is hanging on the string.

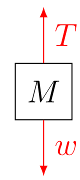


   (a) What will be the acceleration of these masses?

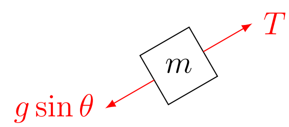
   (b) What will be the tension in the string?

For each moving mass, draw a free-body diagram and write Newton's second law:

- For the hanging mass,
$$T - Mg = Ma.$$
As usual, we are letting up be the positive direction, and down is negative. So if a is positive, then M goes up. This equation has both unknown quantites, a and T in it, so we cannot solve for them yet.



- For the sliding mass, the choice of signs is less clear. If a is positive M goes up, but m goes down the hill. So if we want to use the symbol a for both, we need to say that <u>T is in the negative direction for this mass:</u>



$$mg\sin\theta - T = ma$$

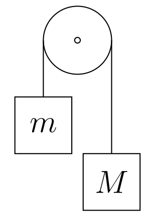
which again has both unknown quantities in it.

From these two equations some algebra will give both unknowns. First, getting T alone in each equation then setting them equal, we have
$$T = Ma + Mg = mg\sin\theta - ma$$
Then putting the terms with a together,
$$a(M+m) = g(m\sin\theta - M)$$
$$\Rightarrow \boxed{a = g\left(\frac{m\sin\theta - M}{M+m}\right).}$$
Hopefully this result should make sense: M goes up or down depending on whether it is smaller or larger than $m\sin\theta$. We can now also solve for T, using what we found for a to write
$$T = Ma + Mg = Mg\left(\frac{m\sin\theta - M}{M+m} + 1\right) = Mg\left(\frac{m\sin\theta - \cancel{M} + \cancel{M} + m}{M+m}\right)$$
$$\Rightarrow \boxed{T = Mg\cdot\frac{m\,(1+\sin\theta)}{M+m}.}$$
Again, this might be larger or smaller than Mg, pulling either M or m upward.

Both of these results were probably not obvious from looking at the original diagram. It was necessary to look at teach of the masses separately, in two free-body diagrams, to get the right pair of equations. It is also often necessary to make sure you are consistent with symbols and signs when you set things up. The algebra can also be challenging, especially with symbols instead of numbers; in this case we still won't know what happens without being given numbers for m, M and θ, but we can see what the possibilities are.

Essentially the same procedure would be used for a situation like this following one:

3. Two masses, m and M are attached by a string that goes over a pulley.



   (a) What will be the acceleration of the masses?

   (b) What will be the tension in the string?

   You will have to decide what you mean by the symbol a, and then use it consistently. You should be able to get

$$a = g\frac{m-M}{m+M} \text{ (or the negative of this) and } T = Mg\frac{2m}{M+m}.$$

   - Think about what these results mean when m and M are equal.
   - What would happen in the previous example if the slope of the hill was increased to $\theta = 90°$?

4.7 Force components and equilibrium

Next, let's look at situations of equilibrium, in which there is no acceleration. According to Newton's second law

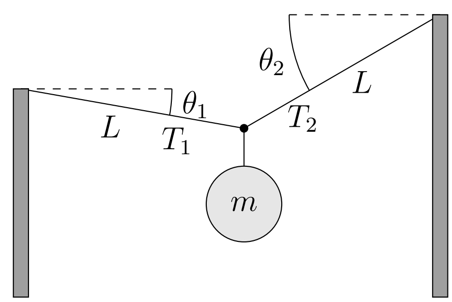
$$\sum \boldsymbol{F} = m\boldsymbol{a}$$

but this means the compnents of $\boldsymbol{F}$ and $\boldsymbol{a}$ must also satisfy

$$F_x = ma_x \text{ and}$$
$$F_y = ma_y.$$

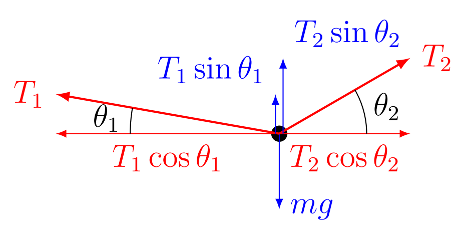
So if an object is not accelerating, we can say that the forces in the x direction and in the y direction separately add to zero. Here are some examples:

1. A mass m is hanging from two wires of length L attached to posts of different heights, as shown. The wires make different angles from horizontal at each end. What are the tensions T_1 and T_2 in the wires?



   - For a free-body diagram, we can use the point where the hanging mass is attached to the wires.
   - The y-components of the tensions must balance the weight of m:

$$\sum F_y = T_1 \sin\theta_1 + T_2 \sin\theta_2 - mg = 0$$



   - And the horizontal components of the tensions must cancel:

$$\sum F_x = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$$

   - Then, some algebra allows us to solve for the two unknown tensions:

$$T_1 = T_2\frac{\cos\theta_2}{\cos\theta_1} \Rightarrow T_2\left(\sin\theta_2 + \frac{\cos\theta_2}{\cos\theta_1}\sin\theta_1\right) = mg \Rightarrow \begin{cases} T_2 = \frac{mg\cos\theta_1}{\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2} = mg\cdot\frac{\cos\theta_1}{\sin(\theta_1+\theta_2)} \\ T_1 = mg\cdot\frac{\cos\theta_2}{\sin(\theta_1+\theta_2)} \end{cases}$$

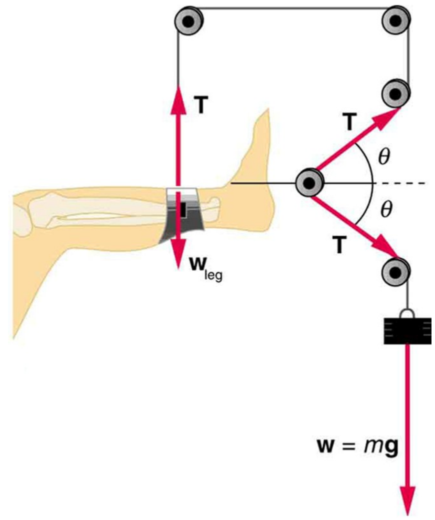
   where the trig identity $\sin A\cos B + \cos A\sin B = \sin(A+B)$ is used to simplify the results.

2. Our text shows a traction system for both supporting a leg horizontally, and applying a horizontal force along the shin bone (the tibia).

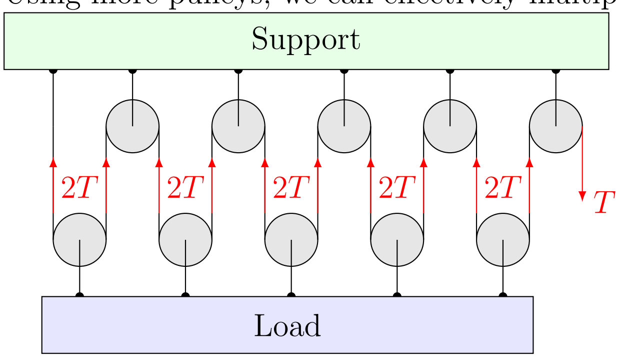
   (a) What is the tension in the rope?
   Answer: $w_{leg} = T = mg$. The tension is the same everywhere along the rope; the pulleys only change the direction.

   (b) What is the force pulling on the tibia?
   Answer: $2T\cos\theta$ because two ropes pull on it at angle θ.

   (c) How could the force on the tibia be increased?
   Answer: smaller θ would give larger horizontal components of T.



3. Using more pulleys, we can effectively multiply the force that a rope will apply to support a load.



- The rope that winds through all these pulleys (assuming no fricition) has the same tension T everywhere.
- The lower pulleys are each pulled upward by two segments of the rope, so there is a force $2T$ upward on each one.
- With 5 pulleys attached to the load, there is an upward force of $10T$ on it.

4.8 Four forces

Read this section if you like, but you won't be tested on any of it. It describes and compares the four kinds of forces known in physics: gravitational, electromagnetic, and two short-range nuclear forces.

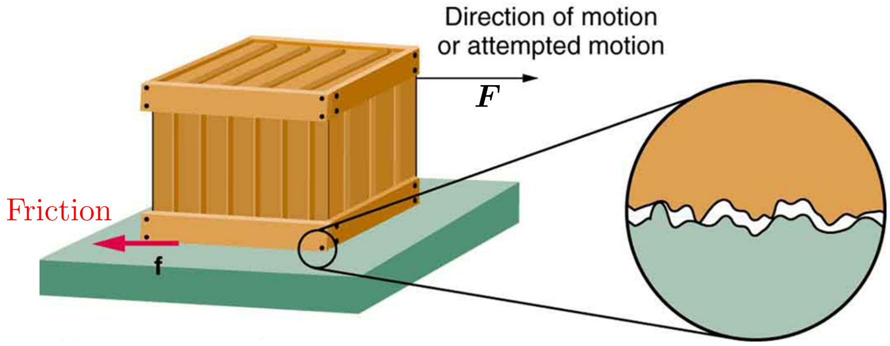
In PHYS 105 topics, the force of gravity is obviously important. The other forces we encounter, such as the normal force between surfaces in close contact, the tension in a rope or other solid material, and the elastic force of a spring or other stretchy material, all turn out to be electromagnetic forces at work. Atoms carry electric charges that repel when pressed togther, and attract when pulled apart.

Day 3, Hour 3: Friction and fluid drag

We know how forces cause acceleration, and have looked at forces that include gravity, tension and the normal force. Next, we will look at two forces that always resist motion.

5.1 Friction

Friction opposes relative motion between two surfaces. If a box is pulled to the right across the floor, friction resists this motion with a force to the left on the box, as illustrated in Fig. 5.2 of our text. The microscopic view shows why this happens: because of the roughness of each surface, when they are in relative motion there are collisions between bumps on the two surfaces.



The larger the force pressing these surfaces together, the larger the amount of friction. As we have already learned, the normal force N is that force of one surface on the other. So it turns out that the friction force f is proportional to N:

$$f \propto N.$$

But instead of a proportionality, an equation is more useful. So there is a number, called the coefficient of friction, designated by μ ("mu" is the Greek letter for the the m sound), that relates f to N:

$$f = \mu N.$$

Since both f and N are measured in newtons, the coefficient of fricition is a unitless number. Here are a few examples from Table 5.1 of the text:

	Coefficients of friction	
Surfaces	Static μ_s	Kinetic μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Teflon on steel	0.04	0.04
Bone joints	0.016	0.015

Notice that there are two columns of μ values.

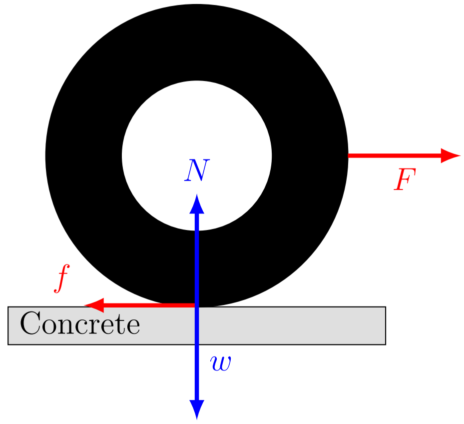
- The static value μ_s tells how well the surfaces can withstand the beginning of relative motion when they are at rest.

$$f_s \leq \mu_s N$$

- The kinetic value μ_k tells how strong is the friction when the surfaces are already in relative motion.

$$f_k = \mu_k N$$

To illustrate how this works, suppose we try to drag a rubber tire weighing 200 N across a dry concrete slab by applying a force F to the right, as shown. For these surfaces $\mu_s = 1$ and $\mu_k = 0.7$.



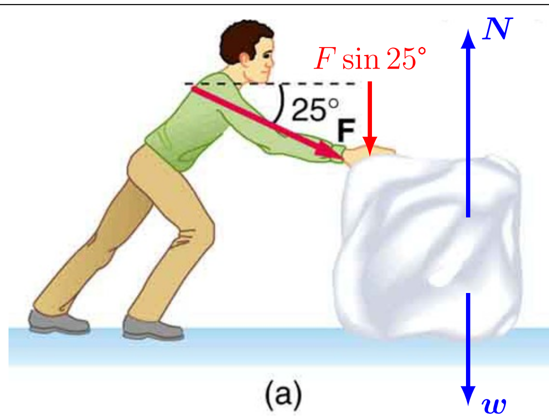
- So it takes a force larger than $\mu_s N = 200\,\text{N}$ to start it moving.
- Then, after it is moving, the force needed to keep it moving is $\mu_k N = (0.7)(200) = 140\,\text{N}$.
- If the horizontal F applied to the tire is less than 200 N, and the tire is at rest, then the static friction force is equal in magnitude to F, and the tire doesn't move.

$$\text{If } F < \mu_s N \text{ then } f_s = F \text{ (so } \sum F_x = F - f_s = 0.)$$

- If the tire is moving at constant speed, then we know $F = f_k = 140\,\text{N}$.
- If the tire is already moving, then a force larger than 140 N will accelerate it. For example, if $F = 180\,\text{N}$ (and the tire is moving) then

$$\sum F = ma \Rightarrow F - f_k = 180 - 140 = 40 = ma = \frac{w}{g}a = \frac{200}{9.8}a \Rightarrow a = \frac{(40)(9.8)}{200} = 1.96\,\text{m/s}^2.$$

It is important to recognize that f is proportional to N, not w, even though these are sometimes the same. Our text shows two scenarios in Fig. 5.21 with $N \neq w$.

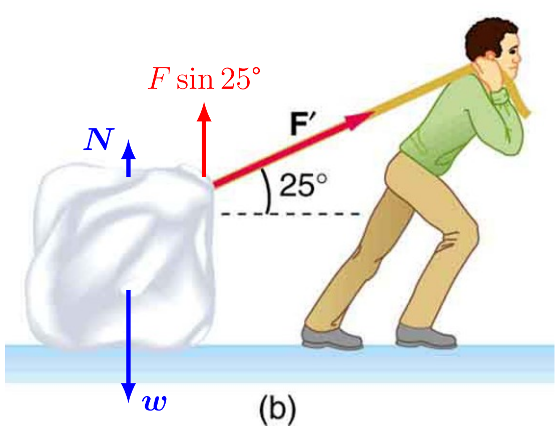


1. In part (a), the pushing force $\boldsymbol{F}$ has a downward component $F \sin 25°$. Therefore

   $$\sum F_y = N - F\sin 25° - w = 0 \Rightarrow \boxed{N = w + F\sin 25°.}$$

   and the friction force opposing the motion is

   $$f = \mu N = \mu(w + F\sin 25°).$$



2. In part (b), the pulling force $\boldsymbol{F}$ has an upward component $F \sin 25°$. Therefore

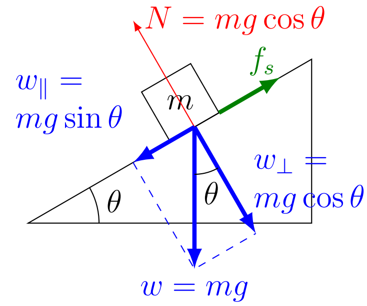
   $$\sum F_y = N + F\sin 25° - w = 0 \Rightarrow \boxed{N = w - F\sin 25°.}$$

   and the friction force opposing the motion is

   $$f = \mu N = \mu(w - F\sin 25°).$$

So there is less friction in the scenario in part (b).

Suppose an object of mass m is on an inclined plane at an angle θ, and there is friction.



- We already know the normal force on the object exerted by the slope:

$$N = w_\perp = mg\cos\theta,$$

and the component of the weight pulling the object downhill is

$$w_\parallel = mg\sin\theta.$$

- Static friction might keep the object from sliding down. It is an uphill force with maximum value

$$f_{s(max)} = \mu_s N = \mu_s mg\cos\theta.$$

- So if the object does not begin to slide, we can say that the sum of forces along the slope is zero:

$$\sum F_\parallel = f_s - mg\sin\theta = 0.$$

- But if the angle θ is increased, the normal force decreases, so friction decreases, while the downhill component of the weight increases. At some angle θ_{max}, the object begins to slide, and then

$$f_{s(max)} = \mu_s mg\cos\theta_{max} = mg\sin\theta_{max},$$

or, rearranging a little, we can see that this angle gives a measurement of μ_s:

$$\mu_s = \frac{\cancel{mg}\sin\theta_{max}}{\cancel{mg}\cos\theta_{max}} = \tan\theta_{max}.$$

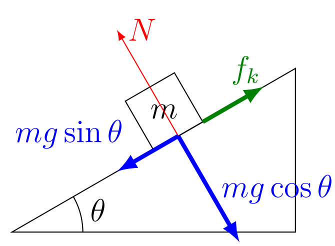
- So if the table of coefficients of friction is correct, the steepest angle you could walk up a wood surface in shoes with $\mu_s = 0.9$ should be

$$\theta_{max} = \arctan(\mu_s) = \arctan(0.9) = 42° \text{ (shoes on wood)}$$

but on ice the limiting angle would be

$$\theta_{max} = \arctan(0.1) = 5.7° \text{ (shoes on ice.)}$$

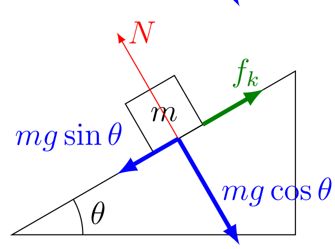
- If the object is sliding down the hill, then the friction is kinetic, and we could find the acceleration:



$$\sum F_\parallel = mg\sin\theta - f_k = mg\sin\theta - \mu_k mg\cos\theta = ma$$

Cancelling out all the m factors gives

$$a = g\left(\sin\theta - \mu_k\cos\theta\right)$$



so friction reduces the acceleration from what gravity would otherwise produce.

- And if at some angle θ_c the object slides at constant speed down the hill, then $a = 0$ and we see that

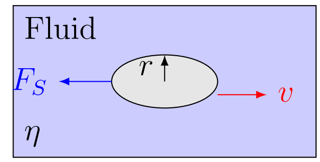
$$\sin\theta_c = \mu_k\cos\theta_c \Rightarrow \mu_k = \tan\theta_c$$

5.2 Drag forces

When we were considering free-fall motion, we ignored the effect of air resistance. But, in fact, when an object moves through a fluid such as air or water, there will be a force resisting the motion.

- Unlike friction, for which we assume a constant coefficient and no speed dependence, for the drag force of a fluid, there is a speed dependence.

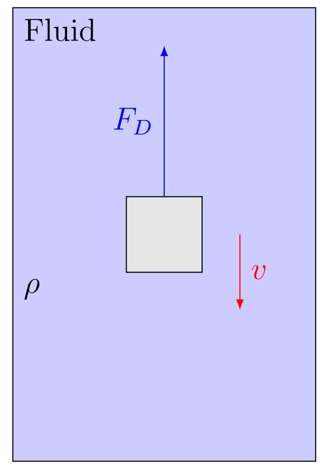
  1. For very small objects moving at slow speeds v, the drag force may be proportional to v: $F_S \propto v$ (Stokes' law) or, as an equation, we have

     $$F_S = 6\pi r \eta v$$

     where r is the radius of the object, and η (another Greek letter, pronounced like "eta") is a property of the fluid called the viscosity, something like the coefficient of friction for layers of the fluid. Bacteria swimming in water would experience Stokes' drag as they move.

  2. For larger objects and higher speeds, usually drag force is proportional to the square of the speed; this case is known as Rayleigh's drag law: $F_D \propto v^2$. The drag is also proportional to the cross-sectional area of the object A, and the density of the fluid ρ, and it is usual to write the equation as

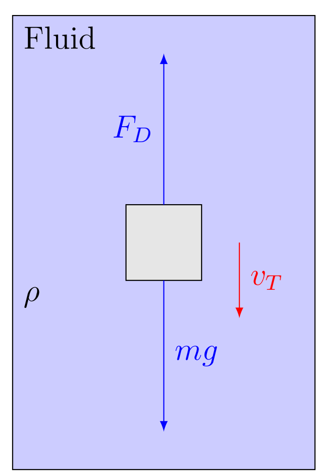
     $$F_D = \frac{1}{2} C \rho A v^2$$

     where the factor C is called the drag coefficient, a unitless number that depends on the shape of the object — especially the shape of the cross-section that travels through the fluid, but also other aspects of the shape, such as how well streamlined it is.

  Our text has a table of C values for objects including cars and skydivers.

Object	C
Sphere	0.45
Cube	1.05
Circular plate	1.12
Horizontal skydiver	1
Vertical skydiver	0.7

- Drag forces give rise to what is called the terminal speed. If gravity is accelerating an object, then it gains speed until the drag force becomes as large as the weight. At that speed, the sum of the forces is zero, and it no longer accelerates but keeps going at this speed. For example, with Rayleigh drag, at terminal speed v_T

  $$\sum F_y = F_D - mg = 0 \Rightarrow \frac{1}{2} C \rho A v_T^2 = mg \Rightarrow v_T = \sqrt{\frac{2mg}{\rho C A}}.$$

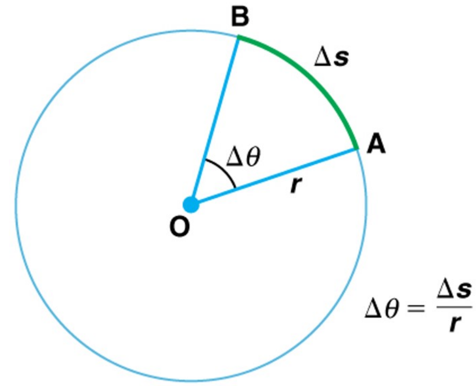
  So, as you might expect, an object has higher terminal speed if it is more massive, has smaller area or a smaller drag coefficient. For a vertically-oriented, 75 kg skydiver, our book does a sample calculation that gives $v_T \approx 98\,\mathrm{m/s} \approx 219$ mph.

Time	Topics	Assignments
12:30 - 1:20	Friction	
1:30 - 2:20	Rotation and Centripetal force	
2:30 - 3:30	Catch up, review, practice exam	

Day 4, Hour 2: Rotation and Centripetal force

We know from Newton's first law that the velocity of an object remains constant unless a force changes it. And Newton's second law says the change is an acceleration proportional to the net force. But so far we have only looked at cases in which the net force keeps a constant direction. Chapter 6 deals with the important case of circular motion due to forces toward the center of the circle. To begin, we look at how to describe such motion, then at different forces that cause circular motion.

6.1 Rotation angle and angular velocity



When an object moves along a circular path, such as from point A to point B in Fig. 6.3 of the text, it travels a distance Δs as shown along the circular curve. This Δs is related to the radius r of the circle, and also to the change of angle $\Delta\theta$. The relationship is

$$\Delta s = r \cdot \Delta\theta$$

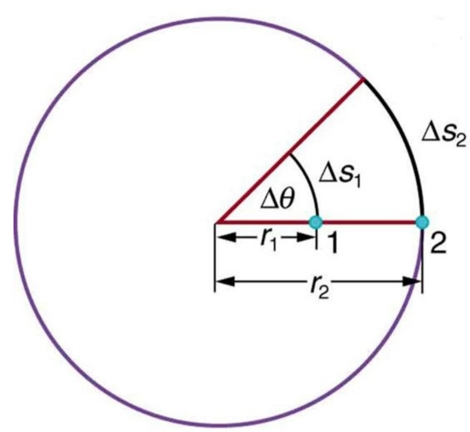
which is very nice, but only works if $\Delta\theta$ is measured in radians.

- You probably know that the circumference of a circle is $C = 2\pi r$; this is the distance a point would travel when making one complete revolution around the circle. So by definition the angle all the way around a circle is $\Delta\theta = 2\pi$ radians $\approx 6.283\,\text{rad}$.
- The circumference formula does not work if the angle is measured in degrees: $C \neq r \cdot 360$.
- The three common ways of measuring angles are related as follows:

$$\boxed{2\pi \text{ radians } = 360° = 1 \text{ revolution.}}$$

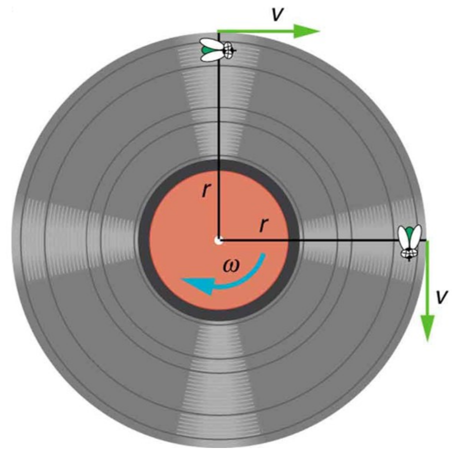
  A 90° angle is therefore 1/4 revolution, or $\pi/2\,\text{rad}$.

- Although radians may be the least familiar, they turn out to be the most convenient angular unit for studying circular motion.
- For example, Figure 6.4 from the text shows two points moving on circles of different radii, r_1 and r_2. The distances they travel Δs_1 and Δs_2 are different too. But the angular displacement is the same $\Delta\theta$ for both:



$$\Delta\theta = \frac{\Delta s_1}{r_1} = \frac{\Delta s_2}{r_2}$$

  - Notice that both Δs and r would be measured in meters, so their ratio is actually unitless, even though it is called the radian. An angle in radians is simply a number.
  - In formulas, anytime you want to put in or leave out the radian unit, you can, because it is unitless.



For straight-line motion we used the velocity definition $v = \frac{\Delta x}{\Delta t}$. For circular motion, velocity is similarly defined as $v = \frac{\Delta s}{\Delta t}$.

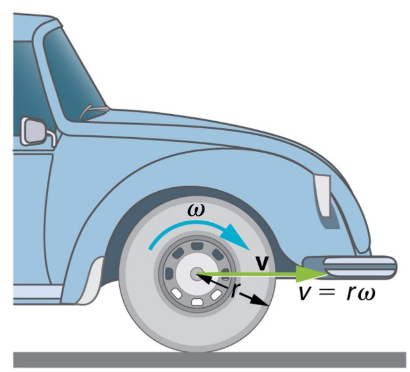
- For example, in Fig. 6.6 of our text, the fly on the rim of a spinning disk has a velocity v along the rim.
- There is another way to describe the fly's motion: it goes through an angular displacement in each time interval too. The angular velocity is defined as

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{1}{r} \cdot \frac{\Delta s}{\Delta t} = \frac{v}{r}.$$

- Notice that the units of v/r would naturally be $(\text{m/s})/(\text{m}) = \text{s}^{-1}$. But the units of $\Delta\theta/\Delta t$ would naturally be rad/s. There is no contradiction though, because the radian is a "unitless" unit that we can leave out or add if we want.
- Using ω is often much easier than using v and r. For example, without knowing the Earth's velocity or the radii of its motions, we can say the Earth's angular velocity as it rotates on its axis is $\omega = \frac{2\pi\,\text{rad}}{day}$, and for its revolution around the Sun $\omega = \frac{2\pi\,\text{rad}}{\text{yr}}$. In standard units:

$$\omega_{rot} = \frac{2\pi\,\text{rad}}{\text{day}} \cdot \frac{\text{day}}{24\,\text{hr}} \cdot \frac{\text{hr}}{60\,\text{min}} \cdot \frac{\text{min}}{60\,\text{s}} = 7.27 \times 10^{-5}\,\text{rad/s} \quad \text{(Earth's rotation on axis)}$$
$$\omega_{rev} = \frac{2\pi\,\text{rad}}{\text{yr}} \cdot \frac{\text{yr}}{365\,\text{day}} \cdot \frac{\text{day}}{24\,\text{hr}} \cdot \frac{\text{hr}}{60\,\text{min}} \cdot \frac{\text{min}}{60\,\text{s}} = 1.99 \times 10^{-7}\,\text{rad/s} \quad \text{(Earth's revolution around Sun.)}$$

Rolling



When a circular wheel rolls, the distance it travels is obviously related to the angle it turns through by that formula we started with:

$$\Delta s = r\Delta\theta.$$

Likewise, the velocity at which it travels along is

$$v = \Delta s/\Delta t = r\Delta\theta/\Delta t = r\omega.$$

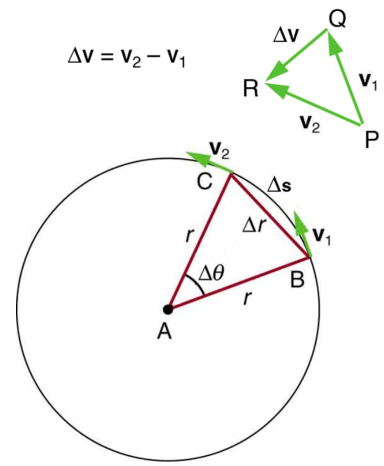
So these relationships apply whether the circular motion is around a stationary center or a moving one.

Example: Boris is riding his bicycle and notices that his wheels are turning exactly 4 revolutions per second. The wheels have a diameter of 700 mm. How fast is Boris moving on his bike?

Answer: The radius is half the diameter. Conversions to standard units are shown.

$$v = r\omega = \frac{700\,\text{mm}}{2} \cdot \frac{4\,\text{rev}}{\text{s}} \cdot \frac{2\pi\,\text{rad}}{\text{rev}} \cdot \frac{1\,\text{m}}{1000\,\text{mm}} = \boxed{8.80\,\text{m/s.}}$$

6.2 Centripetal acceleration



In circular motion, the velocity of any particle is always changing direction. Figure 6.8 of our text shows two such velocity vectors, v_1 and v_2, both tangent to the circle of radius r, and having the same magnitude.

- To follow this path around the circle, a particle must continually turn to the left, and there must be a force in that direction to make it happen. A force toward the center of the circle is needed.
- To help make this plausible, the small vector diagram suggests that the change in velocity, $\Delta v = v_2 - v_1$, is directed inward. It becomes directed exactly toward the center if the time interval shrinks to zero.
- Skipping a few lines of algebra that is in the text, it is found that the acceleration toward the center of the circle is

$$\boxed{a_c = \frac{v^2}{r} \text{ (Centripetal acceleration.)}}$$

- The relationship $v = r\omega$ gives another useful form: $\boxed{a_c = r\omega^2 \text{ (Centripetal acceleration.)}}$
- Because the motion is circular, the distance travelled around a circle is $2\pi r$ so we can say $v = 2\pi r/T_o$, where T_o is the orbital period, the time to go around the circle. So another useful form is

$$a_c = \frac{(2\pi r)^2}{rT_o^2} = \frac{4\pi^2 r}{T_o^2}$$

Example: When Boris rides with his 700 mm-diameter bike tires going around at 4 revs per second, what is the centripetal acceleration of the tires?

Answer: We have $r = 0.7\,\text{m}/2 = 0.35\,\text{m}$, and $\omega = 8\pi\,\text{rad/s}$ so

$$a_c = r\omega^2 = (0.35\,\text{m})\left(\frac{8\pi\,\text{rad}}{\text{s}}\right)^2 = \boxed{221\,\text{m/s}^2.}$$

In case you noticed, the rad^2 unit in the numerator is simply ignored because it is a unitless unit, and we don't want it in a_c.

Another answer: We already found that Boris's velocity was $v = 8.80\,\text{m/s}$ in the previous example, so we could also use it here to write

$$a_c = \frac{v^2}{r} = \frac{(8.8\,\text{m/s})^2}{0.35\,\text{m}} = \boxed{221\,\text{m/s}^2} \text{ again.}$$

Here the units are exactly what we would expect for acceleration.

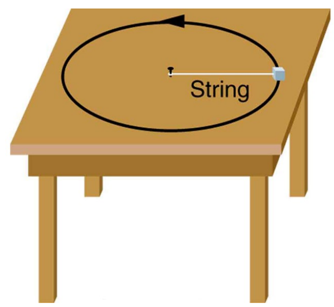
6.3 Centripetal force

We know from Newton's second law that acceleration results from a force in that direction. So whenever circular motion happens, the reason for the centripetal acceleration is a force toward the center of the circle, given by

$$\boxed{F_c = ma_c = m\frac{v^2}{r} = mr\omega^2 = m \cdot 4\pi^2 \frac{r}{T_o^2} \text{ (Centripetal force.)}}$$

This force must be acting on any mass m whose velocity v is along a circular path of radius r. We will look at a few examples of forces that cause circular motion.

<u>**Tension**</u>



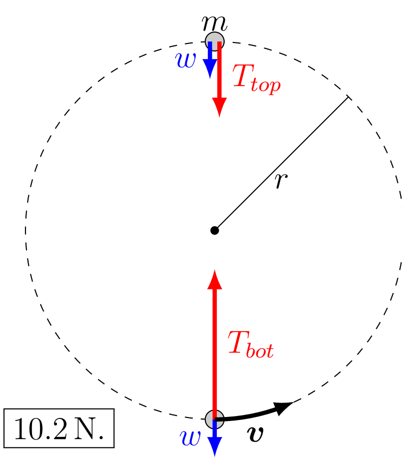
The most obvious example of centripetal force is to have a string fastened to the mass moving along the circle, as in Fig. 6.35 from our text. As the mass goes around, the string provides the tension to keep the mass always changing direction. This tension is along the string toward the center of the circle.

Example: A 50-gram mass on a 60 cm string is twirling around in a <u>horizontal</u> circle with an angular velocity of 3 rev/s. What is the tension in the string?

Answer: Since we have m, r and ω we can convert each quantitiy into standard units

$$m = 50\,\text{gram} \cdot \frac{\text{kg}}{1000\,\text{gram}} = 0.05\,\text{kg}$$
$$r = 60\,\text{cm} \cdot \frac{\text{m}}{100\,\text{cm}} = 0.6\,\text{m}$$
$$\omega = \frac{3\,\text{rev}/}{\text{s}} \cdot \frac{2\pi\,\text{rad}}{\text{rev}} = 6\pi\,\text{rad/s}$$

and then use the formula $F_c = mr\omega^2 = (0.05)(0.6)(6\pi)^2 = \boxed{10.7\,\text{N.}}$



Example: Let the same circular motion as in the previous example be happening on a <u>vertical</u> circle. Find the tension in the string at the top and the bottom of the circle.

Answers: One drawing is probably clear enough to suffice as the free-body diagram for the mass at each position.

- At the top there are two forces downward, the string tension and the weight, so

  $$w + T_{top} = F_c = mr\omega^2 \Rightarrow T_{top} = mr\omega^2 - mg = 10.7 - (0.05)(9.8) = \boxed{10.2\,\text{N.}}$$

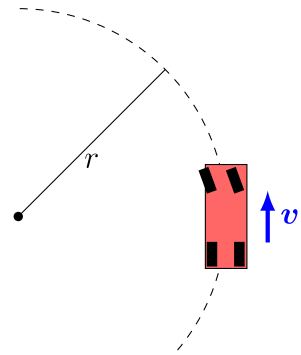
- At the bottom, w is downward and the tension upward, so

  $$T_{bot} - w = F_c = mr\omega^2 \Rightarrow T_{bot} = mr\omega^2 + mg = 10.7 + (0.05)(9.8) = \boxed{11.2\,\text{N.}}$$

  So there is more tension at the bottom of the circle.

Whenever there is circular motion, the force toward the center must be $F_c = mv^2/r = mr\omega^2$. So in this case a combination of weight and string tension provides that force.

Friction

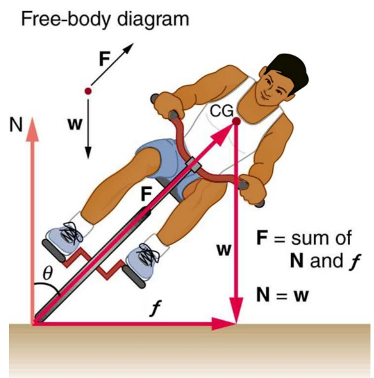


It is also possible to produce circular motion using the force of friction as the centripetal force. Here, from a top view, is a sketch of a red car going along a horizontal circular path. The driver turns the steering wheel to the left, the front tires turn leftward, and the car turns left.

- For the car to go straight, the tires would have to slide across the road. But if there is enough (static) fricition to prevent that, the car changes direction instead, following the curved path.
- When there is not enough friction, the car may slide in a straight line instead of turning.
- If the centripetal force is due to friction we can write $F_c = mv^2/r = \mu mg \Rightarrow v^2/r = \mu g$.

Example: What minimum static friction coefficient is needed for a car going at a speed of 60 km/hr to make a turn on a radius of 30 m?

Answer: $v = \frac{60\,\text{km}}{\text{hr}} \cdot \frac{1000\,\text{m}}{\text{km}} \cdot \frac{\text{hr}}{3600\,\text{s}} = 16.7\,\text{m/s}$, so $\mu_s = \frac{v^2}{rg} = \frac{(16.7\,\text{m/s})^2}{(30\,\text{m})(9.8\,\text{m/s}^2)} = \boxed{0.945.}$ This should work on a concrete road that is dry ($\mu_s \approx 1.0$) but not on a wet road ($\mu_s \approx 0.7$).



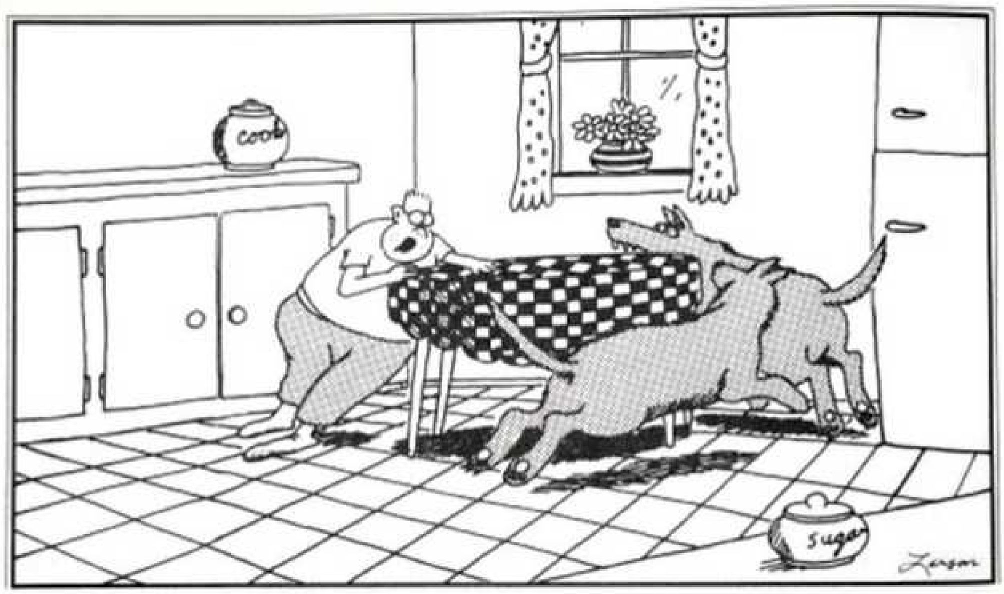
As another example, this figure from our text shows how friction and the normal force allow a biker to lean into a turn on a horizontal road. The normal force on the wheel is vertical, and the friction force is horizontal, providing the centripetal force toward the center of the circle. These are perpendicular components of the net force F that is along the symmetry axis of the bike and rider. We can see from the vector diagram that

$$N = w = mg = F\cos\theta$$
$$f = \mu N = \mu mg = F_c = mv^2/r = F\sin\theta$$
$$\frac{F\sin\theta}{F\cos\theta} = \tan\theta = \frac{mv^2}{mrg} \Rightarrow \tan\theta = \frac{v^2}{rg}$$

So the angle θ will be larger for a higher speed or smaller turn radius.

Example: Boris is riding his bike at 6 m/s and needs to turn at an intersection on a radius of 5 m. At what angle will he be leaning from vertical as he makes the turn?

Answer: $\theta = \arctan\left(\frac{v^2}{rg}\right) = \arctan\left[\frac{6^2}{(5)(9.8)}\right] = \boxed{36.3^\circ.}$ There is no good reason to put this in radians, but if you want to, or your calculator was in that mode, then $36.3^\circ \cdot \left(\frac{\pi\,\text{rad}}{180^\circ}\right) = 0.634\,\text{rad}$.



Luposlipaphobia: The fear of being pursued by timber wolves around a kitchen table while wearing socks on a newly waxed floor.

Example: Gary has *Luposlipaphobia*. What is the shortest time in which he can run around a circle of radius 2 m while wearing socks with a friction coefficient $\mu_s = 0.1$?

Answer: $\mu_s mg = F_c = m \cdot 4\pi^2 \frac{r}{T_o^2} \Rightarrow T_o = \sqrt{\frac{4\pi^2 r}{\mu_s g}} = \sqrt{\frac{4\pi^2(2)}{(0.1)(9.8)}} = 9\,\text{s}$. Probably too slow.

Time	Topics	Assignments
12:30 - 1:20	Test 1, Work, energy	
1:30 - 2:20	Potential and kinetic energy	
2:30 - 3:30	Conservation of energy, Power	

Day 6, Hour 1: Test 1 review, Work and energy

Test 1

Solutions are posted on Canvas.

Now that we have looked at Newton's laws of motion, it might seems like they are sufficient for whatever we might want to figure out. But we will look at two more topics today that are helpful in special situations, and also two of the most important big ideas in physics. These topics are Energy and Momentum. If you want to express the way the universe works in as few laws as possible, two of those laws are that Energy and Momentum are conserved.

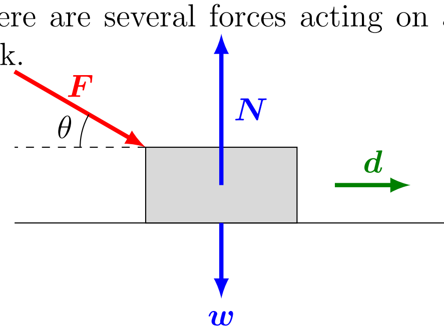
7.1 Work

The concept of energy is a bit vague because it is related to so many things and comes in many kinds. An easier place to start understanding energy is the concept of Work: a common word, but with a scientific definition.

- A very rough definition of Work is to say it is a Force times a Distance. If a force of 10 N moves something 2 m, then the work done is

$$W = \text{Force} \times \text{Distance} = 10\,\text{N} \times 2\,\text{m} = 20\,\text{N} \cdot \text{m}$$

- The standard unit of work is called the Joule, abbreviate as J. So $1\,\text{J} = 1\,\text{N} \cdot \text{m}$.
- A better definition of Work is needed, because sometimes there are several forces acting on an object, and they may or may not be involved in doing the work.
  - For example, a box on the floor has weight w, and the floor pushes up on it with a normal force N. If another force $\boldsymbol{F}$ pushes on it at an angle θ from horizontal, as shown, the box can only move horizontally. If the box is displaced as shown by the vector $\boldsymbol{d}$, then how much work was done by each force?

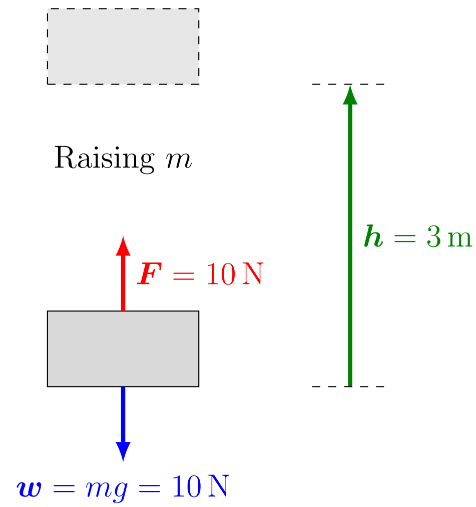


  - It turns out that neither N nor w do any work, and only the part of $\boldsymbol{F}$ aligned with the displacement is doing any work.
- The right definition uses the component of each force in the direction of the displacement.

$$W = (F \cos\theta) \cdot d$$

  where F and d are the magnitudes of the force $\boldsymbol{F}$ and displacement $\boldsymbol{d}$, and θ is the angle between these vectors.
  - In the example above, both N and w are at angles of 90° from the direction of $\boldsymbol{d}$, and $\sin 90° = 0$.
  - Any force with no component in the direction of the displacement does no work.
- Both $\boldsymbol{F}$ and $\boldsymbol{d}$ are vectors, but this way of multiplying them gives a scalar. Work has no direction.

Positive and Negative Work



As another example of work, suppose a force $\boldsymbol{F}$ is able to lift a mass.

1. To be specific, let the weight of this mass be 10 N.
2. A force $\boldsymbol{F} = 10\,\text{N}$ upward could lift this mass at constant speed. If it is displaced vertically a height $h = 3\,\text{m}$, then the work done by the force $\boldsymbol{F}$ is

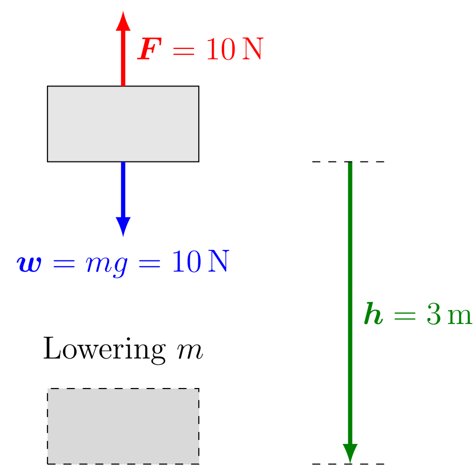
$$W_F = F\cos\theta = (10\,\text{N})(3\,\text{m})\cos 0° = \boxed{30\,\text{J.}}$$

3. But the weight is another force on this mass, the force of gravity, and we can calculate the work this force does too:

$$W_{grav} = mgh\cos 180° = -mgh = \boxed{-30\,\text{J}}$$ ($\theta = 180°$ because $\boldsymbol{w}$ and $\boldsymbol{h}$ are in opposite directions.)

Work can be either positive or negative. It is still a scalar — it has no direction, just a number, but the number may be a negative one or positive.

To emphasize this, suppose now that the 10 N mass is lowered 3 m.



1. The weight of this mass is of course a downward force of 10 N.
2. A force $\boldsymbol{F} = 10\,\text{N}$ upward is used to lower the mass at constant speed. So it is displaced vertically a distance $h = 3\,\text{m}$ downward, as shown. The work done by the force $\boldsymbol{F}$ is

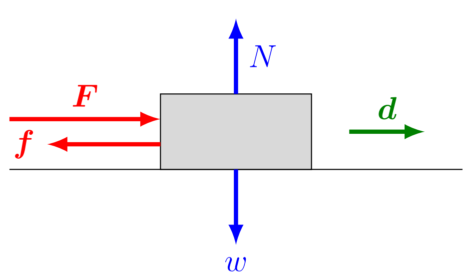
$$W_F = Fh\cos\theta = (10\,\text{N})(3\,\text{m})\cos 180° = \boxed{-30\,\text{J.}}$$

3. But the force of gravity does

$$W_{grav} = wh\cos\theta = (10\,\text{N})(3\,\text{m})\cos 0° = \boxed{30\,\text{J.}}$$

So, there is nothing surprising about either a positive or negative amount of work. It depends on whether the displacement and the force are in the same direction or opposite.

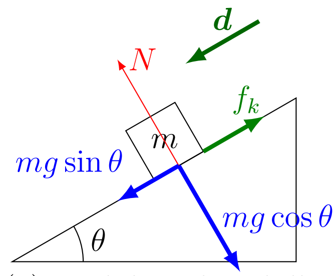
Friction is always in the direction opposite the displacement, as in this situation we looked at previously:



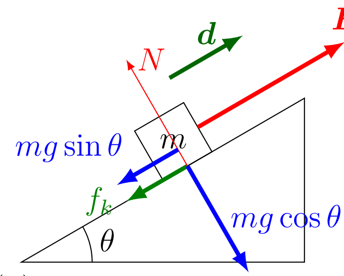
The force of friction is $f = \mu_k N$, and if the force $\boldsymbol{F}$ pushes the box to the right, then the friction force is toward the left, and the work done by fricition is

$$W_f = fd\cos 180° = -fd = -\mu_k Nd.$$

Which of the forces in these diagrams does positive or negative work?



(a) m sliding downhill



(b) m pulled uphill by $\boldsymbol{F}$

(a)
- $W_{grav} = ?$
- $W_f = ?$
- $W_N = ?$

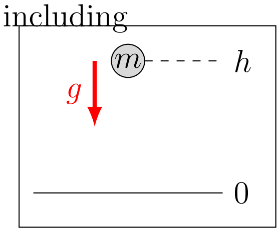
(b)
- $W_{grav} = ?$
- $W_f = ?$
- $W_F = ?$
- $W_N = ?$

Work and Energy

Now that we have some understanding of the concept of work, we could give this definition of energy:

$$\boxed{\text{Energy } = \text{Ability to do Work.}}$$

There are many kinds of energy but we will focus on just a few kinds in PHYS 105, including

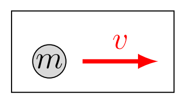


1. A mass raised to a height h has gravitational potential energy

$$PE = mgh$$

Work is done to raise the mass giving it the ability to do this much work.

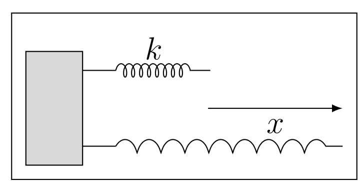
2. A mass is given a speed: this gives it kinetic energy.



$$KE = \frac{1}{2}mv^2$$

Work was done to get it moving, so it has the ability to do this much work.

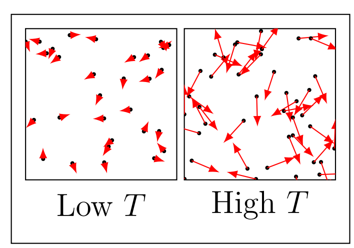
3. A material is stretched or compressed giving it elastic (or spring) potential energy.



$$PE_s = \frac{1}{2}kx^2 \text{ (here } k \text{ is a property of the spring.)}$$

The work done to stretch the spring is the amount of work it can do.

4. Molecules at a temperature T have kinetic energy $\propto T$. For gases,



$$\text{Internal energy } U_{gas} = \frac{3}{2}kT \text{ (here } k \text{ is Boltzmann's constant.)}$$

For solids and liquids, adding heat can change the temperature

$$\text{Heat } Q = mc\Delta T \text{ (} c \text{ is the specific heat of a solid or liquid.)}$$

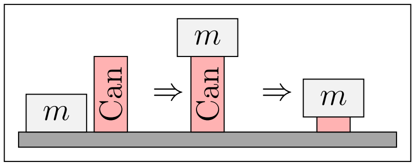
In PHYS 106 you will see other kinds of energy. For example, electric charges that repel or atttract have electrostatic potential energy. Protons and neutrons that form an atomic nucleus have some of their mass converted into nuclear binding energy.

Day 6, Hour 2: Potential and kinetic energy

7.3 Gravitational potential energy *PE*

When a mass is lifted upward, there is work done by the force that lifted it: $W = mgh$. But now this elevated mass has something it did not have before: it has the ability to do work, specifically, the amount of work it can do is also mgh. So this is the energy it gained by being elevated a distance h.

- For example: suppose you need to crush some pop cans. This requires work, because a force must be exerted on the can through a distance to crush it. A sufficiently heavy brick placed on top of the can should do it.

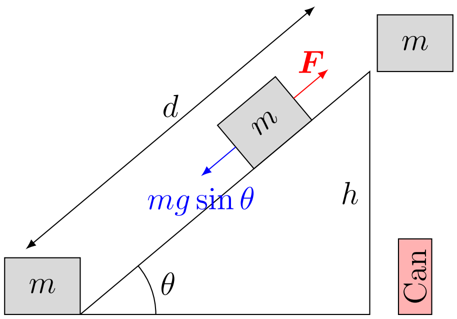


- Our book uses the definition $\Delta PE_g = mgh$, because only changes in potential energy can ever be measured. This is because the starting value for h is wherever you like. Usually it makes sense to say that $h = 0$ at the lowest elevation to which a mass goes. But the Δ and the subscript are awkward to carry along at times, and the book sometimes doesn't use them. I will just use

$$\boxed{PE = mgh.}$$

- Gravitational potential energy does not depend on the path taken to reach elevation h. So, for example, if we slide a brick up a ramp, and it reaches height h, then it has the same PE as it would if it were lifted straight up.
  - For a ramp whose angle is θ, the distance it must go up the ramp is longer than h: it is $d = h/\sin\theta$, but the force needed to move it up the hill is less than the weight: it is $F = mg\sin\theta$. So the work done against gravity is still

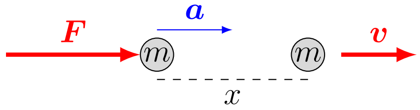
$$W = (mg\sin\theta)d = (mg\sin\theta)\cdot\left(\frac{h}{\sin\theta}\right) = mgh$$



  - If there is friction on the ramp then the total work to raise the brick this way is larger. But the ramp allows a smaller force to raise the brick, so this may be a good method anyway.

7.2 Kinetic energy and the work-energy theorem

How much work is done to get a mass m moving at speed v? Suppose a force F accelerates the mass through a distance x. Then



$$W = F\cdot x = m\cdot a\cdot x.$$

We have an equation that relates the velocity to the acceleration and displacement, from which we can get an expression for $a\cdot x$:

$$v^2 = v_0^2 + 2ax \Rightarrow a\cdot x = \frac{1}{2}v^2 - \frac{1}{2}v_0^2$$

Thus, the work done on the mass can also be expressed as a change in its kinetic energy

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = KE_f - KE_i = \Delta KE$$

using kinetic energy defined as $KE = \frac{1}{2}mv^2$. If work is done to accelerate a mass, it has this much ability to do work. The moving mass can, in turn, exert a force F through a distance x as well.

Example: A baseball has a mass of 0.145 kg, and a fast pitch can have a speed of around 100 mph. What is the kinetic energy of a baseball moving that fast?

Answer:

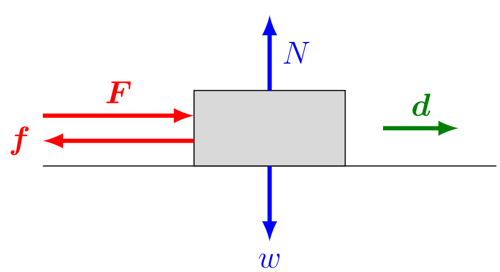
$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.145\,\text{kg})\left(\frac{100\,\text{miles}}{\text{hr}} \cdot \frac{1609\,\text{m}}{\text{mile}} \cdot \frac{\text{hr}}{3600\,\text{s}}\right)^2 = 145\,\text{J}.$$

It would have this much gravitational potential energy mgh if thrown to an altitude of 102 m.

Let's go back a moment to the idea that led to this definition of kinetic energy — the work done by a force to change the speed of the mass. Suppose there are many forces on the object, and $\boldsymbol{F}$ is the net force, the total of all the actual forces. We get the same result for the net force, and so we can say that the net work done by all the forces is

$$W_{net} = \Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

We saw earlier that friction does negative work. Suppose we push a mass forward a distance d using a force F, but at constant speed. This means the friction force f is equal to the pushing force F, but in the opposite direction of course.



In this case, the total work done, or the net work, is zero:

$$W_{net} = \Delta KE = Fd - fd = 0, \text{ (because } \boldsymbol{f} = -\boldsymbol{F}.)$$

This is easy to see when the speed is constant. But whether the forces are equal or different, it turns out that the net work is given by ΔKE.

Example: Boris pulls a wagon that has a mass of 20 kg, accelerating it from rest to a speed of 6 m/s. There was friction opposing him. What was the net work done on the wagon?

Answer: $W_{net} = \Delta KE = \frac{1}{2}mv^2 = \frac{1}{2}(20)(6)^2 = 360$ J by both Boris and friction.

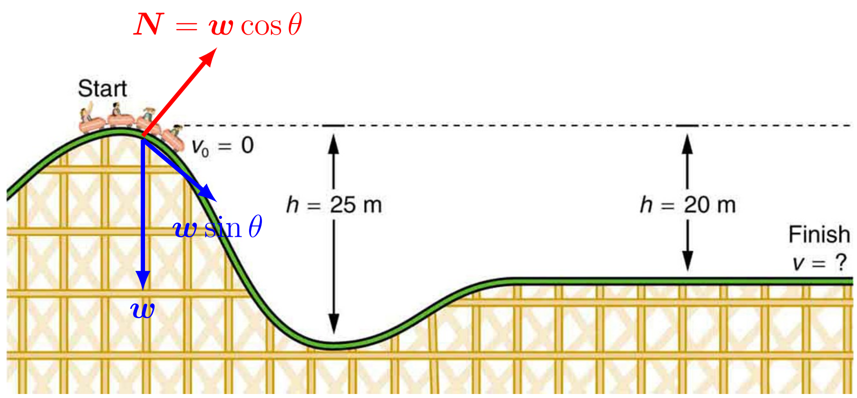
Example: Suppose he pulled the wagon with a constant force of 20 N for a distance of 25 m. What was the magnitude of the friction force?

Answer:

$$W_{net} = 360\,\mathrm{J} = F \cdot d - f \cdot d \Rightarrow f = F - \frac{W_{net}}{d} = 20\,\mathrm{N} - \frac{360\,\mathrm{J}}{25\,\mathrm{m}} = 5.6\,\mathrm{N}.$$

One of the reasons energy is such a useful tool for understanding motion is that energy is a scalar.

- When motion takes place along a curve, such as in Fig. 7.8 from the text, it would be very difficult to keep track of the force vectors and the resulting accelerations as the slope of the track changes, and then use all that to work out a final velocity v at the end.



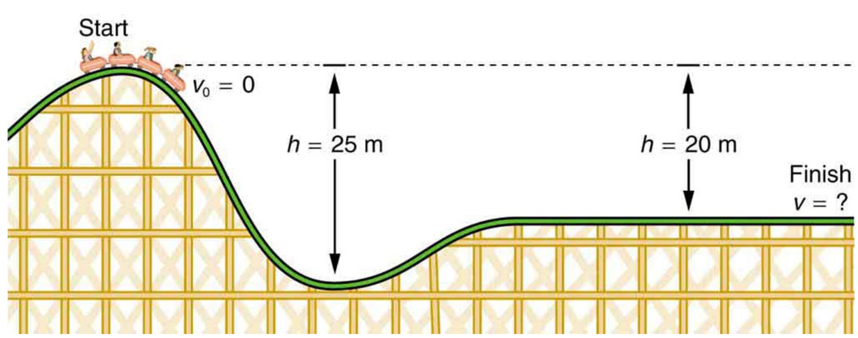
- But we can see that when gravitational potential energy PE is lost, kinetic energy KE is gained, and vice versa.
- If we knew the total of $PE + KE$ stayed constant, then we could simply say

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_0 + 0 = mgh_f + \frac{1}{2}mv_f^2$$

$$\cancel{m}g\,(h_i - h_f) = \frac{1}{2}\cancel{m}v_f^2$$

$$\Rightarrow \boxed{v_f = \sqrt{2g\,(h_i - h_f)}.}$$



  - So at the end where $h_i - h_f = 20\,\mathrm{m}$, $v_f = \sqrt{2(9.8)(20)} = 19.8\,\mathrm{m/s}$.
  - At the low point where $h_i - h_f = 25\,\mathrm{m}$, $v_f = \sqrt{2(9.8)(25)} = 22.1\,\mathrm{m/s}$.

So, an important question is whether or not these kinds of energy are conserved.

- It turns out that, if we include every kind of energy, then energy is conserved:

$$PE_{grav} + KE + PE_{spring} + \text{thermal energy} + \text{chemical energy} + \text{nuclear energy} + ... = \text{constant}.$$

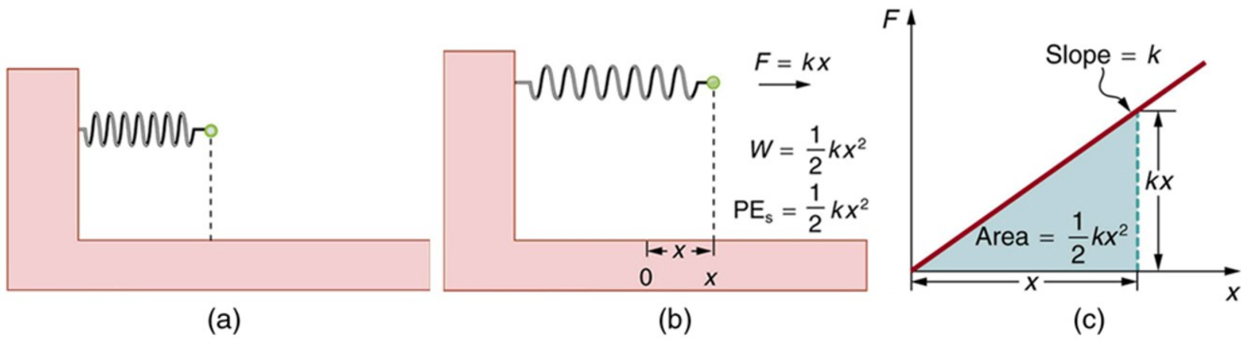
- The energy unit, the Joule, is named after one of the researchers who in the 1840s did the most to prove the conservation of energy.

7.4 Conservative forces and potential energy

Potential energy is energy due to a change of position, if the path taken doesn't matter.

- A mass m lifted to a height h has gravitational potential energy $PE = mgh$ because that is the amount of work done to put it at that elevation.
  - Only vertical changes matter, so any path to the height h gives the same PE. The gravitational force is therefore a conservative force, and PE is a conserved type of energy.
- Another kind is the potential energy of a stretched or compressed spring. If the force needed to change its length by x is $F = k \cdot x$, then the work done is

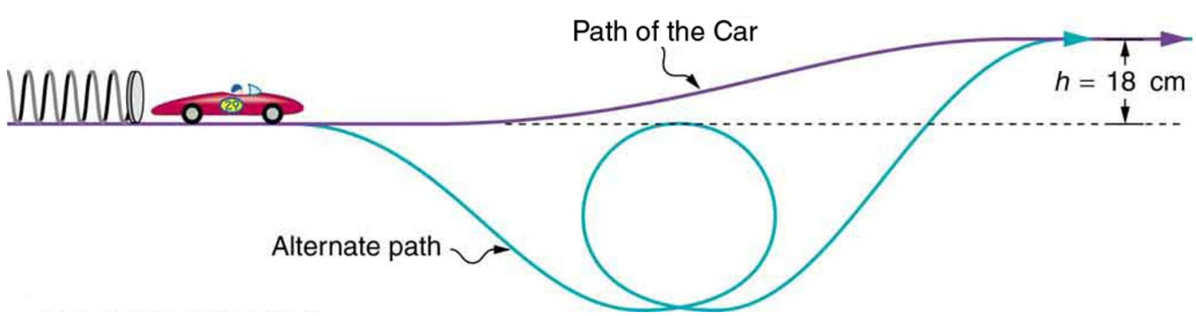
$$PE_s = \frac{1}{2}kx^2.$$



This k is known as the spring's force constant, measured in units of $\text{N/m} = \text{kg/s}^2$. The larger this constant, the harder it is to change the length of the spring. The magnitude of the force increases with x, as shown, and the work done is the average force multiplied by the displacement x.

- It doesn't matter how the spring arrives at x, PE_s only depends on the final value x. So the spring force F is a conservative force, and PE_s is a conserved type of energy.

Example: Figure 7.12 from our text shows a spring that sends a toy car up a hill, converting PE_s to KE to PE. The spring's force constant is $k = 250\,\text{N/m}$, and it is compressed a distance $x = 4\,\text{cm}$ then released. The car's mass is 0.1 kg. What will be the speed of the car when it has reached the point 18 cm higher than it started?



$$PE_s = PE + KE \Rightarrow \frac{1}{2}kx^2 = mgh + \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{kx^2}{m} - 2gh}$$

$$v = \sqrt{\frac{(250)(0.04)^2}{0.1} - 2(9.8)(0.18)} = 0.687\,\text{m/s}.$$

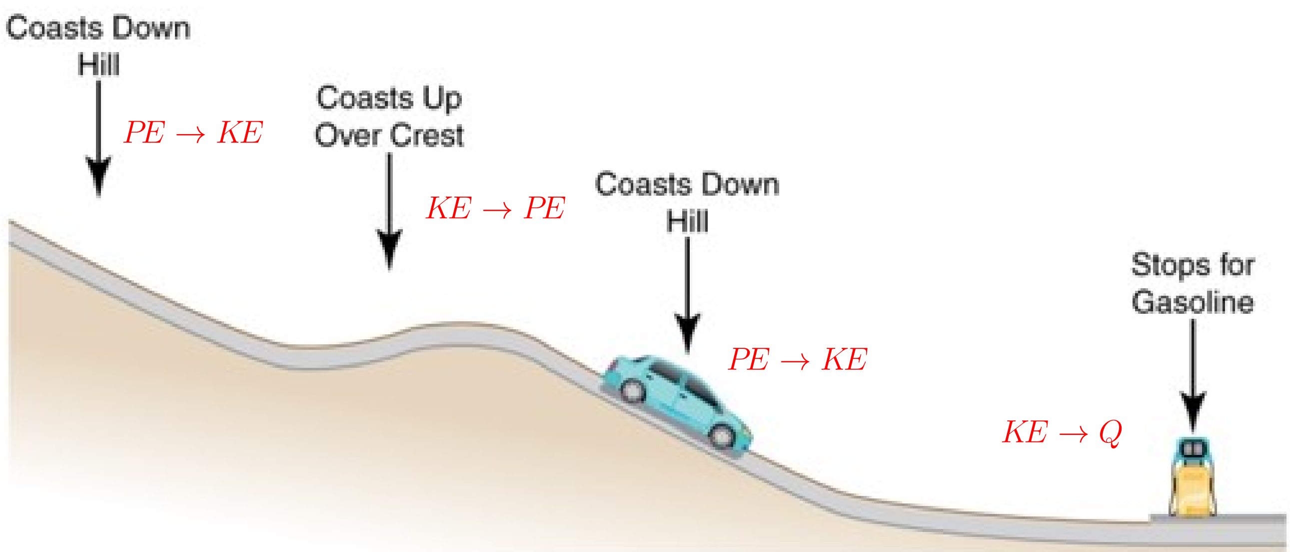
This is independent of the path, because all the forces are conservative ones.

Day 6, Hour 3: Conservation of energy, Power

7.5 Nonconservative forces

Some forces are non-conservative, and the one we will deal with the most is friction.

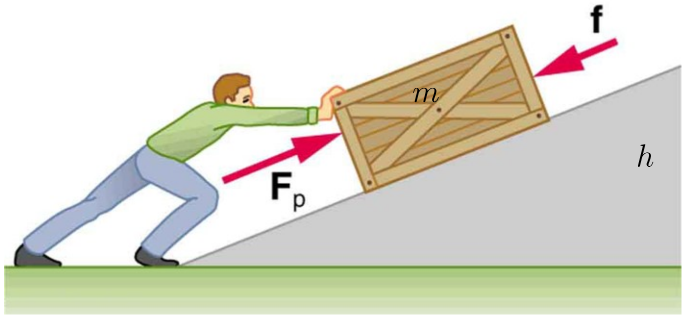
Example: Fig. 7.34 from our text shows a car rolling down over hills to a gas station, where the driver stops for a refill.



- You can see there are some changes in mechanical energy from potential to kinetic and vice versa.
- At the end, by applying the brakes, using friction to stop, the driver converts mechanical energy into heat.
  - Hot brakes are not useful for getting the car moving again.
  - An electric vehicle could convert some of the mechanical energy into stored electricity, which could help the car get moving again.
  - Vehicles such as city buses that stop and start all day convert a great deal of energy into heat. An electrical or mechanical system for storing the energy would be helpful for energy efficiency.
- The work done by friction is always negative. It is not stored as potential energy that can be recovered by returning to the starting place, as was the case for the work done by a spring or gravity.
- The work done by friction is converted to heat, or thermal energy Q, and we can consider it as lost from the mechanical energy of a system.
  - There are devices, heat engines, that can turn heat energy back into mechanical energy, as we will see later in PHYS 105. They are usually not very efficient, and some of the heat energy is always unrecoverable.

7.6 Conservation of energy

The book includes this figure showing a guy pushing a crate up a ramp and opposed by a friction force. He is converting food energy into mechanical work. He does not get the food back if he pushes the crate back down the ramp, so there is something going on here like the non-conservative force of friction. But it is complicated enough that our book chooses to call this sort of energy source *OE*, which stands for "other energy."



- If this crate has 500 J of *PE* at the top of the ramp, then it could be pushed up a frictionless ramp by doing 500 J of work. That is, $W_{by\,guy} = OE = \Delta PE = 500\,\text{J}$.
- But if there is friction, it takes more work to push the crate. Supposing 700 J of work is needed, then −200 J is the work done by friction. So $W_{by\,guy} + W_f = 700 - 200 = 500\,\text{J} = \Delta PE$.
- So when non-conservative forces are involved, we can say that the initial mechanical energy is increased by the amount of work done by non-conservative forces and any other energy added:

$$PE_i + KE_i + W_{nc} + OE = PE_f + KE_f$$

or, equivalently

$$W_{nc} + OE = PE_f - PE_i + KE_f - KE_i$$

which is saying

$$W_{nc} + OE = \Delta PE + \Delta KE.$$

  - Thus, if some of the mechanical energy of a system gets lost, so $\Delta PE + \Delta KE < 0$, we might blame this on non-conservative forces such as friction that do negative work. Or we might find that the mechanical energy has turned into another form such as sound or heat, which can be understood as kinetic energy of molecules.
  - Or, if $\Delta PE + \Delta KE > 0$, it means that some other form of energy, such as your breakfast, has been turned into mechanical energy that got you moving or sent you up a flight of stairs, etc.

7.7 Power

Another important concept related to work and energy is power, a measure of how fast work is being done, or energy is being used:

$$\text{Power} \;= P = \frac{\text{Work}}{\text{time}} \left(\text{Unit: } \frac{\text{J}}{\text{s}} = \text{ Watt} = \text{W.}\right)$$

You have to be careful wit the *W* that represents work, when you now also have the W that represents Watts. You can tell from the context whether it refers to a unit or to work.

The Watt may be familiar to you from the way light bulbs are designated: a 60 W bulb is a common type. A kilowatt is, of course, 1000 W, and you may have heard of this power unit too.

When you buy electricity, you get charged for the amount of energy you use, and the unit for this is usually the kilowatt-hour (kWh) defined as

$$1\,\text{kWh} = 1000\,\text{W} \cdot 1\,\text{hr} \cdot \frac{3600\,\text{s}}{\text{hr}} \cdot \frac{1\,\text{J/s}}{\text{W}} = 3.6 \times 10^6\,\text{J}$$

If you were to turn a crank and keep a 60 W bulb lit for a week, the amount of work it would take is

$$\text{Work} = \left(\frac{60\,\text{J}}{\text{s}}\right) \cdot (7\,\text{days}) \cdot \left(\frac{24\,\text{hr}}{\text{day}}\right) \cdot \left(\frac{3600\,\text{s}}{\text{hr}}\right) \cdot \left(\frac{\text{kWh}}{3.6 \times 10^6\,\text{J}}\right) = 10\,\text{kWh}.$$

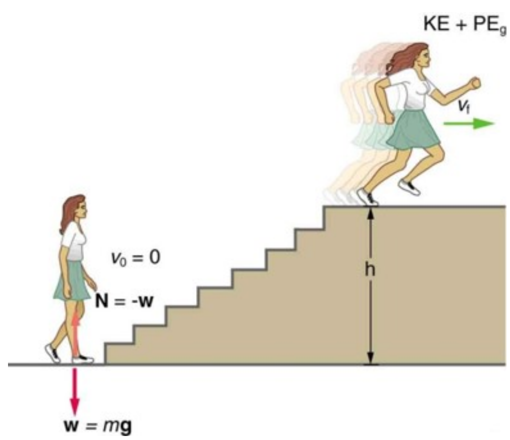
You would probably want to be paid handsomely for doing this job, at least minimum wage. Electric companies charge something like 12 cents per kWh, so you can buy the energy for around $1.20.

And another common unit is the horsepower:

$$\text{Horsepower } = hp = 746\,\text{W}$$

which corresponds roughly to the rate at which a horse can get work done.

If you would like to know your horsepower, try doing something ike the figure below suggests —



Climb a stairs as fast as you can, then divide the work by the time you needed:

$$P = \frac{\text{Work}}{\text{Time}} = \frac{mgh}{t}$$

Suppose a person weighs 1000 N, and can get up a 4 m high flight of stairs in 5 s. Then

$$P = \frac{(1000\,\text{N})(4\,\text{m})}{5\,\text{s}} = 800\,\text{W} \cdot \frac{\text{hp}}{746\,\text{W}} = 1.07\,\text{hp}$$

Another form for Power that is sometimes useful is

$$P = \frac{F \cdot \Delta x}{\Delta t} = F \cdot v$$

for a situation in which a force is moving something at the speed v. For example, a motor that can supply 1 hp is used to lift a load of 500 N. How fast can it lift this load (at constant speed)?

$$v = \frac{P}{F} = \frac{746\,\text{W}}{500\,\text{N}} = 1.49\,\text{m/s}.$$

Time	Topics	Assignments
12:30 - 1:20	Impulse and momentum	
1:30 - 2:20	Collisions	Quiz #4
2:30 - 3:30	Equilibrium, Torque, Statics	

Day 7, Hour 1: Impulse and momentum

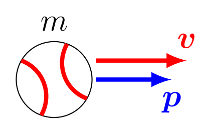
Last time we looked at how one can use conservation of energy to understand how things move. For complicated motion, such as along a curved path, using energy concepts is a better approach than the straightforward use of Newton's laws.

Today we will look at another conserved quantity, momentum, which turns out to be very helpful in situations such as in collisions of objects, when forces are hard to use because they vary with time. In any interaction of objects, momentum is conserved. Momentum is the last of the main topics that make up our study of linear motion.

Next, we will return to the topic of rotational motion, and look brifely at how each of the topics of linear motion has some analogous topic for rotational motion. As a first topic, we will look at torque, the rotational equivalent of a force.

8.1 Linear momentum and force

Momenum is defined as $\boxed{\boldsymbol{p} = m\boldsymbol{v},}$ a vector quantity.
A baseball, with mass $m = 144$ grams moving East at a speed of $45\,\mathrm{m/s}$ would have momentum



$$\boldsymbol{p} = (0.144\,\mathrm{kg})(45\,\mathrm{m/s}) = 6.48\,\mathrm{kg\cdot m/s} \text{ East.}$$

- The units of momentum are just $\mathrm{kg\cdot m/s}$, which is equivalent to $\mathrm{N\cdot s}$; unlike many other combinations of units, this one has not been given anybody's name. Name it after yourself if you like, and see if it catches on.

Newton expressed his second law of motion in terms of momentum by saying

$$\boldsymbol{F}_{\text{net}} = \frac{\Delta \boldsymbol{p}}{\Delta t}$$

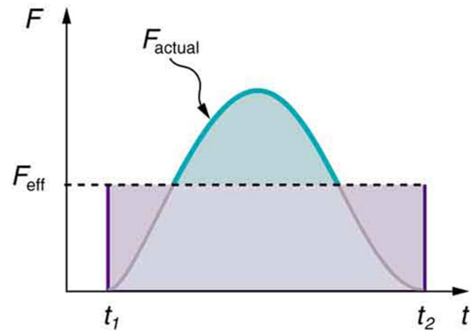
- If m remains constant, then we have the familiar version $\boldsymbol{F}_{\text{net}} = \frac{m\Delta \boldsymbol{v}}{\Delta t} = m\boldsymbol{a}$.
- For cases in which m is also changing, as it does for things like rockets, Newton's expression works too, but that would be rocket science and we won't be dealing with that.
- What we will do, though, is move the Δt from the denominator and put it with $\boldsymbol{F}_{\text{net}}$, and define this quantity as "Impulse" so that Newton's second law says

$$\boxed{\text{Impulse } = \boldsymbol{F}_{\text{net}}\Delta t = \Delta \boldsymbol{p} = \text{ change of momentum.}}$$

  The next section shows how this concept of impulse is useful.

8.2 Impulse

When objects collide, the forces acting between them may be quite complicated, changing in magnitude over time. But the change in momentum that occurs for either object tells us the overall effect, which is the impulse. For example, when a baseball is struck by a bat, it becomes deformed, as in the photo below, and a plot of the force versus time acting on the ball is some sort of curve like the one shown.



- The actual force F_{actual} of the bat on the ball begins at some time t_1, and grows larger as the ball is crushed.
- Then the force begins to decrease as the ball rebounds and finally ends when the ball leaves the bat at time t_2.
- Because the force on the ball changes with time, so does the ball's acceleration.
- If we knew how a varies with time, we could use it, with some difficulty, to calculate the resulting changes in the ball's velocity.
- But it turns out that the same changes would have happened if the force on the ball had been a constant one, shown as the "effective" force F_{eff}, for the same time interval $\Delta t = t_2 - t_1$.
- The impulse is easy to calculate from the F_{eff} function: it is $F_{\text{eff}} \cdot \Delta t$, the area under the force function. The "actual force" curve has the same area under it, hence the same impulse.
- The change in momentum for the ball is equal to this impulse, according to Newton's second law.

Example: A baseball ($m = 144\,\text{grams}$) is pitched at $45\,\text{m/s}$ East, and the batter hits it straight back at $75\,\text{m/s}$ West. What was the impulse on the ball?

Answer: Calling the eastward direction positive,

$$\boldsymbol{F}_{\text{eff}} \cdot \Delta t = \Delta \boldsymbol{p} = \boldsymbol{p}_f - \boldsymbol{p}_i = m\,(\boldsymbol{v}_f - \boldsymbol{v}_i) = (0.144)\,(-75 - 45) = -17.28\,\text{N}\cdot\text{s}$$

This tells us, certainly, that the force of the bat on the ball was in the Westward direction. But it doesn't tell us how the actual force varies with time, or even the average (effective) force $\boldsymbol{F}_{\text{eff}}$, nor does it tell us the time Δt of the collision; we only know the product, the impulse, $\boldsymbol{F}_{\text{eff}} \cdot \Delta t$.

- If we are also given the collision time, say, $\Delta t = 2\,\text{ms} = 0.002\,\text{s}$ then we could also say

  $$\boldsymbol{F}_{\text{eff}} = \frac{\Delta \boldsymbol{p}}{\Delta t} = \frac{-17.28\,\text{N}\cdot\text{s}}{0.002\,\text{s}} = -8640\,\text{N}$$

  which is negative because it is in the Westward direction, as we should expect.

- If the collision time could be made longer, the force would be less.

  Here are some questions related to impulse:

  - What is the reason for air bags in a car?
  - What is a good way to catch a water balloon, or an egg tossed toward you?
  - Why are hammers made of hard materials like steel?

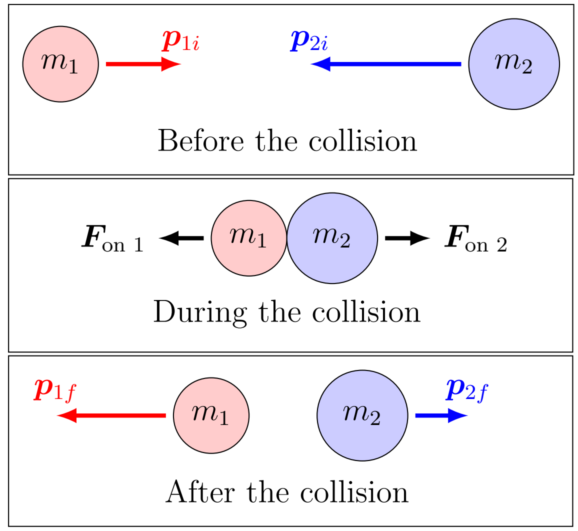
8.3 Conservation of momentum

In the previous example we were looking at how the change in momentum of the baseball told us about the impulse from the bat. Next, let's think about the momentum changes of both objects involved in a collision.

- Before their collision, two masses, m_1 and m_2 are moving with momenta $\boldsymbol{p}_{1i}$ and $\boldsymbol{p}_{2i}$ as shown.
- When they collide, each experiences a force on itself due to the other mass.
- Newton's third law says these forces are opposite but equal in magnitude. And the time of the collision, Δt must be the same for each object. So

$$\Delta \boldsymbol{p}_1 = \boldsymbol{F}_{\text{on }1} \cdot \Delta t = -\boldsymbol{F}_{\text{on }2} \cdot \Delta t = -\Delta \boldsymbol{p}_2$$

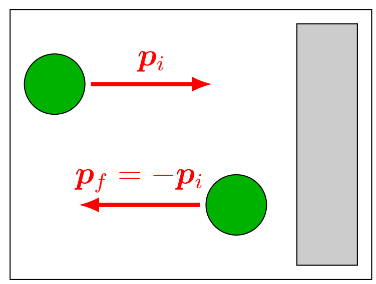
- Each mass has a change in momentum, but the total change for the system of masses is



$$\Delta \boldsymbol{p}_1 + \Delta \boldsymbol{p}_2 = 0 = \Delta(\boldsymbol{p}_1 + \boldsymbol{p}_2)$$

This must be true in any collision or interaction of particles, that the total change in momentum of the particles is zero. Therefore, we can say that

$$\boxed{\boldsymbol{p}_{\text{total}} = \text{constant}} \Rightarrow \boxed{\boldsymbol{p}_{\text{before}} = \boldsymbol{p}_{\text{after}}.}$$



It is not easy to see that this is true in some collisions. For instance, suppose you throw a tennis ball at a brick wall, and it bounces back at you with the same speed. Clearly, the ball has a change of momentum

$$\Delta \boldsymbol{p}_{\text{ball}} = \boldsymbol{p}_f - \boldsymbol{p}_i = -\boldsymbol{p}_i - \boldsymbol{p}_i = -2\boldsymbol{p}_i$$

and the wall doesn't appear to be moving before or after the collision, so it may seem that there is a change in the total momentum of the system.

But the analysis above reminds us that the wall receives an impulse opposite that of the ball: a force of the same magnitude as the ball received also acts on the wall for the same time interval.

- Because the wall is so massive, it doesn't need much velocity to have a momentum as large as required:

$$\boldsymbol{p}_{\text{total}} = \underbrace{\boldsymbol{p}_i + 0}_{\text{before}} = \underbrace{\boldsymbol{p}_{\text{wall}} - 2\boldsymbol{p}_i}_{\text{after}}.$$

- If the wall is attached to the Earth, the system of interacting objects would include the Earth too.
- If you use a less-massive wall, a physics book for example, you can see it recoil, and the book/ball system should have constant momentum:

$$\boldsymbol{p}_{\text{total}} = \boldsymbol{p}_i + 0 = \boldsymbol{p}_f + \boldsymbol{p}_{\text{book}}.$$

- It is also hard to see the conservation of momentum in collisions where friction is at work. Frictional interaction of surfaces is essentially a long-time collision between the surfaces. So we can test some of these ideas better using a nearly frictionless apparatus, an air track.

Day 7, Hour 2: Collisions

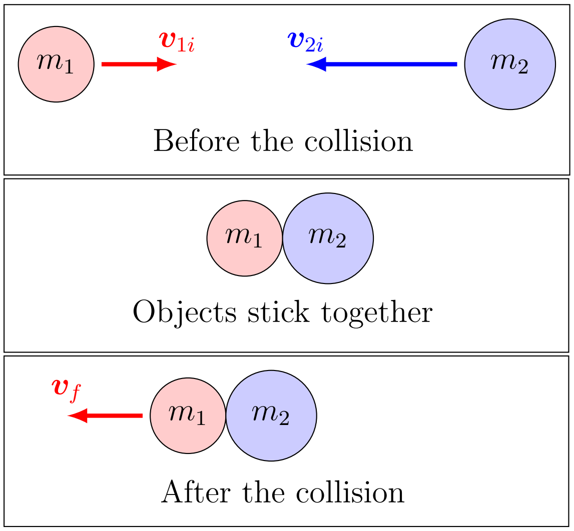
Collisions can be complicated in the real world. To better understand them, it is helpful to consider two idealized types:

- A perfectly inelastic collision is one in which the objects stick together.
- A perfectly elastic collision is one that conserves the total kinetic energy.

In both types, of course, momentum is conserved.

8.5 Inelastic collisions in 1 dimension

A perfectly inelastic collision is one in which the colliding objects stick together.



- They have different initial velocities, $\boldsymbol{v}_{1i}$ and $\boldsymbol{v}_{2i}$, and it is important to correctly specify their signs, because they are vectors. Pick a direction to call positive, and stick with it.
- After they collide, there is a single velocity $\boldsymbol{v}_f$ for both objects.
- There is one equation describing their interaction:

$$\boldsymbol{p}_i = \boldsymbol{p}_f$$

$$m_1\boldsymbol{v}_{1i} + m_2\boldsymbol{v}_{2i} = (m_1 + m_2)\,\boldsymbol{v}_f$$

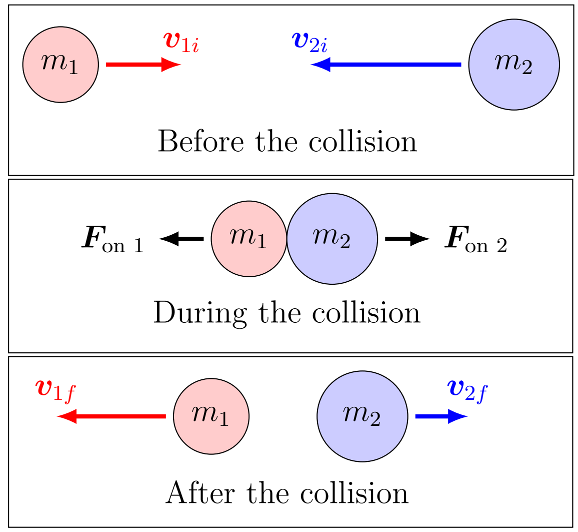
- So if the masses and velocities are known before the collision, then the final velocity can be calculated as

$$\boxed{\boldsymbol{v}_f = \frac{m_1\boldsymbol{v}_{1i} + m_2\boldsymbol{v}_{2i}}{m_1 + m_2} \text{ (Perfectly inelastic collision.)}}$$

Air-track demonstrations

Initial Conditions		Expected
Masses	Velocities	Outcome
$m_1 = m_2$	$v_{2i} = 0$	$v_f = \frac{1}{2}v_{1i}$
	$v_{2i} = -v_{1i}$	$v_f = 0$
$m_1 \ll m_2$	$v_{2i} = 0$	$v_f \approx 0$
	$v_{2i} = -v_{1i}$	$v_f \approx v_{2i}$
$m_1 \gg m_2$	$v_{2i} = 0$	$v_f \approx v_{1i}$
	$v_{2i} = -v_{1i}$	$v_f \approx v_{1i}$

8.4 Elastic collisions in 1 dimension



For a perfectly elastic collision, because kinetic energy is conserved it turns out the two objects must have different final velocities. With some algebra we can derive two equations for the two velocities after the collision. First,

$$KE_f = KE_i \Rightarrow \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2$$

Multiplying by 2, and collecting the m_1 and m_2 parts we have

$$m_1\left(v_{1f}^2 - v_{1i}^2\right) = -m_2\left(v_{2f}^2 - v_{2i}^2\right).$$

But $x^2 - y^2 = (x+y)(x-y)$ so we can write

$$m_1\left(v_{1f} - v_{1i}\right)\left(v_{1f} + v_{1i}\right) = -m_2\left(v_{2f} - v_{2i}\right)\left(v_{2f} + v_{2i}\right)$$

But we can recognize the first terms on each side of the equation as changes of momentum, so

$$\Delta\boldsymbol{p}_1\left(v_{1f} + v_{1i}\right) = -\Delta\boldsymbol{p}_2\left(v_{2f} + v_{2i}\right)$$

and we already saw that $\Delta\boldsymbol{p}_1 = -\Delta\boldsymbol{p}_2$ in collisions to conserve momentum. So we can cancel this term on each side and we are left with

$$v_{1f} + v_{1i} = v_{2f} + v_{2i}.$$

With this equation and the momentum conservation equation,

$$m_1\left(v_{1f} - v_{1i}\right) = -m_2\left(v_{2f} - v_{2i}\right)$$

we can solve for the two unknown velocities. Solving for v_{2f} in each gives

$$v_{2f} = v_{1f} + v_{1i} - v_{2i} \text{ and } v_{2f} = v_{2i} + \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right)$$

so we can set these expressions equal to each other, solve for v_{1f}, then, at last, use v_{1f} to solve for v_{2f}:

$$\boxed{v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i},} \text{ and } \boxed{v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}.}$$

These are not given in the text, but they can be useful in homework, and will be provided for the test.

Air-track demonstrations with $v_{2i} = 0$.

Masses	Expected outcomes: $v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1f}$	Expected outcomes: $v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i}$
$m_1 = m_2$	$v_{1f} = 0$	$v_{2f} = v_{1i}$
$m_1 < m_2$	$v_{1f} < 0$	$0 < v_{2f} < v_{1i}$
$m_1 \ll m_2$	$v_{1f} \approx -v_{1i}$	$v_{2f} \approx 0$
$m_1 > m_2$	$v_{1i} > v_{1f} > 0$	$v_{2f} > v_{1i}$
$m_1 \gg m_2$	$v_{1f} \approx v_{1i}$	$v_{2f} \approx 2v_{1i}$

Quiz #4

Day 7: Hour 3: Rotational motion, Torque

Now that we have gotten through the topics of mechanics for linear motion, our final task is to go back and do it all again for rotational motion. Before we begin, I would like to encourage you that you already can probably guess many of the equations. For many of them, it is simply a matter of trading one set of letters for another. Here is a table that may help:

Linear motion			Rotational Motion			Greek
Name	Symbol/Definition	Unit	Name	Symbol/Definition	Unit	letter
Position	x	m	Angle	θ	rad	theta
Displacement	$\Delta x = x_f - x_i$	m	Angular displacement	$\Delta\theta = \theta_f - \theta_i$	rad	Delta
Velocity	$v = \Delta x/\Delta t$	m/s	Angular velocity	$\omega = \Delta\theta/\Delta t$	rad/s	omega
Acceleration	$a = \Delta v/\Delta t$	$\mathrm{m/s^2}$	Angular acceleration	$\alpha = \Delta\omega/\Delta t$	$\mathrm{rad/s^2}$	alpha
Force	F	N	Torque	τ	$\mathrm{N \cdot m}$	tau
Mass	m	kg	Rotational inertia	I	$\mathrm{kg \cdot m^2}$	
Kinetic energy	$KE = \frac{1}{2}mv^2$	J	Rotational KE	$KE_{\mathrm{rot}} = \frac{1}{2}I\omega^2$	J	
Momentum	$\boldsymbol{p} = m\boldsymbol{v}$	$\mathrm{kg \cdot m/s}$	Angular momentum	$L = I\omega$	$\mathrm{kg \cdot m^2/s}$	

Here are the connections between linear and rotational quantities for describing motion on a circle:

Radius	r
Arc length	$\Delta s = r\Delta\theta$
Velocity	$v = r\omega$
Acceleration	$a = r\alpha$
Centripetal acceleration	$a_c = v^2/r = r\omega^2$
Centripetal force	$F_c = mv^2/r = mr\omega^2$

And here are examples of equations we have used in the linear sense, translated into rotational versions:

	Linear	Rotational
Equations of motion	$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
	$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
Newton's 2nd law:	$F_{\mathrm{net}} = ma$	$\tau_{\mathrm{net}} = I\alpha$
Work:	$W = F \cdot \Delta x$	$W = \tau \cdot \Delta\theta$
Work-energy theorem:	$W_{\mathrm{net}} = \Delta KE$	$W_{\mathrm{net}} = \Delta KE_{\mathrm{rot}}$
Impulse:	$\boldsymbol{F}_{\mathrm{net}} \cdot \Delta t = \Delta \boldsymbol{p}$	$\tau_{\mathrm{net}} \cdot \Delta t = \Delta L$
Momentum conservation:	$\boldsymbol{p}_{\mathrm{total}} = \mathrm{constant}$	$L_{\mathrm{total}} = \mathrm{constant}$

So that's here we're headed in the next couple days; not to re-do everything in detail for rotation, but to get a grasp of what these rotational quantities are, how they correspond to the more familiar linear quantities, and how they affect the way things move.

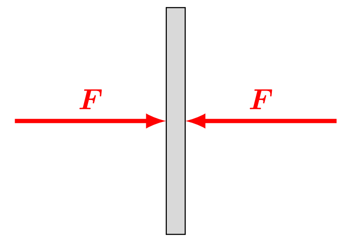
Chapter 9 introduces just one of these new characters, torque,

9.1 First condition for equilibrium: forces

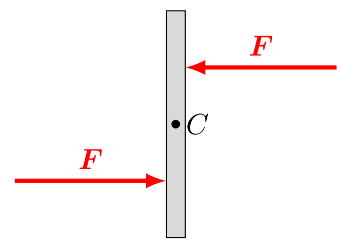
We saw previously in our study of forces that when the net force on an object is zero it does not accelarate:

$$\sum \boldsymbol{F} = 0 \Rightarrow \sum F_x = 0 \text{ and } \sum F_y = 0.$$

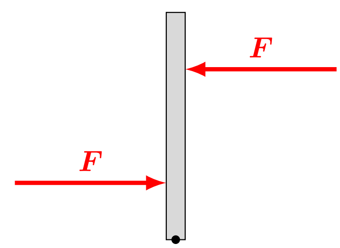
- For example, if an object has opposite forces of equal magnitude on it, as in the upper diagram, we expect that it will not accelerate.



- Nothing should happen. It will not accelerate in the x or y directions, because the net force in both directions is zero.
- But suppose the forces are still opposite in direction, equal in magnitude, but displaced as shown in the middle diagram.



- Now we would expect something to happen instead of nothing. We should expect the object to rotate.
- If it is some kind of uniform object, one with the same distribution of mass everywhere, then we would find that it rotates around its center (marked with a C).
- But suppose one end it is held fixed somehow, so this point can't move. We can call this point the pivot, as our book does. The pivot point is marked with a P in the third diagram. Now what would you expect to happen?
  - Nothing?
  - Rotation clockwise?
  - Rotation counterclockwise?



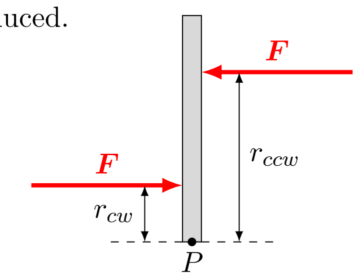
P = pivot point

9.2 Second condition for equilibrium: torques

In the example above, there should be a counterclockwise rotation as a result of the two forces displaced. Each of the two forces produces a torque, which is the rotational version of a force. Torques cause rotational motion similar to the way forces cause linear motion. We will look at the rotational version of Newton's second law next time, but for today we can learn about how a torque is defined and produced.

The definition of torque is made of two quantities, the force applied to the object, and the distance from the pivot point to the place where the force is applied, a distance known as the lever arm $r_\perp$:

$$\tau = r_\perp F \text{ (Unit: N} \cdot \text{m.)}$$



The subcript $\perp$ has an important meaning that can be interpreted in two different ways, as we'll see. But first, take $r_\perp$ as the distance from pivot point P to the place where the force acts on the object.

- In the diagram here, each of the two forces has a lever arm, marked r_{cw} for the lower force, and r_{ccw} for the upper force. The subscripts are to indicate that one force would cause a clockwise torque, that would tend to rotate the object in that direction, and the other force would produce a counterclockwise rotation.
- Which torque wins? Since the forces have equal magnitudes, but the lever arms are different, the larger lever arm produces the larger torque, and the object rotates counterclockwise.

Now we can see that there is a second condition for equilibrium: if there is to be no rotation, then the torques must balance each other. Torques can be summed if we call one direction positive and the other negative. Then the condition for equilibrium is

$$\sum \tau = 0$$

Our book, like most, makes the choice to call counterclockwise torques positive (just as angles are positive in the counterclockwise direction), and clockwise torques are defined as negative. This is arbitrary, of course, like saying up is positive, down is negative, but it can be helpful to have a definition and stick with it.

Lever arm details

Figure 9.6 from the text illustrates how the torque is determined for different situations when a force is applied to a door. The pivot point is the door's hinge, shown connected to a wall.

(a) A force F is applied perpendicular to the door at a distance $r = r_\perp$ from the hinge so

$$\tau_a = rF \text{ counterclockwise}$$

(b) The force is smaller, F', so

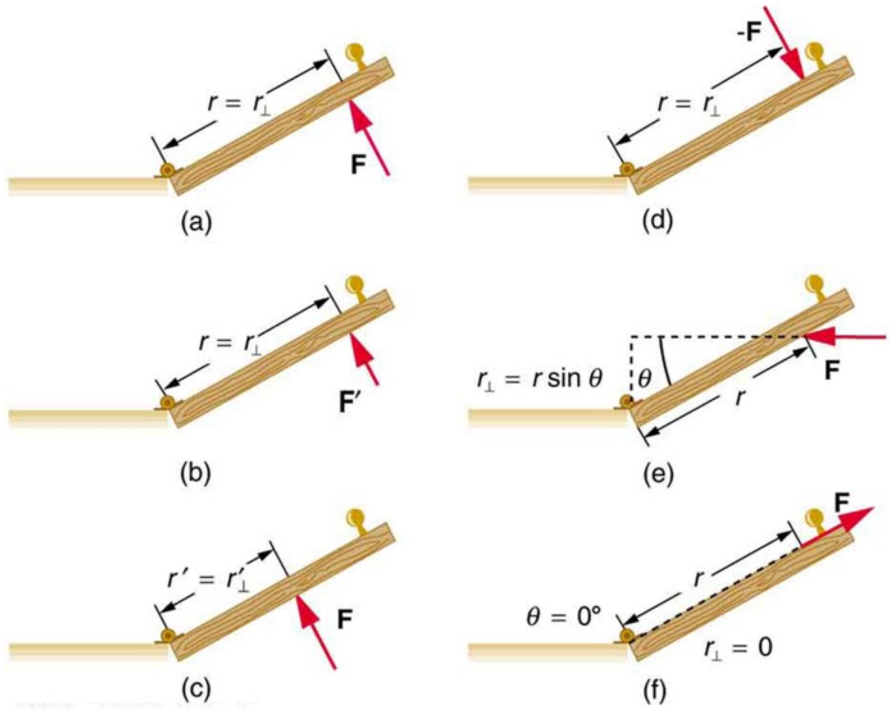
$$\tau_b = rF' < \tau_a$$

(c) The lever arm is smaller, r', so

$$\tau_c < \tau_a$$

(d) The torque is clockwise with F and r, so

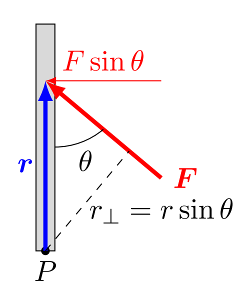
$$\tau_d = -rF = -\tau_a$$



(e) This case shows how to compute the lever arm when the force is not at right angles with the object. The lever arm $r_\perp$ is the perpendicular distance from the pivot point to the line along which the force acts.

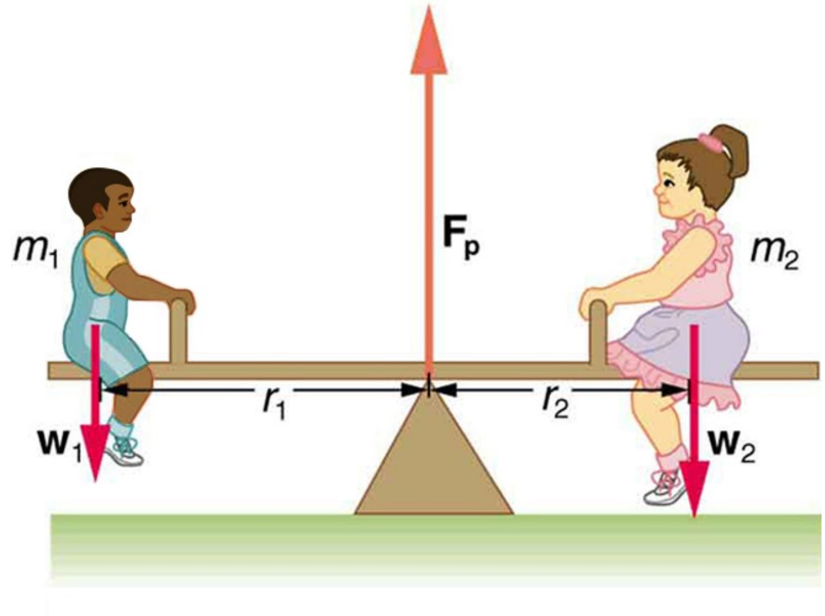
- The easiest general definition is that the force vector $\boldsymbol{F}$ and a vector $\boldsymbol{r}$ from the pivot point to the point where the force acts, make some angle θ, and the torque is $\tau = F \cdot r \cdot \sin\theta$.
- One can say either that the lever arm is the perpendicular distance from P to the line along which the force acts ($r\sin\theta$), or that the force component perpendicular to the vector $\boldsymbol{r}$ is $F\sin\theta$. In either case, the torque is

$$\tau = F \cdot r \cdot \sin\theta.$$



(f) In this diagram, the force is along a line through the pivot point, so there is no lever arm, and no torque. The angle between the vectors $\boldsymbol{F}$ and $\boldsymbol{r}$ in this case is zero, and $\sin 0° = 0$.

Here is a case of equilibrium: two kids on a seesaw which is perfectly balanced.



- There are three forces: the weights of the children, w_1 and w_2, and the normal force at the pivot point, F_P. Because the system is at equilibrium, we can say for the forces,

  $$\sum F_y = F_P - w_1 - w_2 = 0,$$

  and for the torques, with lever arms measured from the pivot point,

  $$\sum \tau = w_1 \cdot r_1 - w_2 \cdot r_2 + F_P \cdot 0 = 0.$$

To show this with numbers, suppose $w_1 = 200\,\mathrm{N}$, $w_2 = 300\,\mathrm{N}$, and $r_1 = 1\,\mathrm{m}$.

- What is F_P?

  Answer: $F_P = w_1 + w_2 = 500\,\mathrm{N}$.

- What is r_2?

  Answer: $w_1 r_1 = w_2 r_2 \Rightarrow r_2 = r_1 \dfrac{w_1}{w_2} = (1)\left(\dfrac{200}{300}\right) = \dfrac{2}{3}\mathrm{m} = 66.6\,\mathrm{cm}.$

Notice that, since the system is at equilibrium, the choice of a pivot point is really arbitrary — the system is not rotating, so it is not rotating around any point we choose. Suppose we choose as pivot point the location of the left seesaw rider.

Now the torque equation would say

$$\sum \tau = w_1(0) + F_p r_1 - w_2(r_1 + r_2) = 0 + (500)(1) - (300)\left(1 + \frac{2}{3}\right) = 500 - 300\left(\frac{5}{3}\right) = 0$$

as it should be.

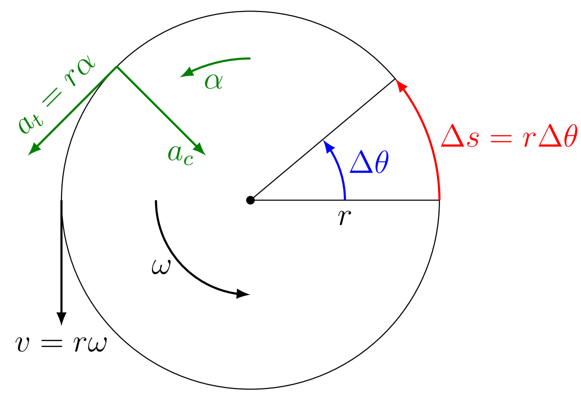
Time	Topics	Assignments
12:30 - 1:20	Rotational kinematics and inertia	
1:30 - 2:20	Rotational energy, Angular mopmentum	Quiz #5
2:30 - 3:30	Collisions, Conservation laws	

Day 8, Hour 1: Rotational kinematics and Inertia

Today we will look in some detail, but briefly, at how the equations of motion, Newton's laws, and the conservation laws for energy and momentum are all applicable to rotational motion. The main goal for today is to learn about the angular quantities, but sometimes it is helpful to also think about their connections to motions tangent to the circle, or radially inward, and to other quantities we have studied previously.

10.1 Angular acceleration

We have looked previously at $\Delta\theta$ and ω, and next we include angular acceleration α (Greek letter "alpha") so that we can describe circular motion similar to the way we did with linear motion.



$$\Delta\theta = \text{ angular displacement. Units: rad.}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \text{ angular velocity. Units: rad/s} = \text{s}^{-1}.$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \text{ angular acceleration. Units: rad/s}^2 = \text{s}^{-2}.$$

This definition pertains to a point on the rim of a circle that is accelerating tangent to the circle because its angular velocity is changing with time. As with linear acceleration, it is certainly possible that α could also be changing with time, but we won't go any further with that possibility. It will be enough for us to assume α is a constant.

Whether there is a non-zero α or not, there is always centripetal acceleration a_c toward the center of the circle. We can put the angular motion quantities together into equations for circular motion that are analogous to our linear equations, such as

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2.$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \omega_{\text{ave}} t \text{ (always.)}$$

$$\omega_{\text{ave}} = \frac{1}{2}(\omega + \omega_0) \text{ (if } \alpha \text{ is constant.)}$$

Now, here are some example problems to try:

1. A phonograph record accelerates from rest to 41 rpm in 6.03 s.
   (a) What is its angular acceleration in rad/s^2)? (Answer: 0.712 rad/s^2.)
   (b) How many revolutions does it go through in the process? (Answer: 2.06 rev.)

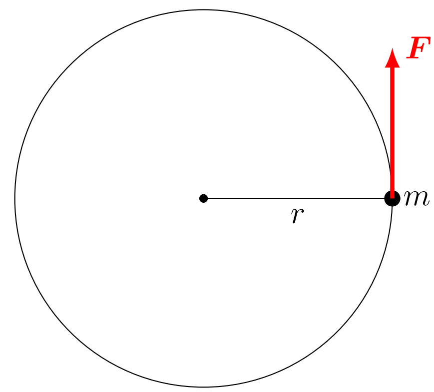
2. In a laundromat, during the spin-dry cycle of a washer, the rotating tub goes from rest to its maximum angular speed of 8.2 rev/s in 9.8 s. You lift the lid of the washer and notice that the tub decelerates and comes to a stop in 16 s. Assuming that the tub rotates with constant angular acceleration while it is starting and stopping, determine the total number of revolutions undergone by the tub during this entire time interval. (Answer: 106 rev.)

10.3 Dynamics of rotational motion: rotational inertia

After learning about the equations of motion for linear motion, we went on to look at Newton's laws to account for why things move as they do, introducing the mass m (linear inertia), the force $\boldsymbol{F}$ and ther relationship $\boldsymbol{F} = m\boldsymbol{a}$.

For rotational motion, the plan is the same: find the inertia of a rotating object, and then express Newton's second law for rotation. The symbol for rotational inertia is I, but what is it?

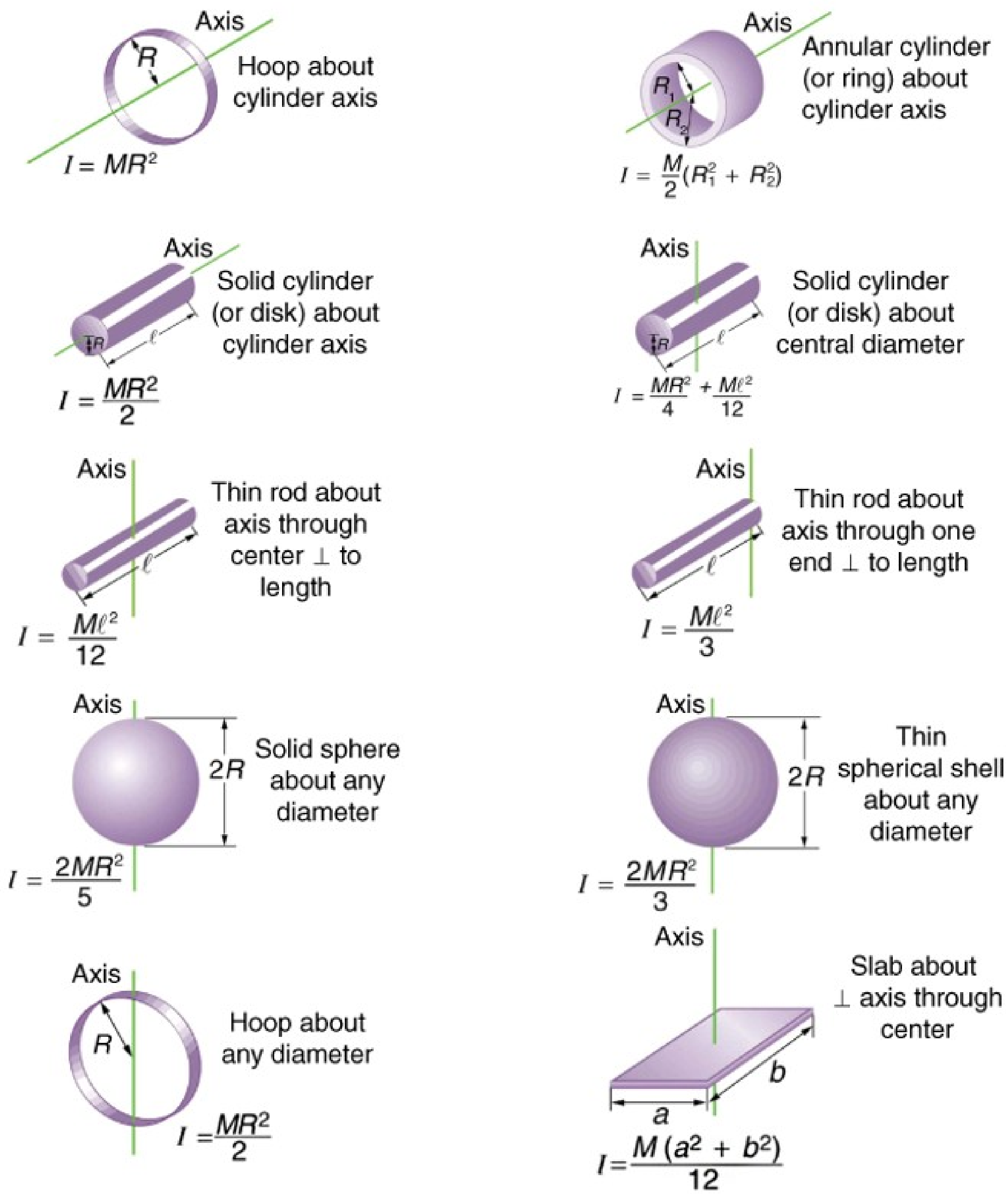
Imagine accelerating a mass around the rim of a circle, at a radius r, using a force $\boldsymbol{F}$ as shown.



- As usual, we can say that $F = ma_t$, where a_t is the tangential acceleration, which is related to the angular acceleration by $a_t = r\alpha$, so
$$F = mr\alpha$$
- Instead of continuing to think about force however, because we are considering circular motion we should find the torque applied to this mass, to make it go around the circle.
- As we saw last time, the torque is the force multiplied by the lever arm; in this case it is simply
$$\tau = rF = r(mr\alpha) = mr^2\alpha.$$
- If we make the substitution
$$I = mr^2 \text{ (Rotational inertia of a point mass } m \text{ at distance } r \text{ from axis of rotation)}$$
then the torque produces an angular acceleration proportional to I:
$$\boxed{\tau = I\alpha.}$$
This is the rotational version of Newton's second law, $F = ma$.

All we have done so far is find the inertia of one particle of mass, but the result tells us the important rule, that each particle adds an amount of inertia that depends not only on its mass, but also its distance from the axis of rotation.

There was nothing that would prevent us from doing the same calculation for each particle of a larger collection of masses. What we would get is
$$I = \sum_i m_i r_i^2,$$
in other words, add the inertias of each individual particle until we have the total for a group of masses. For any complicated object made of molecules, this could be difficult. But if the object is symmetric with a simple shape, it turns out to be easy to do (with calculus mostly). Here are some results:



Notice that

- in each case, the total mass is given as M. If the object is round, R is the outer radius.
- In each case, the axis of rotation is specified; this is necessary so that each particle's distance r from the axis is specified too.
- The first one should be obvious: for a thin ring, every particle is (ideally) at the distance R from the axis, so we just add all the particle masses and multiply by R^2.
- But then, what about a solid cylinder — does it make sense that this has less inertia? Only the outermost layer of molecules will be at distance R, and all the rest are at closer distances, so it should make sense that is is less than the thin ring. But it is amazing that it comes out so simple — just half the inertia of the thin ring.

- Then what would you expect for a sphere with mass M and radius R? Can you see why more of the mass is closer to the axis than it was for the disk? Therefore I turns out smaller again, only $\frac{2}{5}MR^2$.
- One more case: for a thin rod (a meter stick is a fair approximation) rotated around its center, does it make sense that this has mch less inertia ($\frac{1}{12}ML^2$) than for the stick rotated around one end ($\frac{1}{3}ML^2$)? More of the mass is close to an axis through the center.

Here is an example where the rotational inertia plays a role:

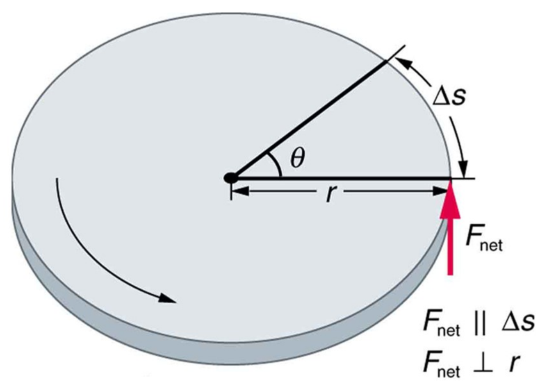
1. Suppose you exert a force of 177 N tangential to a 0.295 m radius, 75 kg grindstone (a solid disk).
   (a) What torque (in N · m) is exerted? (Answer: 52.215 N · m.)
   (b) What is the angular acceleration (in rad/s^2) assuming negligible opposing friction? (Answer: 16 rad/s^2.)
   (c) What is the angular acceleration (in rad/s^2) if there is an opposing frictional force of 19.6 N exerted 2.75 cm from the axis? (Answer: 15.835 rad/s^2.)

Day 8, Hours 2-3: Rotational energy, Angular momentum

There are some surprising behaviors in the rotational motion of objects. We will need to know about the kinetic energies of different objects, so we need to look at that first, and then the angular momentum.

10.4 Rotational kinetic energy and work

Recall the definition we used a couple days ago, that work is a force multiplied by a displacement. Now apply this to angular displacement; we can use Fig. 10.15 from our text, and see that the net force $\boldsymbol{F}_{\text{net}}$ tangent to a circle causes a displacement Δs, so the work done is



$$W = F_{\text{net}} \cdot \Delta s = (r \cdot F_{\text{net}}) \frac{\Delta s}{r} = \tau_{\text{net}} \cdot \Delta\theta.$$

This is just what we would have expected, thinking of torque as analogous to force, and angular displacement analogous to linear displacement.

Keeping these analogies going, we can readily guess that when work has been done to get an object spinning at angular speed ω, it has a kinetic energy equal to the work done:

$$\boxed{W_{\text{net}} = \Delta KE_{\text{rot}},}$$

just as for linear motion, but where the rotational kinetic energy is

$$\boxed{KE_{\text{rot}} = \frac{1}{2} I\omega^2.}$$

We have a table of rotational inertias. Suppose we compare the kinetic energies of a thin ring, a solid disk, and a solid sphere, each with mass M, radius R, and spinning around the axis of symmetry through the center with angular speed ω. Then

Object	I	KE_{rot}
Ring	MR^2	$\frac{1}{2}MR^2\omega^2$
Disk	$\frac{1}{2}MR^2$	$\frac{1}{4}MR^2\omega^2$
Sphere	$\frac{2}{5}MR^2$	$\frac{1}{5}MR^2\omega^2$

Now, let's take what we know about energy conservation, and include rotational kinetic energy as another form of mechanical energy. So, for example, if a circular object rolls down a hill, we can say that its gravitational potential energy is converted into two kinds of kinetic energy, linear and rotational:

$$PE \to KE + KE_{\text{rot}}$$

So, as it gets to the bottom of the hill, rolling all the way,

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2.$$

We can go a couple steps further, because

- there is a connection between v, the linear speed, and ω, the angular speed. Because the object is rolling, the speed it travels is $v = R\omega$, where R is the radius of the rolling object.
- there is a relationship between M and I for a symmetrical rolling object. Let us say that $I = f \cdot MR^2$, where f is some fraction; $f = 1$ for a ring, $\frac{1}{2}$ for a disk, $\frac{2}{5}$ for a solid sphere.

Then

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2} \cdot f \cdot M\left(R^2\omega^2\right) = \frac{1}{2}Mv^2 + \frac{1}{2} \cdot f \cdot Mv^2 = \frac{1}{2}Mv^2\left(1+f\right).$$

It is possible then to express the speed v of the rolling object at the bottom of the hill this way:

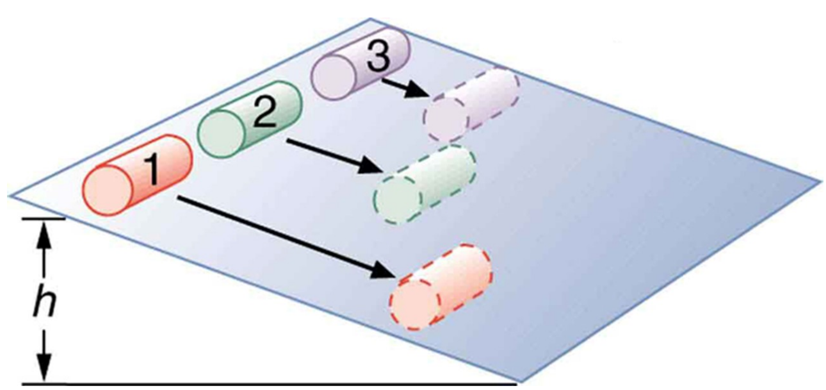
$$v = \sqrt{\frac{2gh}{1+f}},$$ where f is the fraction with MR^2 in the object's inertia I.

Notice that the mass M and the radius R are not in this result — they don't matter, only the type of object matters.

In a rolling race, which will win:

- a ring,
- a solid disk,
- a solid sphere,
- a spherical shell?

In what order will they arrive?

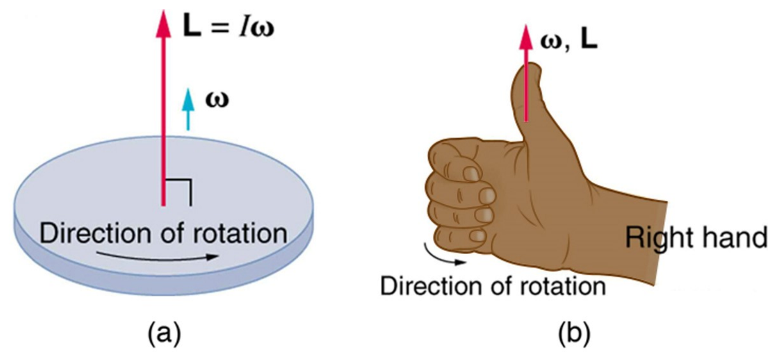


10.5 and 10.7 Angular momentum and conservation

As with the other rotational quantities, we can instantly produce an equation based on analogies with linear motion. A spinning object has a form of momentum analogous to $\boldsymbol{p} = m\boldsymbol{v}$, which is called angular momentum

$$\boldsymbol{L} = I\boldsymbol{\omega}.$$

Both $\boldsymbol{L}$ and $\boldsymbol{\omega}$ are taken to be vector quantities, but it may not be very clear what are their directions, since a spinning object has parts moving in many directions as once. The direction is defined along the axis of rotation, as this figure from our text indicates. The hand in the picture is a right hand, and the rule is one of many right-hand rules in physics. For this case, let your fingers wrap around in the direction of rotation, and your thumb point in the direction of $\boldsymbol{\omega}$ and $\boldsymbol{L}$. So $\boldsymbol{L}$ is perpendicular to the plane of a spinning disk, and it points along the axis of a spinning sphere.



This may seem artificial, but you can feel that it is real if you try spinning a wheel with an axle you can hold onto, and then try changing the direction of the axle; it opposes changes in direction, which hints at the idea of conservation of angular momentum.

Now that we have a definition for $\boldsymbol{L}$, we can write Newton's second law in the form of impulse/change of momentum, just as in linear motion:

$$\text{Impulse } = \boldsymbol{\tau}_{\text{net}} \cdot \Delta t = \Delta \boldsymbol{L}$$

so a net torque applied to an object changes its angular momentum.

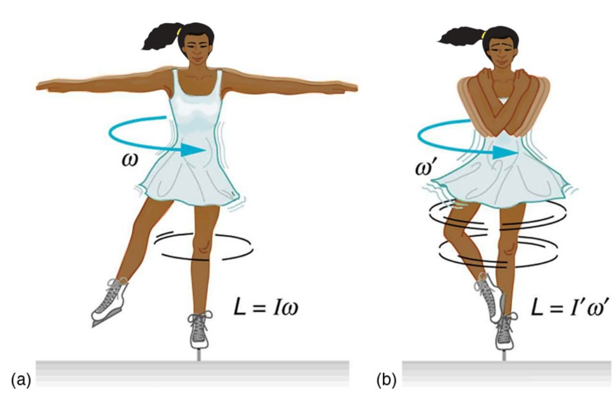
Example; A solid cylinder with a mass of 266 kg and radius 0.065 m is rotating with an angular speed of 89 rad/s about an axis passing through its center and perpendicular to its end faces. The rotation of the cylinder is slowed down by a factor of 5 by applying a tangential frictional force to it for 4.7 s. What is the magnitude, in N, of the friction force applied to the cylinder? (Answer: 131 N.)

We can also express the conservation of angular momentum, just as simply as for linear momentum $\boldsymbol{p} = \text{constant}$:

$$\boxed{\boldsymbol{L} = \text{constant.}}$$

As with linear momentum, this holds true when all the interacting parts of a system are considered. The grindstone in the previous example has a change in angular momentum, so something else must have had an opposite change. We may not be always interested in that bigger picture. But it is a very practical fact in athletics, such as the actions of a figure skater, diver, or gymnast:

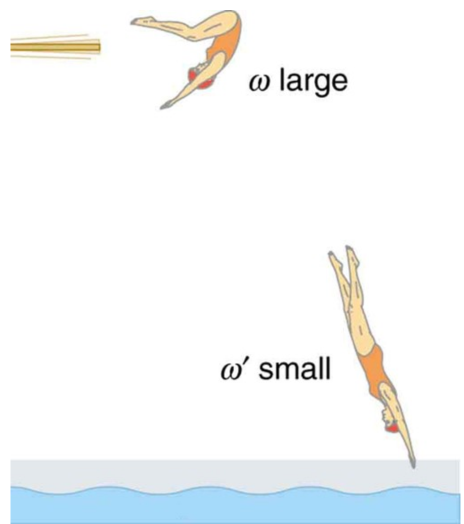
- In part (a) the skater with her arms out has angular speed ω to go with her rotational inertia I and therefore has angular momentum $L = I\omega$.
- By bringing her arms inward, she reduces her rotational inertia to I', and yet, with no external torques applied, her angular momentum must remain L, so



$$L = I\omega = I'\omega' \Rightarrow \omega' = \frac{I}{I'}\omega, \text{ so she spins faster.}$$

The opposite effect is illustrated for the diver in this figure from the text:

- Initially curled in a tuck position, the diver has a small inertia I, and a large angular speed ω around a horizontal axis.
- By straightening out, the diver increases her inertia around the horizontal axis to I', and slows the rotation speed to ω' that is smaller, and hits the water having rotated to the position she intended.



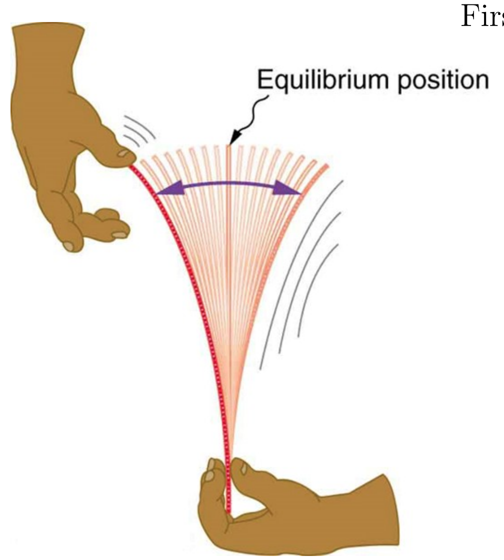
Demonstration

Example: Three children are riding on the edge of a merry-go-round that is 122 kg, has a 1.60 m radius, and is spinning at 19.3 rpm. The children have masses of 17.4, 30.5, and 36.8 kg. If the child who has a mass of 36.8 kg moves to the center of the merry-go-round, what is the new angular velocity in rpm? (Answer: 25.8 rpm.)

Time	Topics	Assignments
12:30 - 1:20	Oscillations, Simple harmonic motion	
1:30 - 2:30	Groups A & C Review, practice exam	
2:30 - 3:30	Group B review, practice exam	

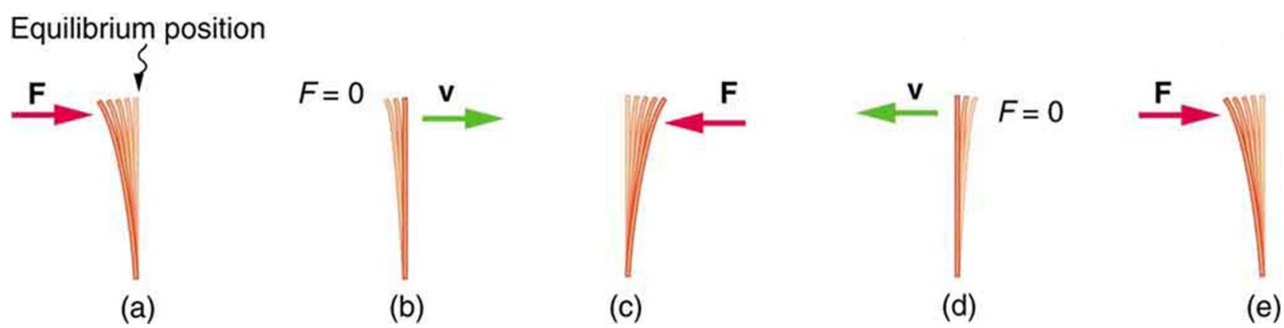
Day 9, Hour 1: Oscillations, Simple harmonic motion

As a last topic in mechanics, we'll take a quick look at oscillations. Lots of things oscillate, and the principles apply to all sorts of things in life, such as hearing, color vision, music, child care, cell phones, and of course physics. We'll look at simple things, but the principles apply to and describe many situations.



First, some simple descriptive terminology:

- Figure 16.2 from our text pictures a plastic ruler waggling back and forth around its equilibrium position.
- It has a displacement that varies with time, and the displacement goes to the positive and negative side; in other words, it oscillates, and it does this at at some regular frequency.
- It oscillates because, when it is displaced, there is a restoring force toward the equilibrium position. If the displacement is to the left, the force is to the right, and vice versa.
- The largest displacement is known as the amplitude. This would remain constant if no energy got lost, but it always does.



16.1 Hooke's law - force constant k

Hooke was one of the great scientists of the 1600s, contemporary with Newton. He worked on theories of gravity, and memory, he designed clocks, and was a keen observer with his telscopes and microscopes. He wrote a book on microscopy in which he is the first to describe the cell as a biological unit. One of his discoveries in physics, now called Hooke's law, said that for an elastic material:

Ut tensio sic vis

in Latin, which in English means "as the extension, so the force." In other words, the more a spring is stretched, the harder it pulls back. Hooke was quite proud of this discovery, and wanted other scientists to know he had discovered it, but also wanted to keep it a secret that only he knew. So he took all the letters from the Latin phrase in alphabetical order, and published it in this form, as a puzzle:

ceiiinosssttuv,

and then two years later published the answer. In two hours you could discover the same thing with an experiment like the one shown in Fig. 16.4:

- Add different masses to a hanging spring, and measure how far its end moves from its equlibrium position.

- Type your data into Excel, dump it into WAPP$^+$, and use the slope of the line as the force constant k in

$$F = kx$$

- The extension, or displacement, x, is proportional to the force.

- This line could go negative as well, like the piece I have added to the plot.

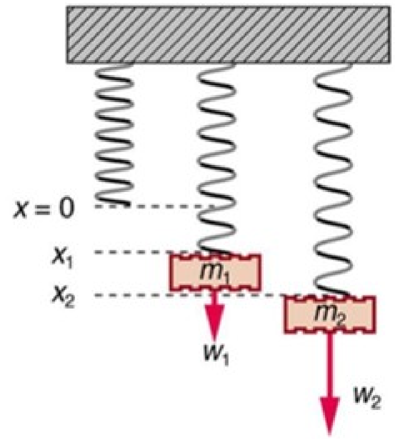
- This means that compressing the spring also requires a force proportional to amount it is compressed.

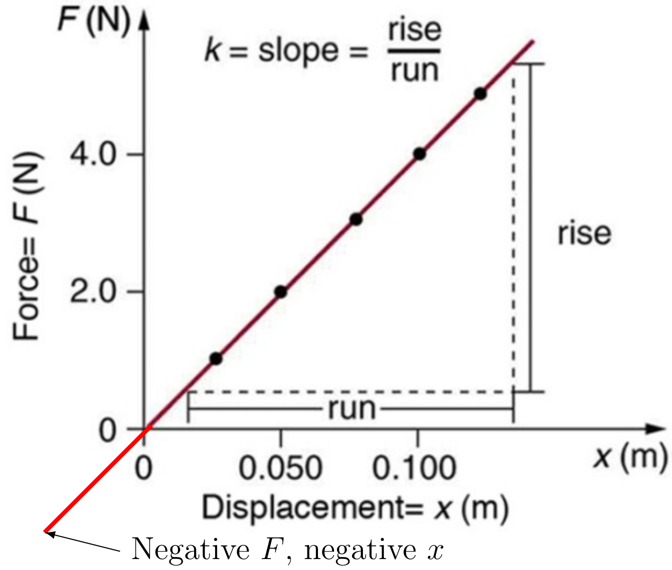
- Whether the spring is compressed or stretched, the force the spring exerts is toward the equilibrium position, so we could say

$$F_s = -kx$$

- This is what is meant by a restoring force, and it is one of the keys to understanding oscillations.

m (kg)	w (N)	x (m)
0.000	0.00	0.000
0.100	0.98	0.025
0.200	1.96	0.050
0.300	2.94	0.076
0.400	3.92	0.099
0.500	4.90	0.127



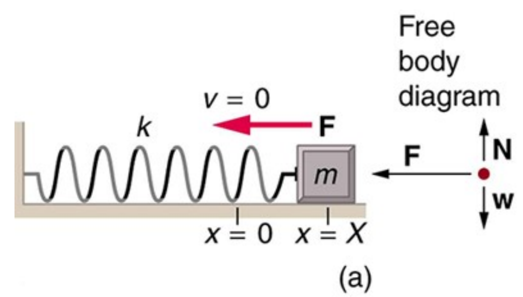


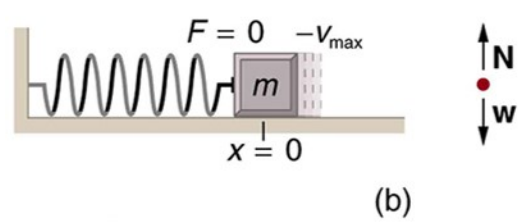
We can make the picture a little simpler by thinking about horizontal motion, and ignoring friction, as in our book's Fig. 16.9. Newton's first and second laws tell us what happens:

- As the little free-body diagram indicates, the weight w and the normal force N remain opposite and equal. If there is a net force on the mass, it is due to the spring.

- When the spring is stretched to a positive x, there is a negative acceleration. From the maximum displacement $x = X$, it begins moving back toward 0.

- The mass is accelerated toward $x = 0$ until it reaches the equlibrium point, so this is where it reaches a maximum velocity in the negative direction. Both x and the acceleration here are zero.



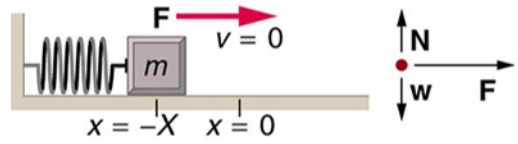


- Of course, with a negative velocity it doesn't stop at the equilibrium point, but keeps going, even while the force decelerates it, until it reaches the opposite extreme at $-X$.

- And from there the compressed spring forces it back toward equilibrium again, where it arrives with maximum speed once more, and keeps going.

- As you'd expect by now, the mass is decelerated by the force of the stretching spring, and finally stops at $+X$ again.

- The cycle of oscillation repeats again and again, with a frequency that depends on the two physical quatntities, m and k.



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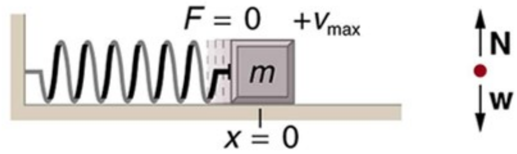
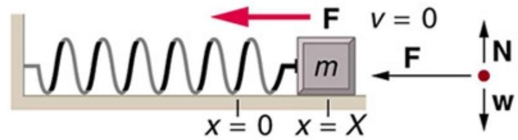


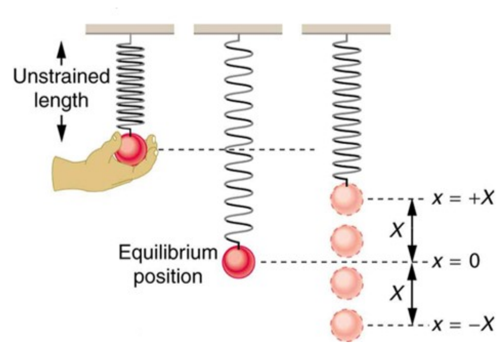
(d)

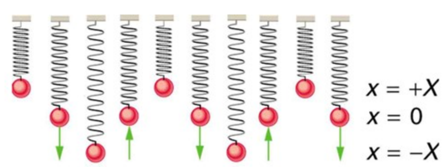


(e)

- As mentioned, Newton's second law describes this oscillating behavior by saying that the force the spring exerts accelerates the mass: $F_s = ma = -kx$, which we can rewrite it the form

$$a = -\frac{k}{m}x$$

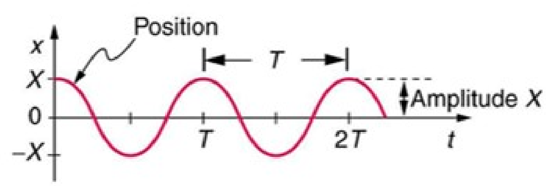




- It would be nice to have an equation of motion for this oscillating mass, and it turns out there is a simple one:

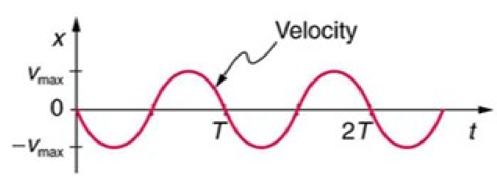
$$\boxed{x(t) = X\cos\left(\frac{2\pi}{T}t\right).}$$

To see that the equation fits, look at the vertical oscillations of m. A cosine function oscillates between ±1, so the function for $x(t)$ oscillates between $\pm X$. The time period for each oscillation is T.



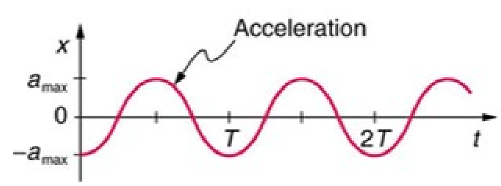
- The velocity, meanwhile, oscillates with the same period T, but not as a cosine function, rather, it is

$$\boxed{v(t) = -v_{max}\sin\left(\frac{2\pi}{T}t\right).}$$



- And the acceleration, as Newton's second law told us, is always in the opposite direction from x: it is written as

$$\boxed{a(t) = -\frac{k}{m}X\cos\left(\frac{2\pi}{T}t\right).}$$



16.2, 16.3, 16.5 Simple Harmonic Motion

Oscillations that go like sine and cosine functions are known as simple harmonic motion, (SHM). This is what always happens when there is a restoring force proportional to the displacement. Because of Newton's second law, the acceleration is in a form like this, proportional to the displacement:

$$a = -(\text{constant})x$$

and the constant determines the behavior of the oscillator. In the example of the mass on a spring, this constant was k/m and we find

1. the period turns out to depend only on the quantities m and k. By measuring the period as m is varied you could discover that

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}.}\ \text{(Unit: s.)}$$

2. The frequency (oscillations per second) is just the inverse of T (seconds per oscillation) so

$$\boxed{f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.}\ \text{(Unit: s}^{-1} = \text{Hertz} = \text{Hz.)}$$

3. The maximum velocity v_{max} turns out to be related to k and m, and also to the amplitude (the maximum displacement) X. It is

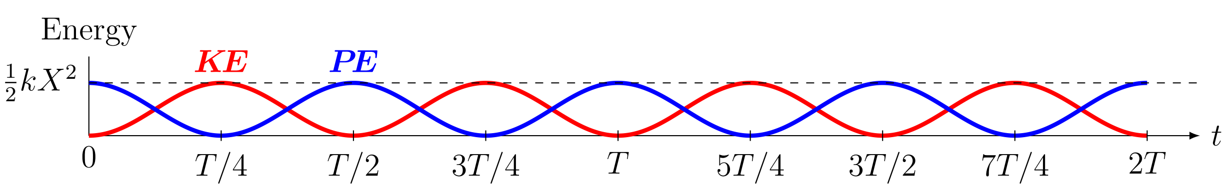
$$\boxed{v_{max} = \sqrt{\frac{k}{m}}X.}\ \text{(Unit: m/s.)}$$

From these expressions we can see how energy is switching between potential and kinetic forms:

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}kX^2\cos^2\left(\frac{2\pi}{T}t\right), \text{ and}$$

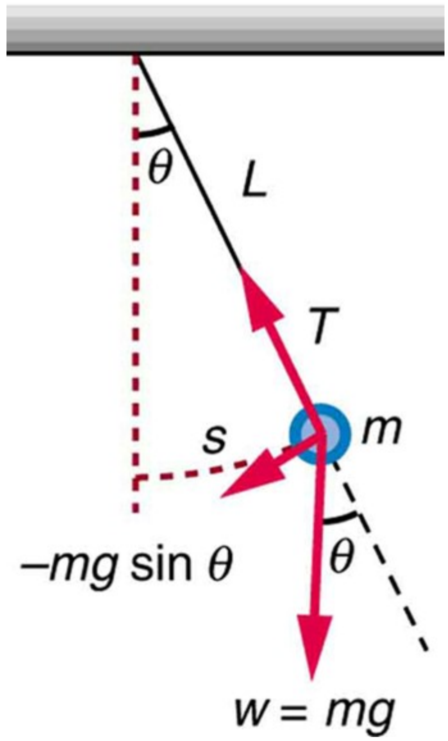
$$KE = \frac{1}{2}mv^2 = \frac{1}{2}mv_{max}^2\sin^2\left(\frac{2\pi}{T}t\right) = \frac{1}{2}m\left(\frac{k}{m}X^2\right)\sin^2\left(\frac{2\pi}{T}t\right) = \frac{1}{2}kX^2\sin^2\left(\frac{2\pi}{T}t\right), \text{ so}$$

$$PE + KE = \frac{1}{2}kX^2\left[\cos^2\left(\frac{2\pi}{T}t\right) + \sin^2\left(\frac{2\pi}{T}t\right)\right] = \boxed{\frac{1}{2}kX^2 = \text{Total energy}} = \text{ constant.}$$



- Initially, when displaced to the amplitude X, the oscillator has potential energy $\frac{1}{2}kX^2$.
- When released, it loses PE until it reaches the equilibrium position, where it has maximum speed and all the energy is KE.
- As it continues on toward $-X$, it loses KE, and all the energy converts back to PE.

16.4 Simple Pendulum



Another important oscillator to understand is the simple pendulum. In this device there is also a switching of potential and kinetic energy at a frequency that depends on how it is constructed.

- Here "simple" means that there is a small mass at the end of a long string that doesn't stretch.
- If the mass was a large sphere, it would get some rotational KE as it swings, and that would change its period.

Figure 16.14 from our text shows a nice diagram of the forces on the mass, and the period of the pendulum is figured out in the book. It may be slightly easier to understand what happens if we look at the torque that the force of gravity produces. We can say either

- $L \sin\theta$ is the lever arm for the force mG, or
- $mg \sin\theta$ is the force perpendicular to L

From either point of view, the torque on the pendulum is

$$\tau = (-mg)(L)\sin\theta.$$

Now, according to Newton's second law,

$$\tau = I\alpha$$

and for a small mass at a distance L from the pivot point, $I = mL^2$, so

$$-mgL\sin\theta = mL^2\alpha \Rightarrow \alpha = -\frac{g}{L}\sin\theta.$$

This would be another example of a simple harmonic motion if instead of $\sin\theta$ the equation had θ. But $\sin\theta \approx \theta$ if the angle θ is restricted to being small, say about 0.25 rad (about 15°) or less. (Try a small angle in radians on your calculator to see that $\sin\theta \approx \theta$.) In this case, we can say

$$\alpha = -(\text{constant})\theta$$

which is the SHM relationship, and this time the constant is g/L. Knowing this, we can write down immediately that

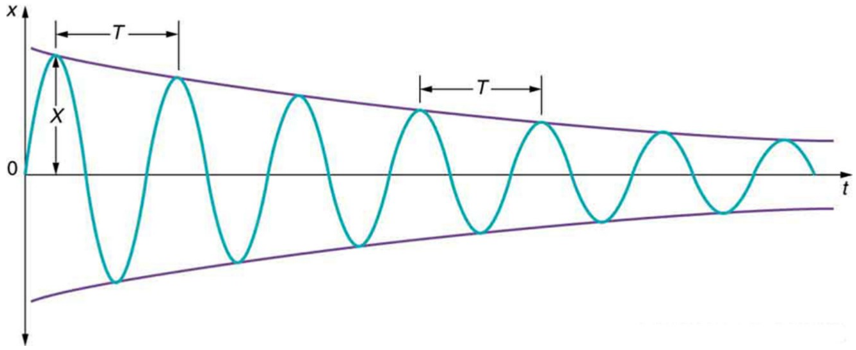
$$\text{Period} = \boxed{T = 2\pi\sqrt{\frac{L}{g}},} \text{ and Frequency } = \boxed{f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}.}$$

16.7 Damped harmonic motion

Endless simple harmoinic motion is an idealization, but for real oscillators there is always some mechanism for energy to be lost, such as friction, or fluid drag. In that case, the theory is more complicated, but the outcome looks like Figure 16.22 from our text: the oscillations grow smaller over time, but with a regular period T. We could apply what we know from energy conservation and say

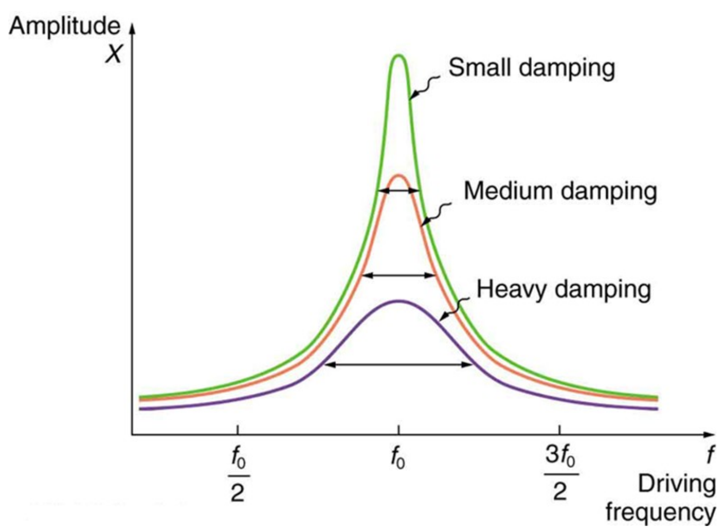
$$\Delta KE + \Delta PE = W_{nc}.$$

A graph like this could represent a mass on a spring or a simple pendulum oscillating with decreasing amplitude. The amplitude at any time gives the amount of mechanical energy remaining, using $\frac{1}{2}kX^2$.



Some oscillators are designed to get rid of energy as quickly as possible. For example, the shock absorbers in a car would ideally bring the vehicle back to the equilibrium position quickly after the car hits a bump, with only one or even zero oscillations. But if they wear out, the passengers may feel the bumps too directly, or oscillate too long afterwards.

16.8 Forced oscillations and resonance



Most oscillators have damping, and in order to keep them oscillating a force must be applied regularly, at approximately the frequency f_0 with which the system naturally oscillates. Figure 16.27 of our text shows what happens, in general, when an oscillator is driven at various frequencies above and below f_0.

- If the driving frequency is too small, the oscillator cannot reach high amplitudes — it gives up energy faster than it receives it.
- And if the driving frequency is too high, the amplitude never gets large either. The pushes will not be synchronized with the displacement as they are in a harmonic oscillator.
- So if a playground swing oscillates with a period of $T_0 = 5\,\text{s}$, frequency $0.2\,\text{Hz}$, then that is the frequency with which you must push to send it to large amplitudes.
- With each push, some energy is added to the oscillator, and if this is more than the energy lost in a period, then it gradually gains energy.
- As the graph shows, when the damping is small, it is possible to reach very large amplitudes when the driving frequency is close to the natural frequency of the oscillator.

Time	Topics	Assignments
12:30 - 1:20	Review Test 2, start Density & Pressure	
1:30 - 3:30	Pascal's principle, Archimedes' principle	

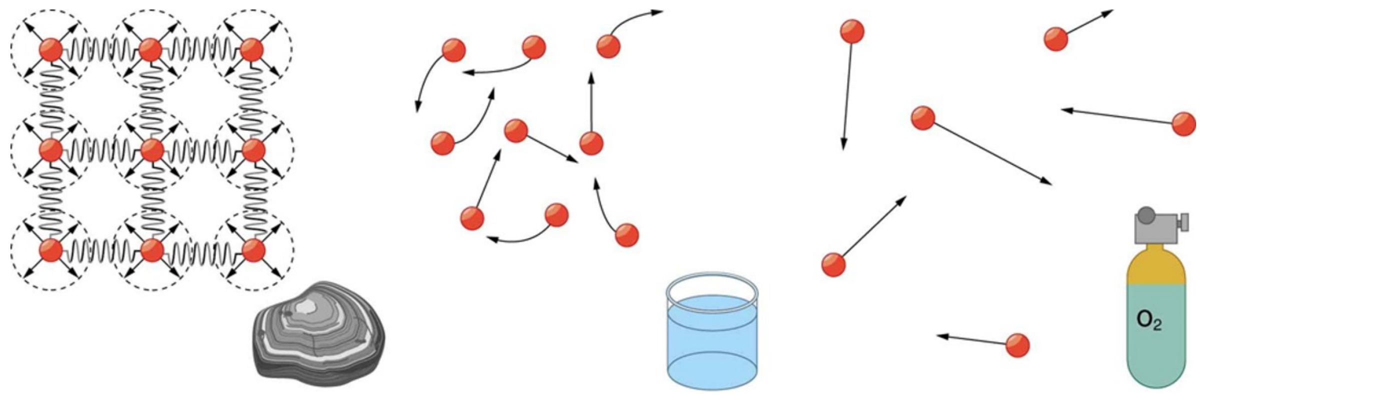
Day 11, Hour 1: Review Test 2, Density & Pressure

Matter is made of atoms that are in constant motion. They tend to attract each other when they are close enough together, but repel if they are pushed too close together.

In the coming chapters we will be looking at properties of matter in three forms: solids, liquids, and gases. The temperature has a lot to do with the form that matter takes, and the temperature is a measure of the kinetic energy of its molecules. For today, we will look at some behaviors that liquids and gases share because their molecules can move around freely.

11.1 What is a fluid?

A fluid is matter that flows: a liquid or gas. Our text shows in Fig. 11.2 these three forms of matter.



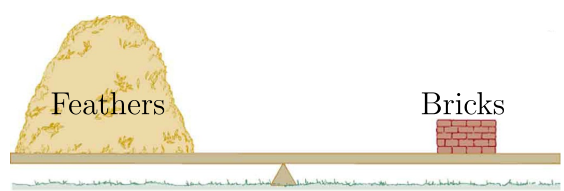
Solid: Form of matter in which molecules are close together, with fixed positions they vibrate around. A solid has a definite shape and volume. At higher temperature a solid melts into a liquid.

Liquid: Matter in which molecules are close but move freely, giving variable shape but definite volume. At higher temperature the liquid vaporizes into a gas.

Gas: Matter in which molecules interact little, moving freely and giving both variable shape and volume to the gas.

11.2 Density

We can weigh a particular object, but it doesn't make sense to ask the weight or mass of a particular substance, such as iron or wood, unless we also say how much iron, how much wood. There is a property that is good for distinguishing the substances themselves, however, density:



$$\text{Density} = \rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{V} \qquad (\text{Unit: } \frac{\text{kg}}{\text{m}^3}.)$$

The letter used for density is "rho," pronounced "row," the equivalent of "r" in Greek. Our book's Table 11.1 lists densities for some different substances, and a few will be useful to have in mind, such as in this shorter table.

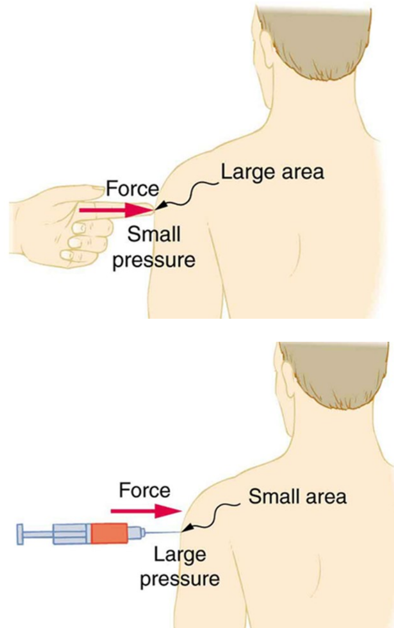
- Gases, such as air and helium, are much less dense than liquids or solids.
- The density is a property that is involved in almost all the fluid behaviors we will be looking at.

Substance	ρ (kg/m^3)
Mercury (Hg)	13 600
Lead	11 300
Iron	7800
Seawater	1025
Water (4 °C)	1000
Ice (0 °C)	917
Air (sea level, 20 °C)	1.29
Helium	0.18

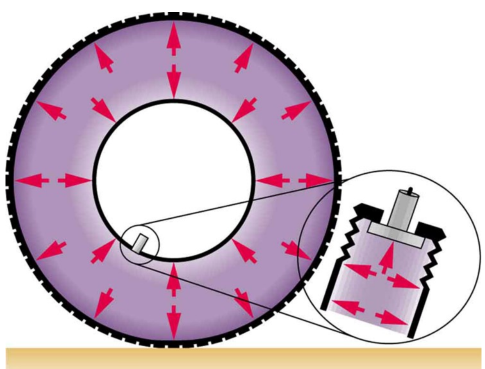
11.3 Pressure

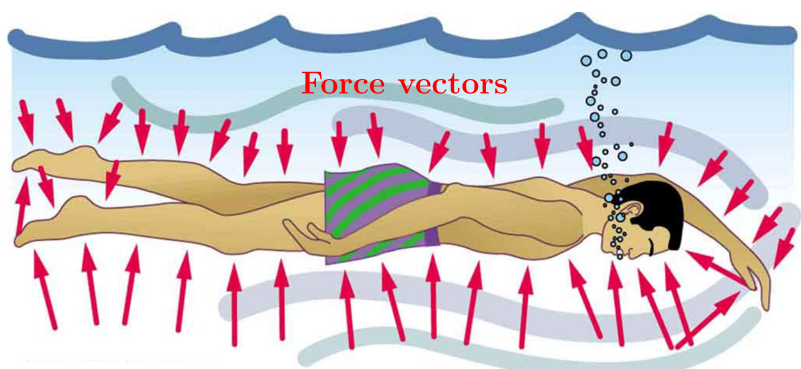
Another quantity that is important for understanding the behavior of fluids is pressure. Pressure is defined as

$$\text{Pressure} = P = \frac{\text{force}}{\text{area}} = \frac{F}{A} \qquad (\text{Unit: } \frac{\text{N}}{\text{m}^2} = \text{Pascal} = \text{Pa.})$$



(Another common unit for pressure is psi = pounds/square inch. We will see some others too.) As Fig. 11.6 from the text suggests, a force applied over different areas may have different effects. Touch receptors in the skin are alarmed by too much pressure. A hypodermic needle can penetrate the skin with only a small force applied over a very small area.
Although force is a vector, having a direction, area also has a direction (an orientation, at least, and a direction normal to the area shows how it is oriented). When these two quantities are combined as pressure, we get a scalar quantity. In fluids, pressure has no direction, but it produces a force normal to any surface it is in contact with.



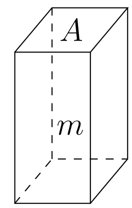


The pressure of the air in an inflated tire is just a number that can be read on a pressure gauge. But the force exerted by the air is normal to the surface of the container, the tire. Likewise, the swimmer shown immersed in water has force vectors normal to his entire surface when immersed in water.

- If you go swimming in a deep pool, and the pressure makes your ears hurt, does it help to turn your head a different direction?
- Would a spherical balloon brought to the bottom of the pool become flatter or just smaller?

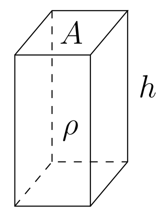
11.4 Variation of pressure with depth in a fluid

It is easy to see how pressure could be determined for a solid object, such as the pressure beneath an iron block on a table: gravity produces a downward force mg, and this is distributed over the area A, so



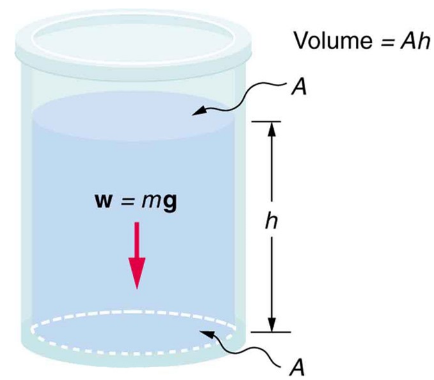
$$P = \frac{F}{A} = \frac{mg}{A}.$$

But there is another way to find P that will be even more useful for fluids, which have no fixed shape. If we know the material density ρ, then instead of m we can use $m = \rho V$, where V is the volume. And if we know the height h of the material, then $V = A \cdot h$, so



$$P = \frac{F}{A} = \frac{\rho V g}{A} = \frac{\rho g A h}{A} = \rho g h.$$

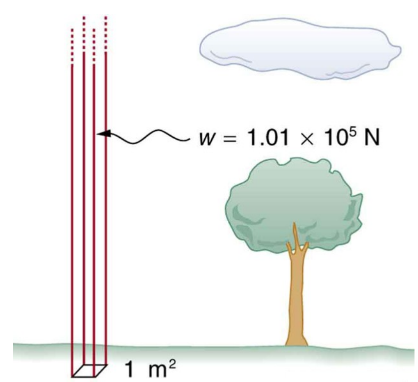
It may seem a little strange that pressure, which is defined as force over area, can be calculated without knowing either the force or the area. Especially when we are dealing with fluids, it is the depth and density that determine the pressure.
So, for example, if you go to a depth of 10 m under water, the pressure there will be



$$P = \rho g h = \left(1000\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)(10\,\mathrm{m}) = 9.8 \times 10^4\,\mathrm{Pa} \approx 1 \times 10^5\,\mathrm{Pa}.$$

That is a pretty large pressure, and the force on your eardrums would be painful if you did not equalize the inside pressure behind your eardrums.

It is perhaps a bit surprising that we walk around all the time with about this same amount of pressure due to the tall pile of air above us. The density of air decreases with altitude, but the atmosphere extends upward something like 120 km and that much air weighs a lot. The weight of all the air gives a pressure on the ground known as atmospheric pressure, which is standardized at 1 atmosphere, or



$$\boxed{P_{\mathrm{atm}} = 1\,\mathrm{atm} = 1.013 \times 10^5\,\mathrm{Pa} = 14.7\,\mathrm{psi}.}$$

So this is the pressure inside our ears normally, and on everything around us. Even a flat tire has air at atmospheric pressure in it.

- When a bicycle tire is inflated to 100 psi, the pressure gauge reads 100 psi, so this is known as gauge pressure.
- But the absolute pressure of the air in the tire is

$$P_{\mathrm{abs}} = P_{\mathrm{atm}} + P_{\mathrm{gauge}} = 114.7\,\mathrm{psi}.$$

Day 11, Hour 2: Pascal's principle, Archimedes' principle

Now that we know something about pressure in fluids, we can look at two related principles with a long history and many applications.

11.5 Pascal's principle

Pascal is another of those 1600s scientists who did a huge variety of things. For Pascal this included discoveries in physics and mathematics, designing calculating machines, and writings on theology and philosophy. His work guides us in philosophical quandries such as:

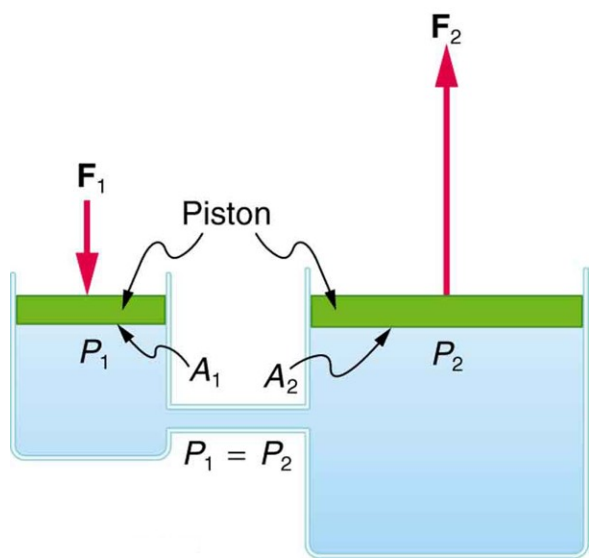
- Isn't is strange how you get toothpaste to come out of the tube? Why does the toothpaste come out the end when you squeeze on the sides?
- How can you lift things that are too heavy for you to lift?

The answers are found in a property of fluids. Pascal's principle states that

Pressure applied to an enclosed fluid is transmitted equally throughout the fluid.

When a force is applied to an enclosed fluid, the pressure goes up everywhere in the fluid. Figure 11.3 from our text illustrates this with a force $\boldsymbol{F}_1$ pushing on the left-side piston, with area A_1. The pressure goes up everywhere in the fluid by an amount

$$P = \frac{F_1}{A_1}.$$



But there is another moveable piston on the right side of the system. This one has area A_2, so the force exerted on it by the additional fluid pressure P is

$$F_2 = P \cdot A_2 = F_1 \cdot \frac{A_2}{A_1}.$$

This pair of different-sized pistons with a fluid between them could serve as a hydraulic jack, a device that converts a small force into a large one.

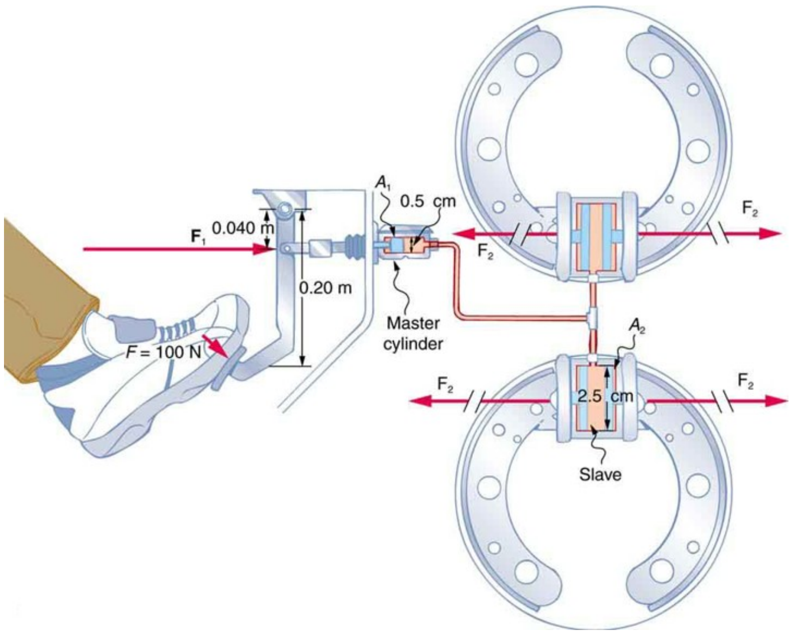
There is a tradeoff, of course. Energy, or work is conserved. The distance that piston 1 goes down is related to the distance that piston 2 goes up, becasue the volume of fluid that moves from one side to the other is

$$V = A_1 \cdot d_1 = A_2 \cdot d_2 \Rightarrow d_2 = d_1 \cdot \frac{A_1}{A_2}.$$

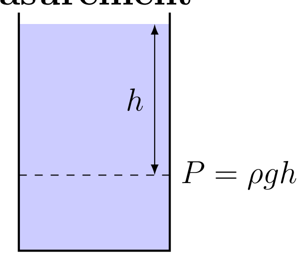
So the small piston must push a larger distance than the big one. And the work done at each piston turns out the same:

$$W = F_2 \cdot d_2 = \left(F_1 \cdot \frac{A_2}{A_1}\right) \cdot \left(d_1 \cdot \frac{A_1}{A_2}\right) = F_1 \cdot d_1.$$

But the device is very useful in all sorts of hydraulic-power machines, where a pump can push fluid into a large piston and generate a large force. You probably used a similar system to operate the brakes on any car you have driven. Our book's Fig. 11.14 shows a hydraulic brake system that also uses a long and short lever arms on the brake pedal to multiply the force applied on the fluid by a factor of 5.

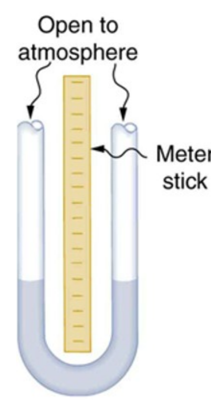


11.6 Gauge pressure, Absolute pressure, Pressure measurement



Another implication of Pascal's principle is that the pressure in a fluid must be the same everywhere in the fluid at a given depth.

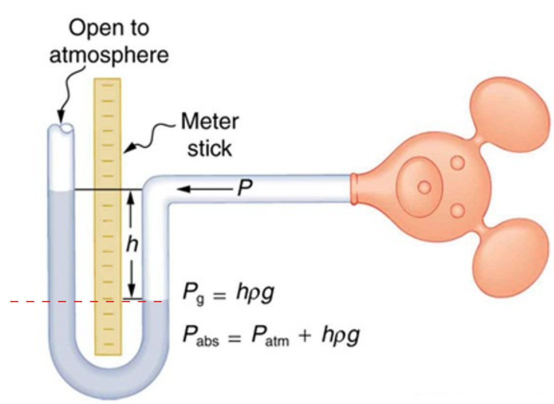
- Obviously the pressure should be the same everywhere at a depth h in a simple container of water.
- It is also fairly obvious that the pressures must be equal on the top surfaces, or at any depth for the liquid in this U-shaped tube. Each side has atmospheric pressure applied to the liquid surface, and P is equal on both sides at a depth h as well.



- Pascal's principle also accounts for the fluid behavior in the system which has one end of a U-tube open to the atmosphere, but the other end connected to a container of gas at pressure P. This device is called a manometer.
- Choosing the lower liquid surface on the right-hand side of the tube (where a red dashed line is added), we can say that the pressure there is P due to the gas in the chamber. But at the same level, the pressure on the fluid in the left-hand tube must also be P, according to Pascal, but

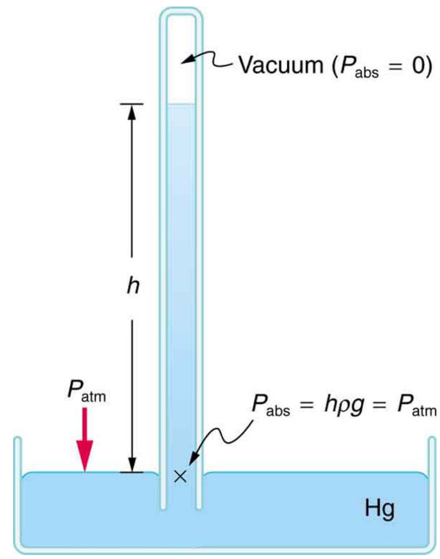
$$P = P_{\text{atm}} + \rho g h$$

due to the atmosphere and the pile of liquid above that level. So the height of the liquid gives a measurement of the gas pressure in the container.



- The same reasoning from Pascal's principle explains how a barometer measures atmospheric pressure, which changes regularly.
  - Looking at the top surface of the liquid in a dish open to the atmosphere, we can say that the pressure on it is, obviously P_{atm}. But to measure this, we can also look at the height of the same liquid as it stands in a tube with a vacuum at the top end. The pressure at the same level as the open surface of the liquid is just $\rho g h$, since there is no other pressure inside the vacuum region. Hence

$$P_{\text{atm}} = \rho g h.$$



- If the liquid is mercury, which is extremely dense for a liquid, then the height of the mercury column, when the atmospheric pressure is the standard amount, would be

$$h = \frac{P_{\text{atm}}}{\rho g} = \frac{1.013 \times 10^5\,\text{Pa}}{(13\,600\,\text{kg/m}^3)\,(9.8\,\text{m/s}^2)} = 0.76\,\text{m} = 76\,\text{cm} = 760\,\text{mm}.$$

- For that reason, another unit for pressure that is used in connection with the atmosphere or other gases is the "mmHg." So we have several pressure units to work with.

$$P_{\text{atm}} = 1\,\text{atm} = 1.013 \times 10^5\,\text{Pa} = 760\,\text{mmHg} = 14.7\,\text{psi}.$$

11.7 Archimedes' principle

Archimedes lived in the 200s BC, and is famous for many inventions and discoveries in mathematics and science. He was asked by a king to determine whether or not a crown was made of pure gold, and found a way to compare its density with the density of gold.

When a balloon full of air is immersed in water, why does it go up, instead of down? It has the same weight in either case.

- The difference is another force, the upward buoyant force F_B exerted on it by the water.
- For a floating object, F_B is larger than the weight w.
- This buoyant force occurs on objects that sink too, but is not large enough to overcome their weight.

Archimedes discovered that

The buoyant force is equal to the weight of the displaced fluid.

- For any certain volume of water within the larger tank of water, the buoyant force on this part of the water equals its own weight.

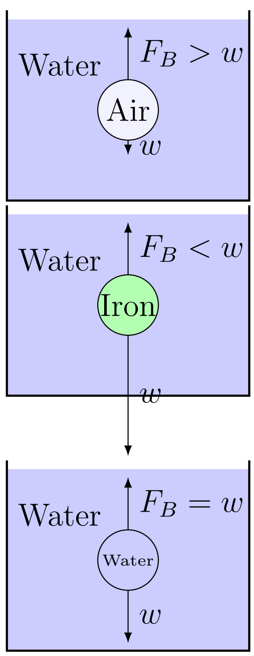
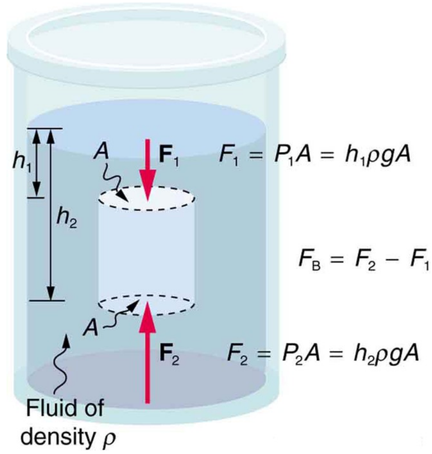


Figure 11.20 from our text is meant to show why this buoyant force exists.

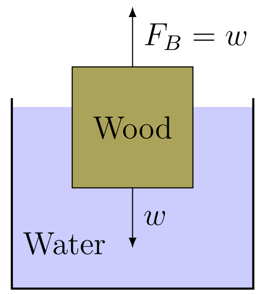
- At any depth h in the fluid, there is a pressure $P = \rho g h$.
- This pressure causes a force on any surface in contact with the fluid.
- At a lower depth h_2 the pressure is larger than at the shallower depth h_1.
- So the force F_2 upward beneath the object is larger than the force F_1 downward on the top of the object.
- The buoyant force is the difference:



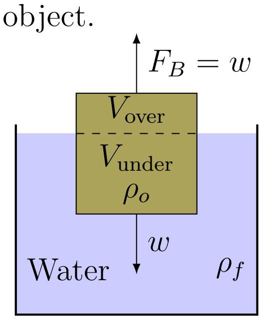
$$F_B = F_2 - F_1 = \rho g (h_2 - h_1) A = \rho g V$$

where V is the volume of the fluid displaced by the object.

In the examples shown so far, the volume of the displaced fluid equaled the volume of the object, because the objects were shown completely submerged. But that is not always the case. In particular, we have to be careful, when considering a floating object, to use Archimedes' principle correctly.



- A floating object, such as the block of wood pictured here, is supported by a buoyant force that is equal to the weight of the object, and also equal to the weight of the displaced fluid.
- But the volume of the displaced fluid is less than the volume of the object.



- Let's divide the object into two volumes: V_{over} for the volume above the water surface, and V_{under} for the volume below the water surface.
- The densities of the fluid and object are, respectively, ρ_f and ρ_o.
- Using Archimedes' principle we can say that the buoyant force equals the weight of the displaced fluid, and equals the weight of the floating object:

$$F_B = \text{Weight of displaced fluid} = \rho_f g \cdot V_{\text{under}} = \text{Weight of object} = \rho_o g \left(V_{\text{under}} + V_{\text{over}}\right)$$

which simplifies to

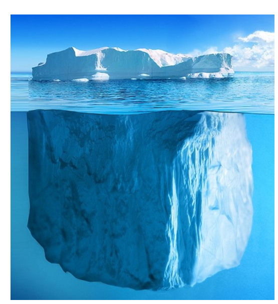
$$\boxed{\frac{V_{\text{under}}}{V_{\text{under}} + V_{\text{over}}} = \frac{\rho_o}{\rho_f}} \text{ for a floating object.}$$

So an object floats if its density is less than that of the fluid it is placed in.

- For freshwater ice in seawater, we can easily calculate that

$$\frac{V_{\text{under}}}{V_{\text{under}} + V_{\text{over}}} = \frac{917}{1025} = 0.895$$

so almost 90% of an iceberg is underwater.



We know that fluids include gases, and that we are immersed in air. Fortunately our density is greater than that of air, so we don't float away. But it can be done using gases less dense than air.

Up!

Following the example of Larry Walters, who in 1982 went up over over Long Beach California, Ken Couch in 2007 drifted 193 miles across Oregon in a lawn chair supported by 105 helium-filled weather balloons.

For a volume V of helium, the weight is $w_{He} = \rho_{He} V g$, while the buoyant force is the weight of the displace air, $F_B = w_{air} = \rho_{\text{air}} V g$, so the net upward force is

$$F_{\text{net}} = F_B - w_{He} = (\rho_{\text{air}} - \rho_{He})\, V_{He} \cdot g = (1.29 - 0.18)\, V_{He} \cdot (9.8) = 10.8\,\text{N/m}^3 \cdot V_{He}.$$

A typical weather balloon might have a diameter of 2.5 m, so its volume is

$$V_{He} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\,(1.25)^3 = 8.2\,\text{m}^3,$$

thus each ballon would give a lift of about $(10.8)(8.2) = 90\,\text{N} \approx 20$ pounds. If you are planning a trip, of course, don't forget to calculate the weight of the ballon material in addition to any equipment and passengers you are hoping to send up.

Time	Topics	Assignments
12:30 - 1:30	Fluid dynamics: continuity, Bernoulli, viscosity	
1:40 - 2:20	Temerature, thermal expansion	
2:30 - 3:30	Ideal gas law	Quiz #6

Day 12, Hour 1:

Last time we looked at the basics of fluids at rest: fluid statics. Today, the much more complicated topic is fluid dynamics: fluids in motion. It is one of the more difficult areas of physics, and we are not going to be able to go much beyond three basic ideas.

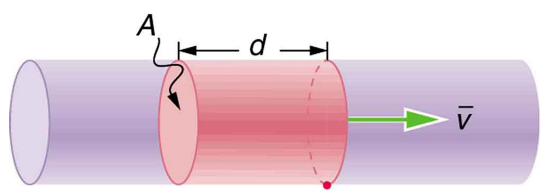
1. If density stays constant (though it might not), then the continuity equation relates the speed of a fluid to its area.

2. If mechanical energy is conserved (though it might not be), then Bernoulli's equation relates pressure, speed and height of a fluid.

3. If there is friction (viscosity) in a fluid, then the pressure needed to move a fluid through a tube depends strongly on the tube radius.

12.1 Flow rate

When fluids are transported from place to place, one of the measures we might be interested in is the flow rate:

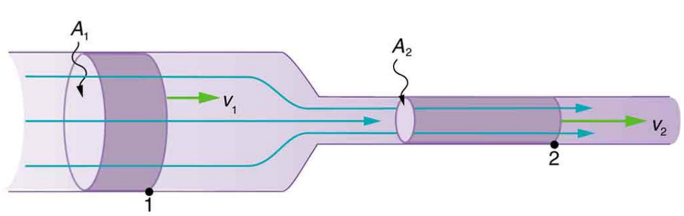


$$Q = \text{flow rate} = \frac{\text{Volume}}{\text{time}} \text{ (Unit} = \text{m}^3/\text{s.)}$$

It would tell us how much time is needed for fluid to fill a container, for instance, or measure the volume of blood pumped by the heart. One step beyond this definition, there is a way to relate Q to the area of the tube carrying the fluid. If a fluid is not compressible, then the same volume of fluid that goes in one end of a tube must come out the other end. The area of the tube is A, and in a time interval t the length of the fluid moving along the tube is $d = v \cdot t$, so $V = A \cdot d$ so

$$Q = \frac{A \cdot d}{t} = \frac{A \cdot v \cdot t}{t} = A \cdot v = \text{constant}.$$

This is the continuity equation. It says that when the area of a tube decreases, the speed of the fluid must increase, and vice versa, to keep the flow rate constant, as pictured below. The speed v_2 is larger than v_1 becasue the area A_2 is smaller than A_1.



- Why is it fun to squeeze the end of a hose, or block part of it with your thumb, when water is flowing through it?

- Why do squirt guns have such a small hole for the water to come out?

Our book shows an example of estimating the number of capillaries in a human cardiovascular system. If each capillary has an area A_c, and the aorta has area A_a, and measured values of the speeds in the aorta and capillaries are v_a and v_c, then we can say

$$Q = A_a v_a = N_c \cdot A_c \cdot v_c \Rightarrow N_c = \frac{A_a}{A_c} \cdot \frac{v_c}{v_a}$$

which turns out to be a very big number, about 5 billion. The flow speed is much smaller in a capillary compared to the aorta, but the total cross-sectional area of the capillary system is $N_c \cdot A_c$ which is much larger than A_a.

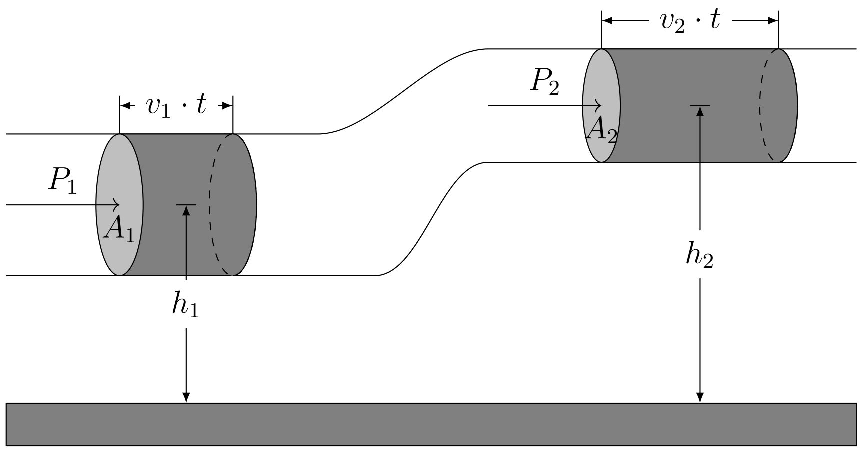
12.2, 12.3 Bernoulli's equation

Along with changes in area for transport of fluids, changes in height may also be significant, and different pressures may be applied in different places. Bernoulli's equation takes these variables into account and assumes energy is conserved. Here is a sketch to go with Bernoulli's equation:

A pressure P_1 is applied to the fluid at height h_1 where the tube has area A_1 and the fluid speed is v_1.

Elsewhere along the tube, the variables are P_2, h_2 and A_2.

The continuity equation says that the speed v varies with area A, as we saw earlier.



Conservation of energy requires that

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant},$$

or

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2.$$

- The terms with $\frac{1}{2}\rho v^2$ resemble kinetic energy, but are KE per unit of volume.
- Likewise, the $\rho g h$ terms are the PE per unit of fluid volume.
- The pressure term is $P = \frac{F}{A} = \frac{F \cdot d}{A \cdot d} = \frac{\text{Work added}}{\text{Volume}}$.

This all looks complicated, and it can be, even though it is a simplification of reality (ignoring friction for instance). But Bernoulli's equation is quite useful and fairly easy to use with a little practice.

First, a few qualitative examples of related fluid-dynamics effects:

- Try blowing air under a page of your book. Can you flip a page this way?
- Try blowing between two hanging sheets of paper. What happens?
- A bicyclist feels a sideways push into the airstream that follows a truck on the highway.
- A strong wind blowing over a roof causes a lifting force on it.

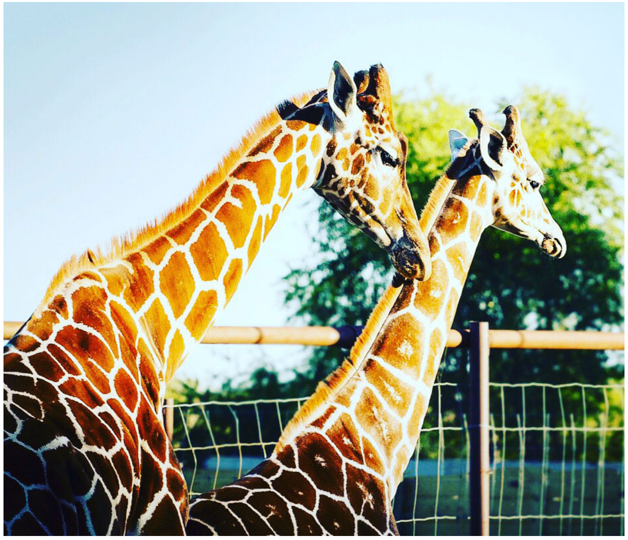
- A stream of air can support a ball and keep it in the stream.

These are examples of the interplay between fluid speed and pressure. When air moves faster the pressure decreases.

Fluids that change height also do interesting things.

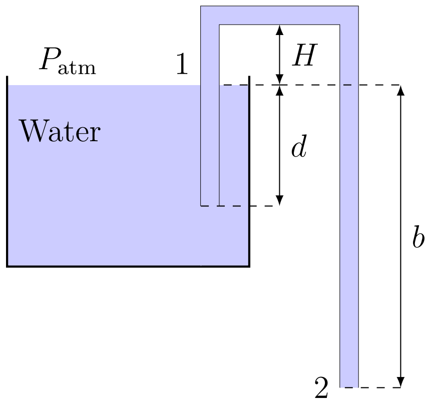
- The question of how a giraffe can raise and lower its head while maintaining proper blood pressure to its brain is apparently not well understood by humans. (Note: there are two giraffes less than 20 miles from SJU.) The process seems to involve reservoirs and blood vessels that change diameter, along with an exceptionally thick-walled heart and tight skin over the legs.

- To drain a barrel of water, a hose out the top of the barrel might be all you need (a siphon).



A siphon is pictured here: a tank of water has a hose that starts at some depth d in the water, rises a height H above, and ends at a bottom distance b below the top of the water in the tank. Bernoulli's principle lets us say that the water at every place obeys the equation

$$\rho\frac{v^2}{2} + \rho gh + P = \text{constant}.$$



So we can pick places in the fluid that have properties we know or care about, such as the top surface (labled 1) and the exit hole at the bottom end of the hose (labeled 2). We can say

$$\rho\frac{v_1^2}{2} + \rho gb + P_{\text{atm}} = \rho\frac{v_2^2}{2} - \rho g\cdot 0 + P_{\text{atm}} + \rho_{\text{air}}gb \Rightarrow v_1^2 + 2gb = v_2^2 + \frac{\rho_{\text{air}}}{\rho}gb.$$

If we know the areas of the tank and the hose, we can use the continuity equation to eliminate one of the speeds, such as

$$v_1^2 = v_2^2\left(\frac{A_2}{A_1}\right)^2$$

and solve for the other. But it is common in a situation like this to make a reasonable approximation. If the tank is very wide compared to the hose diameter, then $A_1 >> A_2$ and the ratio squared is totally negligible. So $v_1 << v_2$ and we can neglect it. Also, the slight pressure difference due to the air depth b is pretty small too, and it would make sense to ignore it as well.

We find

$$v_2^2 = 2\cdot g\cdot b \Rightarrow v_2 = \sqrt{2gb}.$$

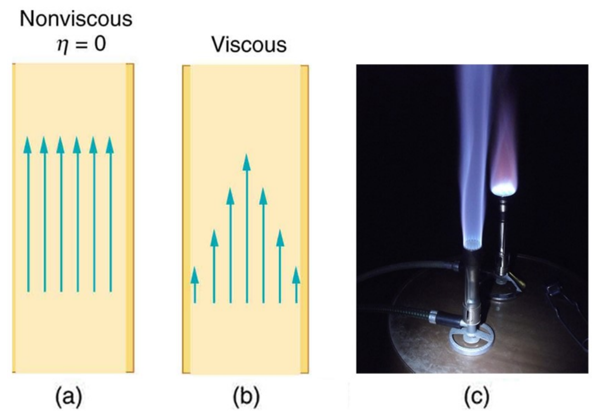
This is the same speed an object gains from falling a distance b, converting potential to kinetic energy.

So as long as the low end of the hose is below the water level in the tank, water should flow out the hose. There does turn out to be a practical limit on how high the elevation H can be, but that is a more complicated matter.

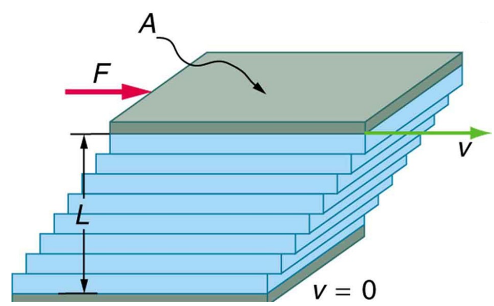
Several examples are worked out in the text to show how Bernoulli's equation can be applied, sometimes with approximations, to explain fluid motions that do not require attention to frictional losses of energy.

12.4 - 12.6 Viscosity

When there is friction in a fluid, its behavior is more complicated in several ways. For instance, the flow along a tube is not uniform, as was assumed in the continuity equation. Fluid friction is different from the friction of solid surfaces, and the correct word for this property of fluids is viscosity. A viscous fluid is at rest along the walls of a tube, but moves with less resistance farther away from the walls. Figure 12.13 shows the velocity vectors for a viscous fluid moving in a pipe, and shows the effect in a Bunsen burner flame.



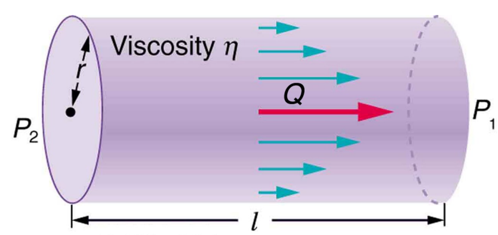
To measure the viscosity of a fluid, one can measure the force needed to pull one surface relative to another at a distance L away, as shown in Fig. 12.12. For constant velocity, the force F applied must equal the resisting force of viscosity. The quantity that results is



$$\eta = \text{viscosity} = \frac{FL}{vA} \qquad \text{(Units: Pa} \cdot \text{s.)}$$

The text has a table 12.1 of viscosities. I will just note that for water, $\eta = 1 \times 10^{-3}\,\text{Pa} \cdot \text{s}$, while blood has a viscosity about three times as large.

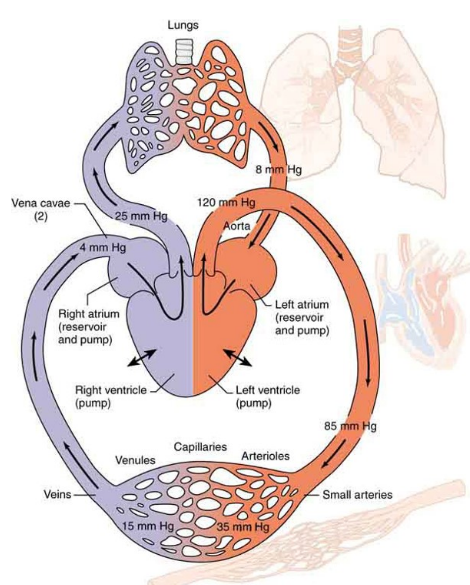
An important result of viscosity is its effect on the flow rate of a fluid, which we called $Q = \frac{V}{t} = A \cdot v$ at the start of the chapter. Now, the speed varies across a tube, and the flow rate becomes more complicated. It still depends on the tube area, but also its length ℓ:



$$Q = \frac{(P_2 - P_1)}{8\eta\ell} \cdot \pi r^4.$$

There needs to be a pressure difference between the ends of a tube in order for a viscous fluid to move along, and the flow rate depends very strongly on the radius of the tube.

The text briefly describes the design of the circulatory system in regard to the various sizes of blood vessels and the pressure differences, matched to the needs of the organs they supply.



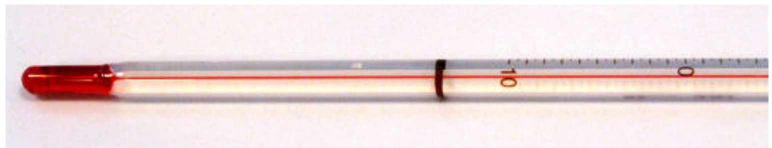
Day 12, Hour 2: Temperature and thermal expansion

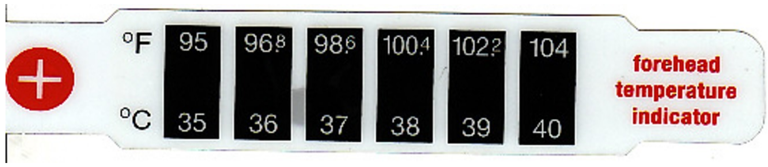
13.1 Temperature

Hot and Cold are relative terms. You perhaps like your coffee hot and milkshakes cold. But a hot day in August is not as hot as your coffee, and a day in January that is milkshake-cold might be thought of as rather a warm day for that season.

There are all sorts of ways to be more specific about temperature, because there are so many things that vary with temperature. You could measure temperature by observing:

- The number of people wearing shorts.
- The volume of a liquid, solid or gas.
- The pressure of a gas.
- The color of a hot object.
- The electrical resistance of a wire.
- The viscosity of a fluid.





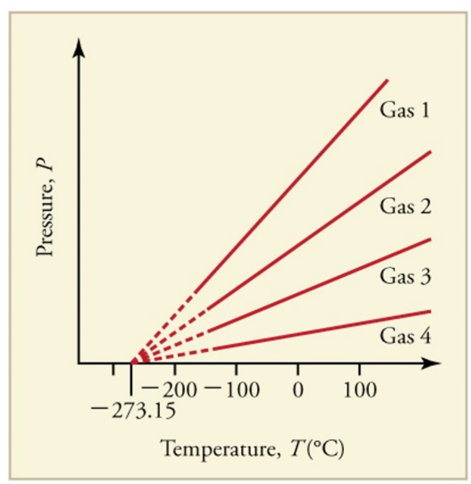
- The color of liquid crystals. This picture shows two important temperature scales.

1. One famous scale for temperature measurements was introduced by Fahrenheit in 1704, based on what he thought were two easy reference points:
   - 0°F was a salt/ice mixture which he thought was about as cold as you could make something in a laboratory, and 96°F corresponded to a healthy male armpit.

   Now we are accustomed to defining the Fahrenheit scale by the freezing and boiling points of water, calling them 32°F and 212°F.

2. The other common scale, certainly more widely used in science if not always in weather reporting, is the Celsius scale: water freezes at 0 °C and boils at 100 °C.

3. The third, and for us, most important temperature scale is the Kelvin scale, based on the pressure of a gas.

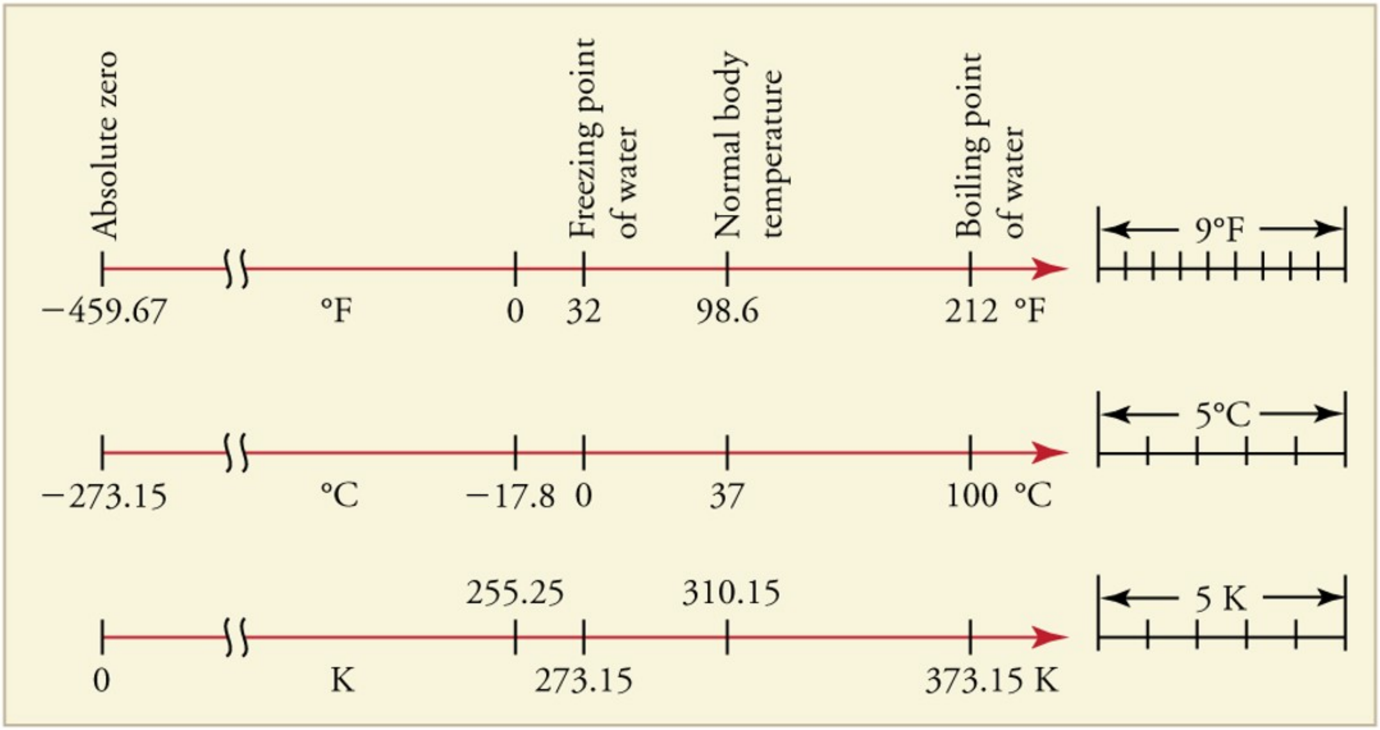


   - Any gas, when cooled, will have a lower pressure, and the change is very linear as the graph shows.
   - Different gases have different slopes, but they all have the same intercept on the T axis, at −273.15 °C.
   - No real gas gets all the way to this temperature while remining a gas; they all liquify at some temperature. But if there were such a gas, that could stay a gas all the way to zero pressure, an ideal gas, then the equation for its pressure using Celsius temperature would be

$$P \propto (T_C + 273.15) \Rightarrow \boxed{P \propto T_K,} \text{ where } T_K = T_C + 273.15 \text{ is the Kelvin-scale temperature.}$$

- This simple gas law is one reason for the Kelvin temperature scale, in which the degrees are the same size as Celsius degrees, but shifted by 273.15 from the Celsius temperature.

This Figure 13.6 from our text shows the three temperature systems compared:



and formulas that relate them are:

$$\boxed{T_F = \frac{9}{5}T_C + 32^\circ} \Leftrightarrow \boxed{T_C = \frac{5}{9}\left(T_F - 32\right)}$$

$$\boxed{T_C = T_K - 273.15} \Leftrightarrow \boxed{T_K = T_C + 273.15}$$

13.2 Thermal expansion

Solids and liquids generally expand when heated. This is why bridges have expansion joints like these pictured in Fig. 13.10. The equation used for thermal expansion of a solid is

$$\Delta L = \alpha L \, \Delta T$$

where L is the length of the solid at some temperature, and ΔL is the change in length resulting from a temperature change ΔT. The factor α here is the coefficient of linear expansion, which depends on the particular substance. Our book has a big table of these coefficients. For instance, for steel, $\alpha = 1.2 \times 10^{-5}$ /°C, so a meter of steel whose temperature is raised by 1 Celsius degree would grow longer by 1.2×10^{-5} m.

An example calculation for the Golden Gate Bridge, with $L = 1275\,\text{m}$ at its coldest temperature, and assuming a maximum $\Delta T = 55\,°\text{C}$ results in $\Delta L = 0.84\,\text{m}$.

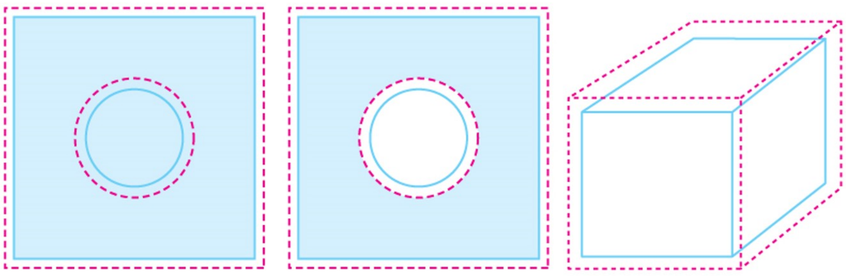
The area of a solid will also expand, and we can find a formula using $A = L^2$ and finding $A + \Delta A = (L + \Delta L)^2$. The result is (to a very good approximation, ignoring tiny terms that are squared)

$$\Delta A = 2\alpha A\,\Delta T.$$

We could do this again for a volume $V = L^3$ and find

$$\Delta V = 3\alpha V\,\Delta T.$$

- What would happen to a hole in a piece of metal? Would it shrink or grow if the metal is heated?

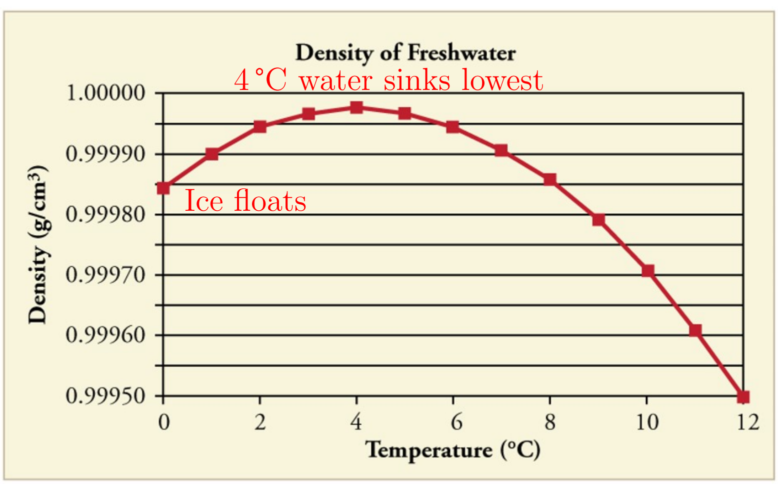


For liquids, which have no definite shape, there is only volume expansion, and the coefficient is called β (typically about e − 4 for common liquids), so

$$\Delta V = \beta V\,\Delta T.$$

Water is exceptional in many ways, and it is good that it is exceptional in regard to its thermal behavior at cold temperatures.

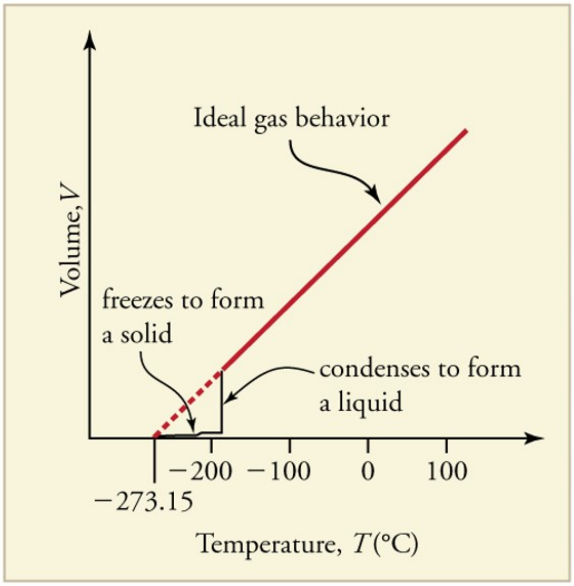
- Figure 13.12 shows that the density of water increases as it is cooled, toward 4 °C. This is what most liquids do, because they contract to a smaller volume, increasing their density.



- But at 4 °C, the water begins to expand as it is cooled further, lowering its density.
- As we learned from Archimedes' principle, this means the coldest water, just above 0 °C, rises to the surface of a lake, where it can form a layer of ice for us to play on.
- But the "warm" (4 °C) water drops to the bottom, and remains water for the fish to swim in.

Day 12, Hour 3: Ideal gas law

13.3 Ideal gas law



The behavior of different gases is remarkably similar in some ways. If the pressure is not too great, and the temperature is not too small, then, as we've seen, the pressure is proportional to T, and as this Figure 13.27 from out text shows, the volume varies proportional to T as well.

So for many purposes, real gas behavior is well described by this ideal gas law:

$$\boxed{P \cdot V = nRT.}$$

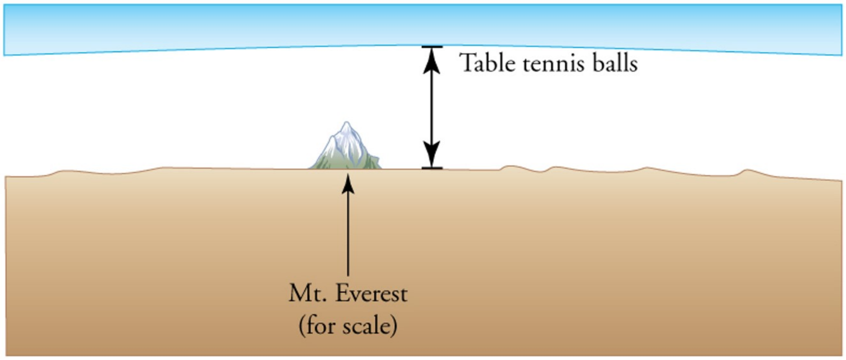
where

- P is the pressure, usually measured in Pa or atmospheres (atm $= 1.013 \times 10^5$ Pa).
- V is the volume, usually measured in m^3 or liters ($\mathrm{L} = 10^{-3}\,\mathrm{m}^3$.)
- T is temperature, and must be in Kelvins for the law to work correctly as it is.
- n is the number of moles of the gas, where a mole is Avagadro's number of molecules:

$$N_A = \text{Avagadro's number} = 6.02 \times 10^{23}\,/\text{mol}$$

- R is a constant needed to make the units agree.
  - When Pa and m^3 are used then $\boxed{R = 8.314\,\mathrm{J/(mol \cdot K)}.}$
  - When atm and L are used, then $\boxed{R = 0.0821\,\mathrm{L \cdot atm/(mol \cdot K)}.}$

Avagadro's number is a big number. As Fig. 13.19 from the book illustrates, this number of ping-pong balls would coat the Earth to a very high elevation, about 40 km.



The gas law can also be expressed in terms of the actual number of molecules, $N = n \cdot N_A$, in which case a new constant is needed:

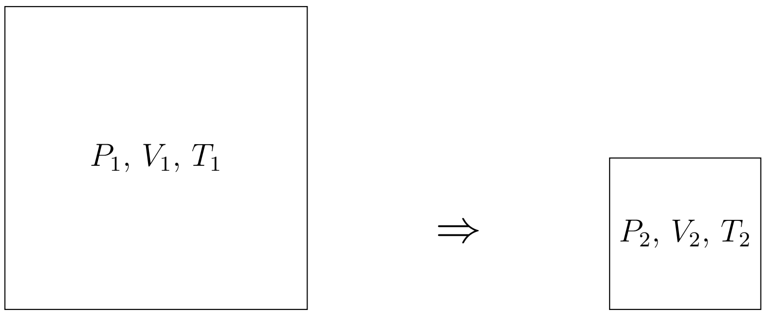
$$\boxed{P \cdot V = NkT,} \quad \text{where} \quad \boxed{k = 1.38 \times 10^{-23}\,\mathrm{J/K} = \text{Boltzmann's constant.}}$$

We will make use of the gas law very often in the remainder of the class, and both forms are useful. First, it is helpful to have a good grasp of what it means.

- If you squeeze the gas with a higher pressure, but keep T constant, what happens?
- To get the gas to a larger volume, but with constant pressure, what must you do?
- If you can keep the volume constant, what happens when you lower the temperature?

Suppose you work with all three variables P, V and T:

- What happens if a gas is at temperature T_1, pressure P_1 and volume V_1, then you triple the pressure, and reduce the volume to half of what it was? What will be the new temperature T_2?



You might want to work with numbers, but it is probably simpler to use ratios as much as possible first. A good way to begin is by collecting the things that remain constant:

$$\frac{P_1V_1}{T_1} = nR = \text{constant} = \frac{P_2V_2}{T_2} \Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

and then write an equation for the parameter you want in terms of the ones you have:

$$T_2 = T_1 \cdot \frac{P_2}{P_1} \cdot \frac{V_2}{V_1} = T_1(3)\left(\frac{1}{2}\right) \Rightarrow T_2 = \frac{3}{2}T_1.$$

So if the gas was at temperature $T_1 = 0\,^\circ\text{C}$, what is the new temperature in Celsius? (Answer: 137 °C. Do you see why?)

- Suppose you did the same thing again, but also let half the gas escape the container. What would be the final temperature?

$$\frac{P_1V_1}{n_1T_1} = R = \text{constant} = \frac{P_2V_2}{n_2T_2} \Rightarrow \frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} \Rightarrow T_2 = T_1 \cdot \frac{P_2}{P_1} \cdot \frac{V_2}{V_1} \cdot \frac{n_1}{n_2}$$

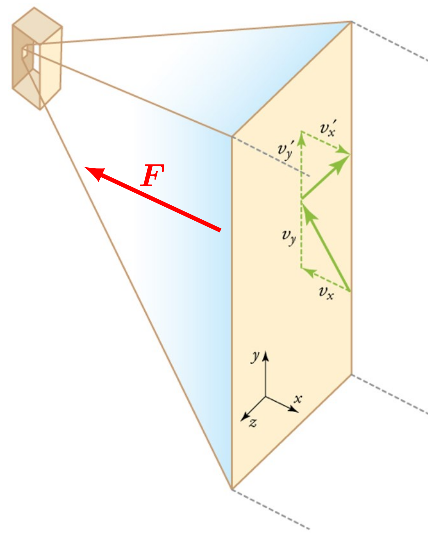
$$T_2 = T_1 \cdot (3)\left(\frac{1}{2}\right)(2) = 3T_1 \Rightarrow 3(273.15\,\text{K}) = 819\,\text{K} \Rightarrow T_C = 819 - 273 = 546\,^\circ\text{C}.$$

Next time we will look at more details of the physics and meaning behind the gas law.

Quiz #6.

Time	Topics	Assignments
12:30 - 1:30	Kinetic theory, Phase changes	
1:40 - 2:20	Heat, Heat capacity, Latent heat	
2:30 - 3:30	Heat transfer mechanisms	Quiz #7

Day 13, Hour 1:



13.4 Kinetic theory of pressure and temperature

From the 1700s and through the 1800s, physicists worked on ways to understand the behavior of gases based on the idea that a gas is made of particles in motion that obey Newton's laws, colliding with the walls of their container.
If a single molecule of mass m bounces off a wall, it exerts a force outward on the wall. The impulse is found from the molecule's change of momentum in the x direction, normal to the wall:

$$m\Delta v = m(v_f - v_i) = m[v_x - (-v_x)] = 2mv_x = F\Delta t.$$

If the box walls are a distance ℓ apart, such a collision happens every time the particle bounces across the box and back, a distance 2ℓ, at the speed v_x, so setting the time interval to $\Delta t = 2\ell/v_x$ gives

$$F = \frac{2mv_x}{2\ell/v_x} = \frac{mv_x^2}{\ell}.$$

With N molecules in the box with a variety of speeds, the average of v_x^2 is used, and the force is multiplied by N, giving a force

$$F = N\frac{m\overline{v_x^2}}{\ell}.$$

If the particle velocity components x, y and z are equally likely, then the magnitude of $\boldsymbol{v}$ squared will be

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3\overline{v_x^2}$$

so the force on any wall of the box, in terms of the average speed will be

$$F = N\frac{m\overline{v^2}}{3\ell}.$$

Since the gas pressure is related to the force on the box walls by $P = F/A$, we get a step closer to the gas law:

$$P = N\frac{m\overline{v^2}}{3(\ell \cdot A)} = N\frac{m\overline{v^2}}{3V} \Rightarrow P \cdot V = \frac{1}{3}Nm\overline{v^2} = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) \Rightarrow P \cdot V = N \cdot \frac{2}{3} \cdot \overline{KE}$$

where the average kinetic energy has appeared. This looks like the form of the ideal gas law

$$P \cdot V = NkT$$

and leads to the connection of the molecular kinetic energy with the temperature of the gas:

$$\boxed{\frac{1}{2}m\overline{v^2} = \overline{KE} = \frac{3}{2}kT.}$$

So the root-mean-square speed (the square root of the average of the squared speeds) of molecules is

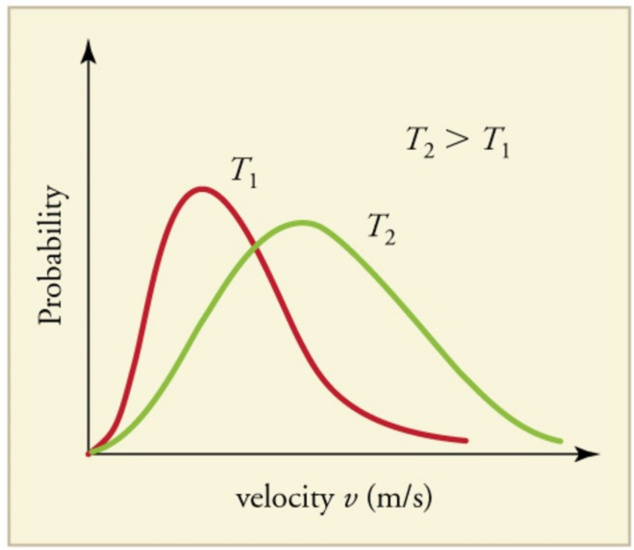
$$\boxed{v_{rms} = \sqrt{\frac{3kT}{m}}} \text{ or } \boxed{v_{rms} = \sqrt{\frac{3RT}{M}}.}$$

where the second form uses the molar mass M instead of the mass of a molecule m, and the gas constant R instead of Boltzmann's constant ($k = R/N_A$).

Molecules are, of course, not very massive, so their speeds can be quite large. For example, nitrogen molecules (N_2) in air at 20 °C have a molar mass $M = 28\,\text{g/mol} = 28 \times 10^{-3}\,\text{kg/mol}$, so

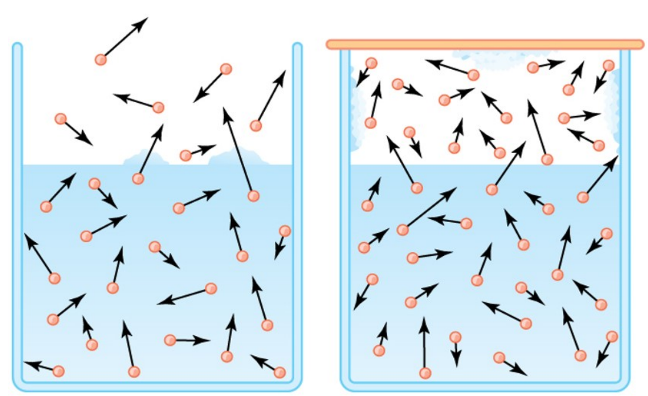
$$v_{rms} = \sqrt{\frac{3\,(8.31\,\text{J/mol}\cdot\text{K})\,(293\,\text{K})}{28 \times 10^{-3}\,\text{kg/mol}}} = 511\,\text{m/s}.$$

It is important to note that this doesn't mean all the molecules of a gas have the same speed. The v_{rms} value is based on the average for all the particles; there will be many with lower speeds, and many with higher speeds than v_{rms}. James Clerk Maxwell was able to find a probability function for the speeds of molecules, and this is shown in our book's Fig. 13.24 at the right. The most-probable speed (the peak of the curve) increases with temperature, and the width of the distribution also increases with temperature.



13.5 - 13.6 Phase changes

So far this kinetic theory has been addressing molecules in a gas, which interact with each other only in collisions. But to some extent it also applies to molecules in solids and liquids, even though these also experience forces binding them to other molecules. These binding forces give the molecules negative PE in addition to their temperature-dependent KE. But with enough KE one could escape to join the gas.



Evaporation Saturated vapor

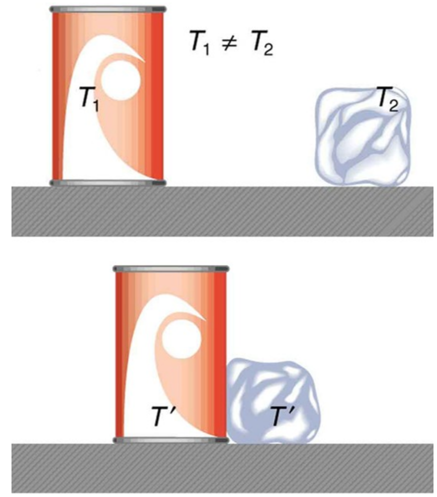
The Maxwell speed distribution helps explain how fast-moving molecules at a temperature below the melting or boiling temperature of a substance might nevertheless escape from a solid or liquid and become free molecules in a gas or vapor, as in Fig. 13.33.

Sections 13.5 and 13.6 are worth reading for a qualitative picture of how the molecules of a substance take on the different forms of solid-liquid-gas as a result of the temperature, pressure and volume conditions applied to it. But we will deal with the physics of phase changes in more detail in chapter 14.

Day 13, Hour 2:

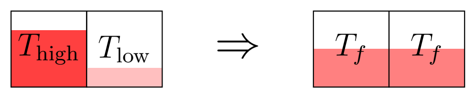
14.1 Heat energy

In our study of energy conservation, the fact came up often that one form of energy is heat. We sometimes accounted for missing mechanical energy, such as in a collision, by saying that it was converted into heat. We can imagine the molecules of a baseball and a brick wall each gaining *KE* in a collision, and the temperature of both increasing.



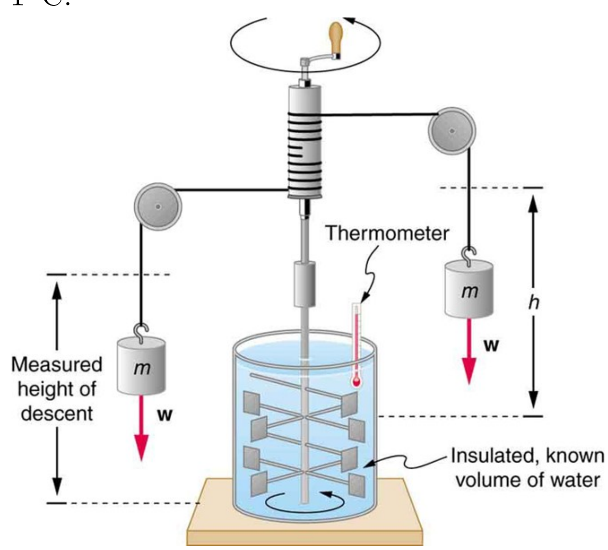
But the transfer of energy from one object to another can take place gradually through many collisions of molecules. When the molecules of objects at different temperatures are in contact, as in Figure 14.2 from the text, energy flows from the object at higher temperature to the one at lower temperature by collisions that transfer molecular *KE*. When the objects reach the same temperature the net transfer of energy stops.

Like other forms of energy, heat can be measured in units of Joules. There is also another common unit, the calorie, whose name came from an older theory about heat. In the caloric theory, heat was considered a fluid that would flow from an object at higher temperature to one at lower temperature until their caloric fluid levels were equalized. The caloric theory had some problems, such as that it seemed possible to extract an infinite amount of heat from an object by friction. Regardless, the calorie became the name of a heat unit, defined as the heat needed to raise the temperature of a gram of water by 1 °C.



In the 1840s, Joule did experiments such as the one illustrated in Fig. 14.3 to connect mechanical work with heat energy. Water was stirred by a paddle wheel, and its temperature was observed to increase. The increase was proportional to the mechanical work done to raise masses that would operate the paddle wheel when allowed to descend. Joule's experiments led to accpetance of the idea of energy conservation, and also to this connection of energy units:

$$\boxed{4.186\,\text{J} = 1\,\text{cal.}}$$



The energy available in food is measured in a unit also called the Calorie (with a capital C), which is 1000 cal or 1 kcal, so

$$\boxed{4186\,\text{J} = 1\,\text{kcal} = 1000\,\text{cal} = 1\,\text{Calorie} = 1\text{ food calorie.}}$$

This seems to suggest that if you eat 2000 Calories of food, you might be able to do a lot of work.

Converting this all into gravitational potential energy, a person weighing 800 N (about 180 lbs) would be able to climb a distance h given by

$$2000\,\text{kcal} \cdot \frac{4186\,\text{J}}{\text{kcal}} \approx 8 \times 10^6\,\text{J} = PE = mgh = h \cdot (800\,\text{N}) \Rightarrow h \approx \frac{8 \times 10^6\,\text{J}}{8 \times 10^2\,\text{N}} = 10^4\,\text{m}.$$

That's considerably above Mt. Everest. Of course, not all the food you eat can be turned into mechanical work. Lists indicate that even activities like sitting use around 100 kcal/hr; energy is being used to maintain body temperature, pump blood, breathe, etc.

14.2 Temperature change and heat capacity

When heat energy is added to an object, one thing that can happen is a change in the kinetic energy of molecules, hence a change in temperature. Using Q (now) to represent an amount of heat, the relationship with temperature change is

$$Q = m \cdot c \cdot \Delta T$$

where m is the mass of the object, and c is the specific heat, a property of the material to which the heat is added.

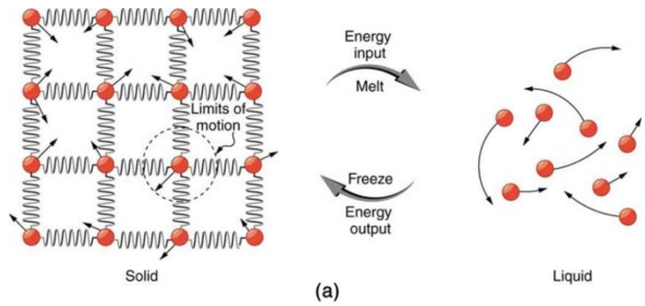
- With mass in kg, Q in J, and ΔT in °C (or K; the degrees are the same size, so either works for the change in temperature), the units of c must be J/(kg · °C) or J/(kg · K).
- If the heat is measured in kcal, then c is needed in kcal/(kg · °C) = kcal/(kg · K).

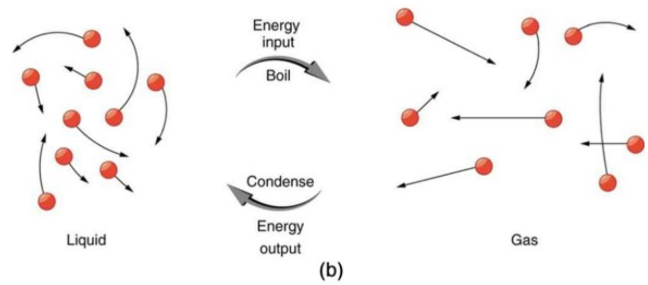
Our book has a table of c values, Table 14.1. As examples of specific heats, a few are listed here in the two common unit systems. They all vary with temperature to some extent, but that would be complicated to deal with, so assume they are constant in homework and exams.

	Specific heat c	
Substance	J/(kg · °C)	kcal/(kg · °C)
Aluminum	900	0.215
Copper	387	0.0924
Iron	452	0.108
Wood	1700	0.4
Water	4186	1.000
Ice	2090	0.50
Steam (constant V)	1520	0.363
Steam (constant P)	2020	0.482

So the same amount of heat added to different materials will result in different changes of temperature. It takes much more heat to get the same ΔT for a kg of water than for a kg of copper, for instance.

14.3 Phase change and latent heat





The other thing that can happen to a material when heat is added is a phase change. This means changing the potential energy of molecules, by breaking bonds of a solid to form a liquid, or of a liquid to form a gas, or either process in reverse.

When the phase of the material changes, its temperature does not. The equation describing a phase change is in a form like $Q = m \cdot L$, where m is the mass and L stands for latent heat, a property of the substance. The latent heats also depend on which phase change is to happen, so there are two symbols used:

- L_v is the latent heat of vaporization, for a liquid to be boiled into gas or a gas to condense into liquid.
- L_f is the latent heat of fusion, for a solid to be melted into liquid or a liquid to freeze into a solid.

With these we can write

$$Q = m \cdot L_f \text{ (melting a solid / freezing a liquid)},$$
$$Q = m \cdot L_v \text{ (boiling a liquid /condensing a gas)}.$$

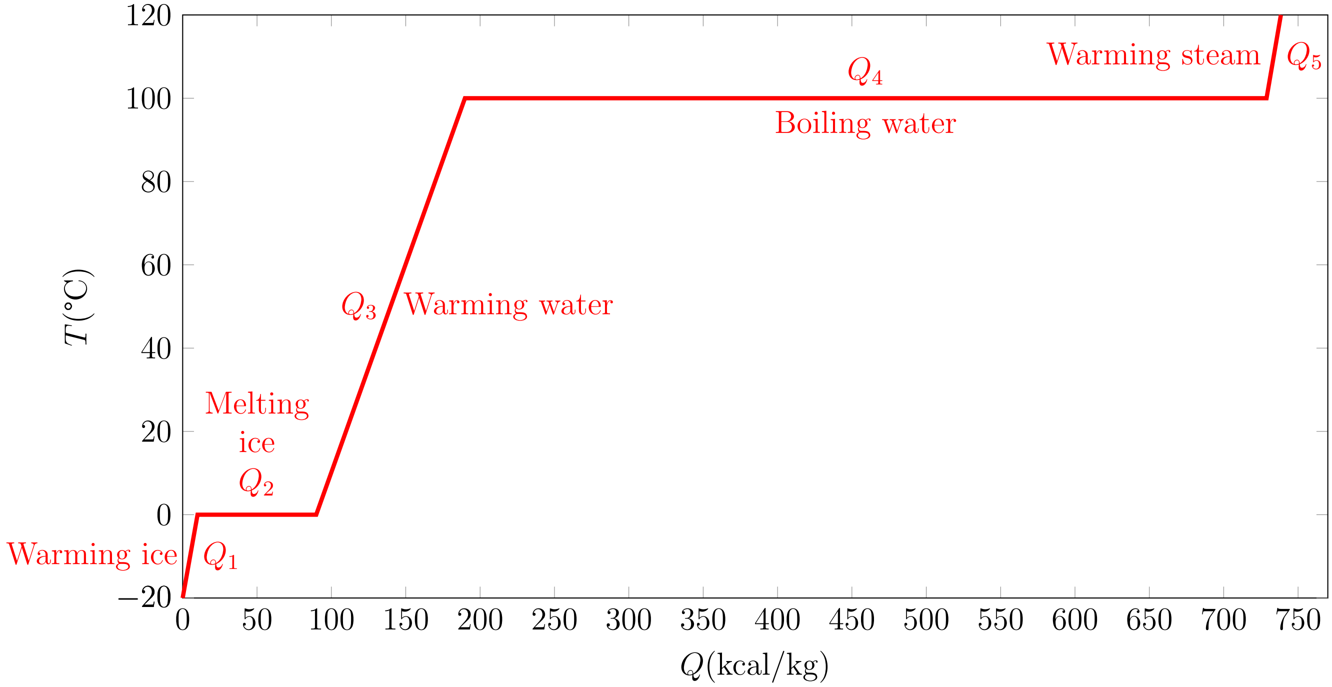
Our book also has a nice table of these latent heats and the phase change temperatures for different substances, Table 14.2. A few example values are listed here.

	Melting	L_f		Boiling	L_v	
Substance	T(°C)	kJ/kg	kcal/kg	T(°C)	kJ/kg	kcal/kg
Nitrogen	−210	25.5	6.09	−195.8	201	48.0
Oxygen	−218.8	13.8	3.30	−183.0	213	50.9
Lead	327	24.5	5.85	1750	871	208
Water	0	334	79.8	100	2256	539
Aluminum	660	380	90	2450	11 400	2720

To illustrate both temperature and phase changes, suppose we have 1 kg of ice at $T = -20\,°\text{C}$ and wish to convert it to steam at $T = 120\,°\text{C}$. We can begin adding heat to do each of the following steps.

1. Warm the ice to 0 °C: $Q_1 = mc_i\Delta T = (1)(0.5)(20) = 10\,\text{kcal}$.
2. Melt the ice to water: $Q_2 = mL_f = (1)(79.8) = 79.8\,\text{kcal}$.
3. Warm the melted ice (now water) to 100 °C: $Q_3 = mc_w\Delta T = (1)(1)(100) = 100\,\text{kcal}$.
4. Boil the water to steam: $Q_4 = mL_v = (1)(539) = 539\,\text{kcal}$.
5. Warm the steam to 120 °C (at constant P let's say): $Q_5 = mc_s\Delta T = (1)(0.482)(20) = 9.64\,\text{kcal}$.

The total Q needed comes out to about 738 kcal, with the majority being the phase change to steam.



The same processes can operate in the opposite directions, so that 738 kcal would have to be removed from 1 kg of steam at 120 °C to convert it to ice at −20 °C.

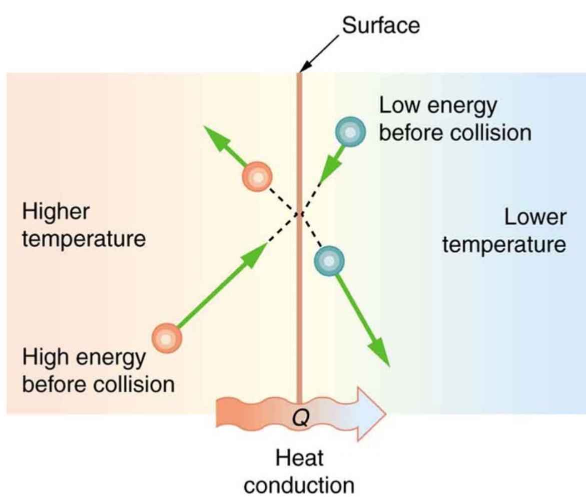
Day 13, Hour 3:

14.4 Heat transfer methods

How does heat transfer into or out of a system? There are three mechanisms: conduction, convection, and radiation. We will look at each one briefly.

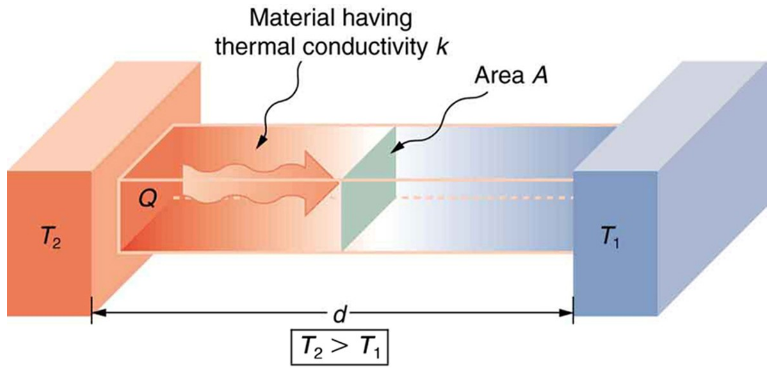
14.5 Conduction

The mechanism for conduction is like what we saw in the kinetic theory: molecules colliding with a surface. But now we can think of a substance on each side of the surface, and having different temperatures, hence the molecules on each side of the surface have different kinetic energies.



- Molecules at higher temperature tend to lose more energy in the collisions, and the lower-temperature molecules gain energy.
- So heat is gradually transferred across the surface.
- The process continues until the two sides reach the same temperature.
- Or, if the temperature difference is maintained somehow, heat continues to flow across the surface.

The rate of heat flow by conduction through a material is often the quantity of most interest. It depends on the temperature difference $T_2 - T_1$ at the two ends, and the area A of the surface through which the heat must pass. But it also depends on the material and how far the heat must travel. The equation is



$$\frac{Q}{t} = \frac{kA\,(T_2 - T_1)}{d} \quad \text{(Units: J/s = W.)}$$

where k is the thermal conductivity of a material, and d is its thickness, as shown. Some k values from our book's Table 14.3 are listed here.

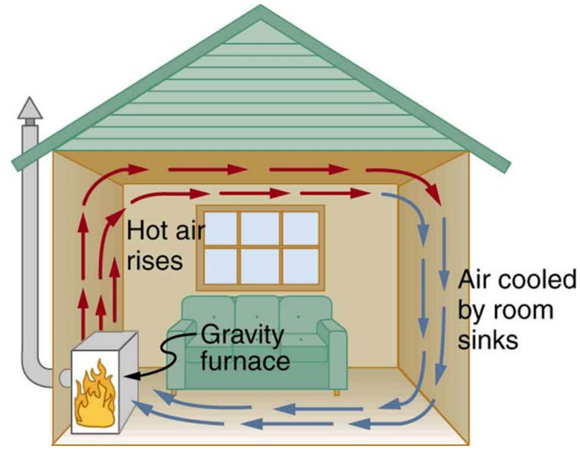
Substance	k [W/(m · °C)]
Copper	390
Aluminum	220
Iron	80
Ice	2.2
Water	0.6
Wood	0.08 – 0.16
Fiberglass wool	0.042
Down feathers	0.025
Air	0.023
Styrofoam	0.010

- So if you want to slow the flow of heat, use a material with low conductivity, and make it thick.
  - A lofty down-filled jacket will help you stay warm in the winter, when the outside temperature is lower than your body temperature.
- The combination of k and d is often combined into one quantity, the thermal resistance, or R-factor, defined as $R = d/k$. In that case we can write

$$\frac{Q}{t} = \frac{A\,(T_2 - T_1)}{R}$$

  Building materials are rated by their R-factor so it is easy to figure out what the total of several layers will furnish.
  - A 2-inch-thick piece of styrofoam has $R = \frac{2\,\text{in}}{0.01\,\text{W/(m°C)}} \times \frac{2.54 \times 10^{-2}\,\text{m}}{\text{in}} = 5\,\text{m}^2 \cdot \text{°C/W}$.
  - Building codes for new houses around here recommend at least $R = 5$ for wall insulation to reduce the flow of heat to the outside.
- You should not stick your tongue on a metal flagpole when the temperature is below 0 °C. What will happen, and why?

14.6 Convection

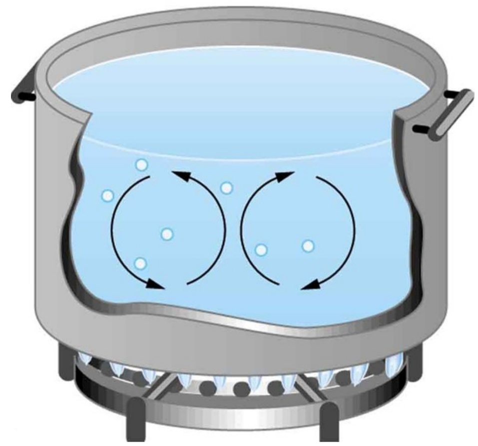


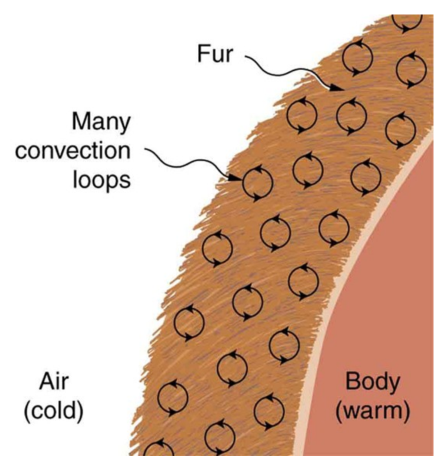
Convection is heat transfer by movement of a fluid.

- Warm air rises, because it is less dense than cooler air, and so it carries heat upward.
- A pot of water heated on a stove shows similar fluid motion, circulating the heat throughout the water.
- Car engines are cooled by the flow of a liquid coolant through pathways in the engine and its moving parts.
- The wind can significantly increase the loss of body heat, which is nice in the summer when you're hot, or a problem in hte winter when you're trying to stay warm enough.
  - The book's Example 14.8 shows how to calculate the amount of heat carried away by evaporation of sweat, using

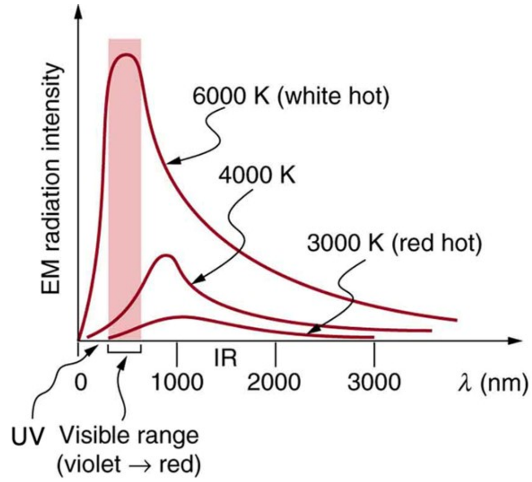
    $$\frac{Q}{t} = \frac{mL_v}{t}$$

    to estimate the mass m of water that must be given up in time t to get rid of an amount Q of heat.
- To reduce the effects of convection, the fluid motion should be limited somehow.
  - Block the cracks around doors and windows.
  - Use windows with multiple layers of glass with air or another gas between layers. This reduces the size of circulating currents from the inside to outside surfaces.
  - The book's Fig. 14.19 shows how feathers or fur reduce the size of convection loops to reduce the flow of heat by convection.





14.7 Radiation



Electric charges in motion generate electromagnetic waves, as you will learn more about in PHYS 106. These waves carry energy away at the speed of light, because they are a form of light. This is how heat is transferred by radiation.

- The energy we receive from the Sun is delivered by radiation. There is obviously no convection or conduction through empty space.
- The electromagnetic waves we can see are in a small range of possible wavelengths, from about 400 nm for violet light to about 800 nm for red light. But the motion of atoms in a hot object produces wavelengths over a much wider range than this, depending on the temperature T of the object.
- Objects like ourselves, with temperatures of 300 K, emit a peak wavelength that is in the infra-red range of the spectrum.
- Cameras able to detect these IR wavelengths can show variations in temperature, such as this thermographic image of a house. The colors are artificial, but the red colors show highest temperatures, blue shows the lowest.

- The higher the temperature of an object, the shorter is the peak wavelength in the spectrum it emits.
- Objects also absorb energy by radiation from their surrounding environment.
- The rate at which energy is gained by an object depends on its temperature T_1 and the surrounding temperature T_2 according to

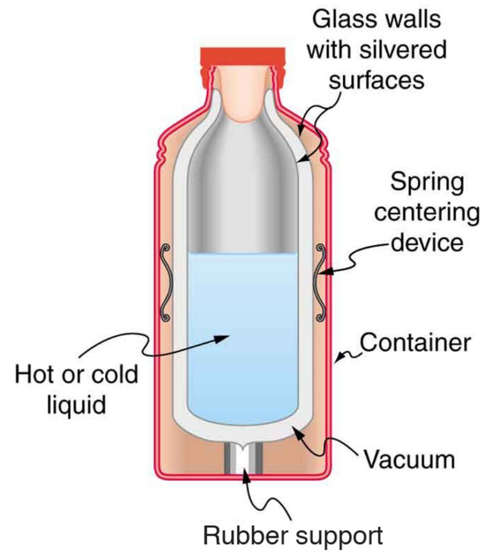
$$\frac{Q_{\text{net}}}{t} = \sigma e A \left(T_2^4 - T_1^4\right)$$

  where

  - $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/(m^2 \cdot K)}$ is a constant.
  - e is the emissivity of the object, a number somewhere between 0 and 1 that tells how well this object converts heat energy into radiation. A perfect absorber and emitter has $e = 1$, while an object that reflects all radiation has $e = 0$ and also emits no radiation.
    - ∗ The tungsten filament in a light bulb has $e \approx 0.5$.
  - A is the surface area of the object, so a larger surface area allows more radiation to occur, and more absorption of radiation as well.
  - The temperatures T_1 and T_2 are in the Kelvin scale.

Now that we know about the three mechanisms of heat transfer, what would be a good way to prevent heat transfer in all its forms?

- Suppose you wanted to keep a liquid cold, or hot, for as long as possible. In other words, you would like to reduce $\frac{Q}{t}$ as well as possible.
- What features of this thermos bottle (Dewar) can you identify that are helpful, and why?



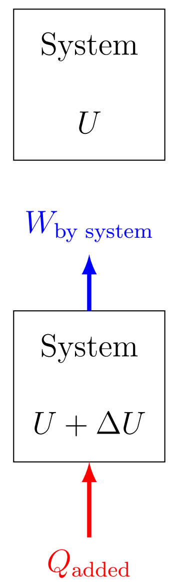
Quiz #7

Time	Topics	Assignments
12:30 - 1:30	First law of thermodynamics: energy	
1:40 - 2:20	Second law of thermodynamics: entropy	
2:30 - 3:30	Catch up, review, practice exam	

Day 14, Hour 1: First law of thermodynamics: energy

It should seem plausible by now that one of the laws of thermodynamics is conservation of energy. We will look at how this conservation law can be applied when it includes both heat energy and mechanical energy. One of the main goals is to understand systems for converting heat energy into useful work.

15.1 The first law of thermodynamics



The first law of thermodynamics simply says that energy is conserved. It applies to everything, but we will apply it especially to thermodynamic systems, that is, groups of molecules that can gain or lose energy in different ways, and can do work. Picturing a system as a box of molecules with total energy U, we can say

- When heat energy is added to the system then, logically, this energy will either be found in the system or it will come out as work done by the system.
- To express this mathematically, the heat added to a system is Q, and the work done by the system is W. The change of energy within the system is ΔU. So

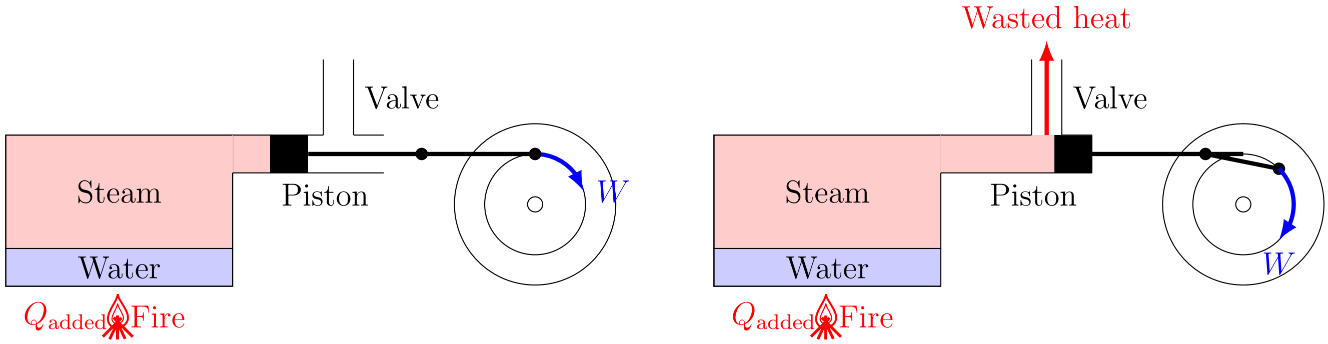
$$\boxed{Q = \Delta U + W \text{ (1st law of thermodynamics.)}}$$

15.2 The first law of thermodynamics and some simple processes

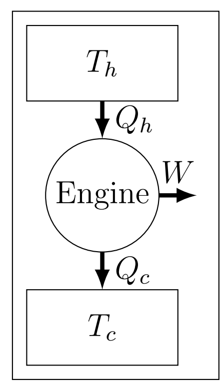
Heat engines

- A solid or a liquid, when heated, generally expands, and so it could do work using heat.
- If you need a rock lifted, you could place it on a piece of iron, then heat the iron to get some thermal expansion: $\Delta L = \alpha L \Delta T$.
  - The work done on the rock by lifting it a distance ΔL would be $W = mg\Delta L$.
  - Because α is small, you would get relatively little work done by the heat added to this system.
  - Most of the added heat is going into internal energy: $Q = mc\Delta T$. Higher T means molecules with more KE, so the system mostly just gains internal energy, ΔU.
- Gases are much more effective at turning heat into work than solids or liquids.

Allowing a heated gas to expand is a good way to convert heat into work. A gas cylinder with a moveable piston is one of the ingredients for many heat engines, along with some means for heating or cooling the gas.



- In this cartoon sketch of a basic steam engine, a fire provides the heat to make steam, and the steam provides the pressure to push a piston and turn a wheel somehow.
- But the piston could only be pushed once unless there is also a valve to let the steam escape at some point; then the piston can be returned (by the momentum of the wheel, somehow).
- It is too complicated to draw engines like this, but it shows some essentials.
  - They have a source of heat at some high temperature T_h.
  - They have a place for unused heat to escape at a lower temperature T_c.
  - They have a mechanism for doing work with some of the heat energy.
  - They have some way of repeating their action, working in a cycle of some kind.
- A heat engine can be sketched without all the mechanical details like this:



  - The high-temperature source of heat is a reservoir at temperature T_h.
  - Heat Q_h goes into the engine from the hot source.
  - Some work W is done by the engine.
  - The unused heat is Q_c expelled to a cold reservoir at temperature T_c.

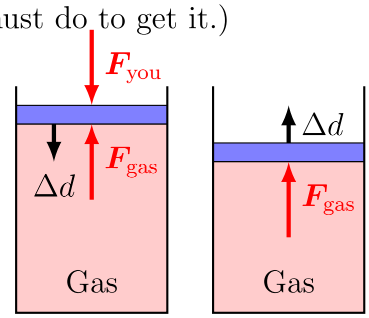
  According to the first law, since energy is conserved, the work done by the engine is

$$\boxed{W = Q_h - Q_c.}$$

  Engine efficiency is defined as the amount of work produced per unit of heat provided:

$$\boxed{Eff = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}.}$$ (What you want/What you must do to get it.)

Now we can look at the thermodynamics of a gas as the basis of a heat engine.



- Suppose you apply a force to a piston to compress a gas in a cylinder. When it is being compressed the gas does negative work.
- And if the piston goes up, as the gas expands it does positive work.

For gases, the pressure produces the force on the piston, according to the definition $P = F/A$, where A is the piston area, so the work done by the gas is

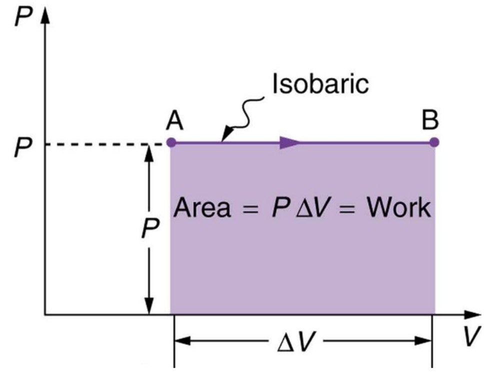
$$F \cdot \Delta d = P \cdot A \cdot \Delta d = \boxed{P \cdot \Delta V = W_{\text{by gas}}.}$$

PV diagrams and work

Since pressure and volume are so important in this process, it is helpful to plot them together on a PV diagram. P is plotted on the vertical axis, and V on the horizontal axis. Sometimes it is also apparent what is happening to the temperature T as well, even though it is not plotted. We will look at some of the simple possibilities for useful behaviors of gases.

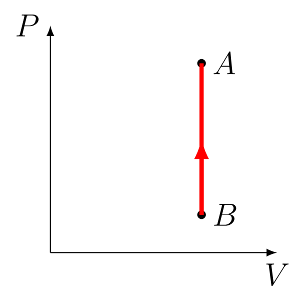
1. Isobaric (constant pressure) process
   Suppose a gas is made to expand at constant pressure, as sketched in Fig. 15.10 from our text.

   - The work $W = P \cdot \Delta V$, is the area under the line that represents the volume change.



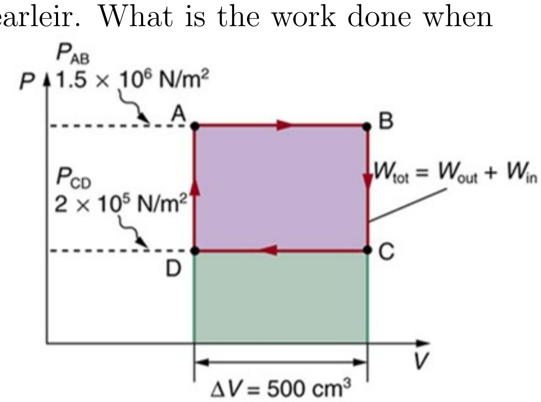
   - If the volume increases, $W > 0$.
     - For this process to happen, there must be an increase in temperature from T_A to a larger T_B.
     - Heat must be added to the gas, and some of it goes into an increase of the internal energy.
     - The actual amounts of Q and ΔU would depend on the type of gas used.
   - For an isobaric compression, $W < 0$, the temperature decreases, $\Delta U < 0$ and $Q < 0$. Heat must be removed from the gas.

2. Isochoric (constant volume) process
   - If the volume is made to stay constant while pressure changes, there is no area under the curve, and no work done.
   - There must be a temperature change happening too. In this sketch, the temperature must increase, so the heat added is all going into internal energy:
   $$Q = \Delta U.$$



3. Cyclic processes
   The importance of a cycle for a heat engine was mentioned earleir. What is the work done when a process returns the gas to its starting condition?

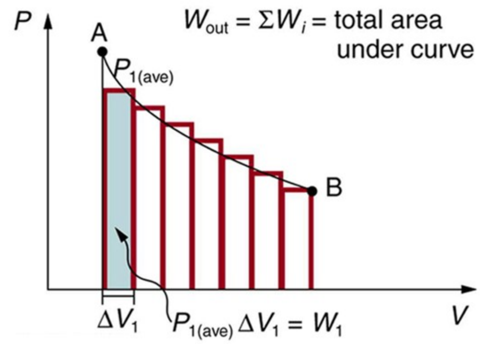
   - From A to B the volume increases, and $W_{AB} > 0$ is the area under the line AB.
   - From B to C, no work is done.
   - From C to D, negative work is done, in an amount equal to the area under line CD.
   - From D back to A, again, no work is done.



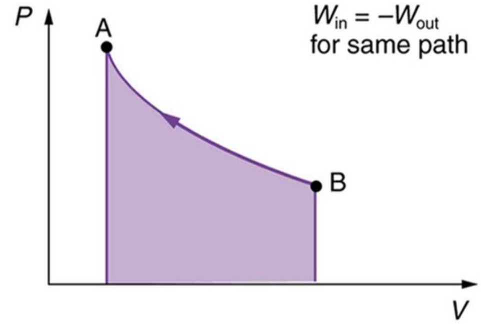
   - The total work done, including the positive and negative amounts, is equal to the area inside the cycle $ABCD$.
     - If the cycle is clockwise, $W_{\text{tot}} > 0$. If the cycle goes counterclockwise, then $W_{\text{tot}} < 0$.

4. Unspecified process

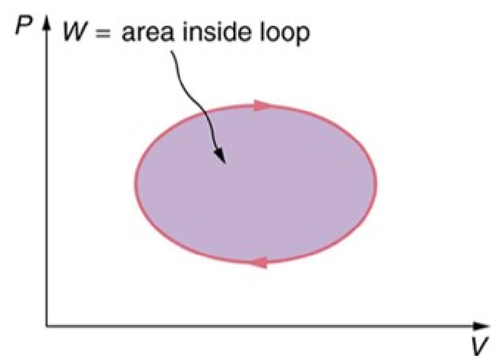
Figure 15.11 shows some unspecified process happening, in which both P and V are made to vary. The work done in any process still turns out to be the area under the curve, even though it may not be simple to calculate.



- Along the path from A to B, if the volume change is divided into infintely thin, infinitely many small changes, there is some pressure for each, and the work done in that interval must be $P \cdot \Delta V$. But this is the area under that segment of the curve.



- Adding the work in all the intervals gives the total.
- If the path is an expansion of the gas, $W > 0$. If the gas is compressed then $W < 0$.
- If the path forms a closed loop, as before, the area inside the loop is the work done in the cyclic process.
  - the work $W > 0$ for a clockwise cycle, and $W < 0$ for a counterclockwise cycle.



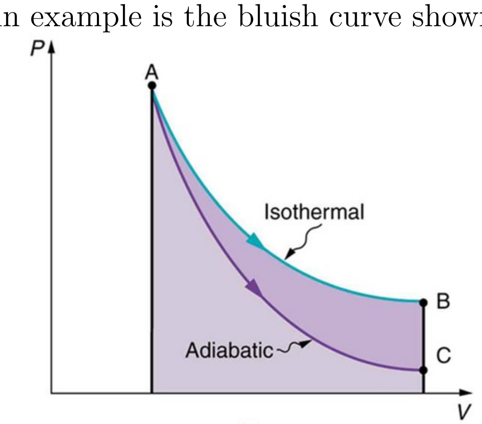
5. Isothermal process

A process that keeps T constant is called isothermal. An example is the bluish curve shown from A to B in our book's Fig. 15.13.

- Using $PV = nRT =$ constant, an isothermic curve on the PV diagram satisfies the equation

$$P = \frac{\text{constant}}{V},$$

where the constant depends on T of course.



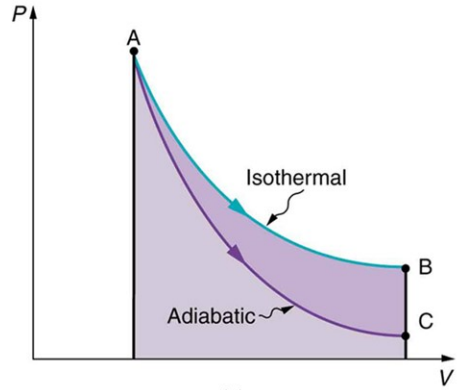
- Since there is no change of internal energy, all the added heat goes into doing work:

$$Q = W.$$

- An isothermal process must be relatively slow, to allow the gas to maintain a constant T throughout the system. This is in contrast to the next process.

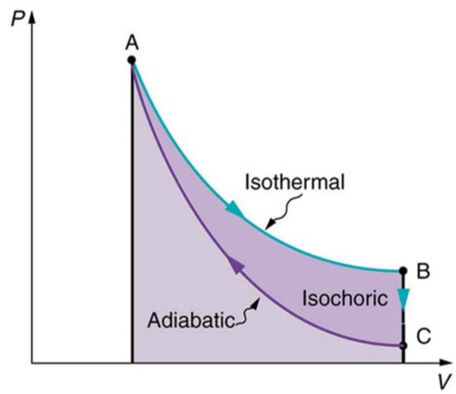
6. Adiabatic process (usually relatively fast, giving no time for heat to enter or leave the system.)

A process that occurs with no heat added to the gas is called adiabatic. An example is the violet curve from A to C in Fig. 15.13 from the text.



- Because $Q = 0$, the work that is done to expand the gas in this A-to-C process must come from the internal energy of the gas, so $W = -\Delta U$.
- Comparing with the isotherm above it, it should be evident that the gas goes to a lower temperature along the adiabat. The gas at C has a lower pressure than B but the same volume, therefore $T_C < T_B$.

These are the kinds of processes that make up an engine cycle. For example, in the cycle shown here,



1. Heat is added along the AB isotherm, and it all turns into work: $W_{AB} = Q_{AB}$ and $\Delta U_{AB} = 0$.
2. No work is done on the isochoric side, but the gas is cooled and its pressure is lowered; it loses internal energy and that is the heat that is removed: $W_{BC} = 0$, $Q_{BC} = -\Delta U_{BC}$.
3. No heat transfer takes place along the adiabat, but the gas is compressed by raising its pressure, allowing some increase in temperature too. Negative work is done, equal to the amount of internal energy the gas gains. $Q_{CA} = 0$, $W_{CA} = -\Delta U_{CA}$.

Other examples are shown in the text, such as the Otto cycle, corresponding to an internal combustion engine used in cars. It uses two adiabats and two isochoric processes. You might find it intersting to read about.

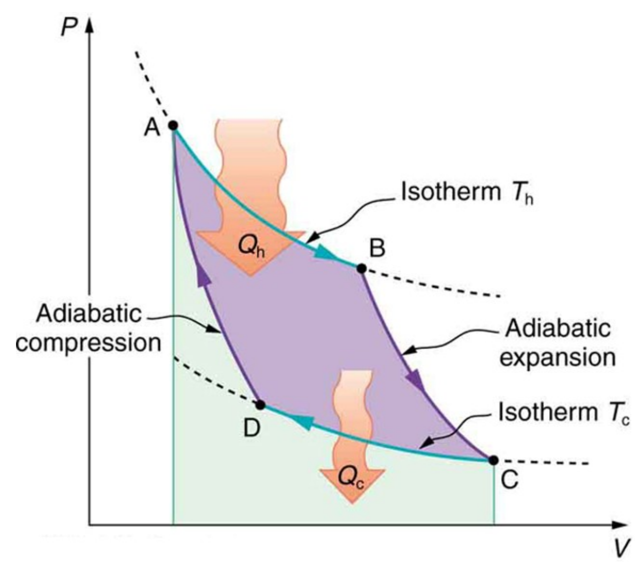
The one famous cycle you should know about is the Carnot cycle: two isotherms and two adiabats. It turns out that the Carnot engine is the best possible at converting heat into work.

- Heat Q_h goes in along the isotherm AB.
- Heat is removed along isotherm CD.
- These amounts of heat, Q_h and Q_c are proportional to the Kelvin temperatures at which they happen:

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}.$$

- Since $Eff = 1 - \frac{Q_h}{Q_c}$ for any engine, for the Carnot engine this becomes

$$\boxed{Eff_C = 1 - \frac{T_c}{T_h}}.$$



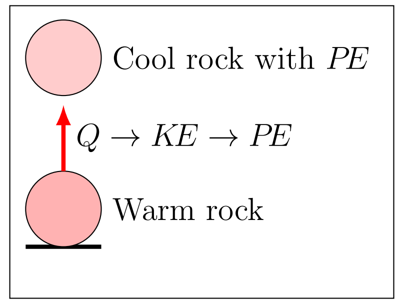
- Efficiency is a number between 0 and 1, theoretically.
- But in practice, it can never reach 1, because that would require $T_c = 0\,\mathrm{K}$. That is called a perfect engine, because it would convert all the added heat into work; there would be no wasted heat Q_c from a perfect engine.
- The Carnot engine is sometimes referred to as an ideal engine, because it does the most possible work with the temperatures available.

Day 14, Hour 2: Second law of thermodynamics: entropy

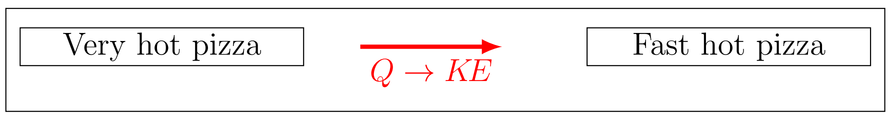
If heat Q and work W are just different forms of energy, interchangeable with U in a system, and the first law simply requires that they add correctly:

$$Q = \Delta U + W,$$

then why don't we see things like this happen?



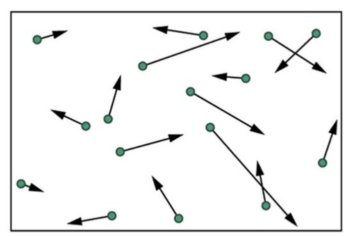
Or, better yet, a self-delivering pizza:

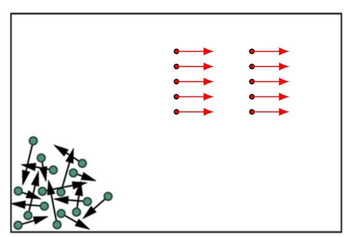


$$Q = mc\Delta T = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2c\Delta T}$$

Say $c = 500\,\mathrm{J/(kg\cdot °C)}$ and $\Delta T = 1\,°\mathrm{C}$. Then $v = \sqrt{2(500)(1)} = 32\,\mathrm{m/s} \approx 71\,\mathrm{mph}$.

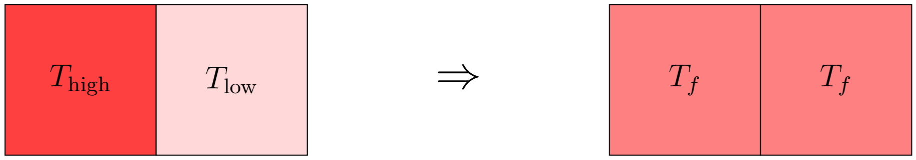
For these kinds of things to happen, some of the random velocity of molecules in objects would have to change into a concerted motion of all the molecules in the same direction. This turns out to be a very unlikely accident when dealing with large numbers of molecules.


(a) Likely


(b) Highly unlikely

- The 1st law of thermodynamics does not have any problem with it.
- But the 2nd law of thermodynamics says it will not happen accidentally.
- If a baseball flies past, you know someone threw it; it's not a lucky accident of random motion.
- If you see half the water molecules in the Red Sea go left, and the other half go right, you know it was no accident. Someone did that.
- The probability is based on the large numbers of molecules involved.
- The 2nd law deals with the entropy, or disorder of a system, and says that entropy does not decrease for the system as a whole.

As a simple example, you might recall from last time the idea that heat naturally goes from the higher-temperature object to the lower, until they reach an equilibrium temperature. The second law easily can be used in a quantitative way to account for this.



A change in entropy is measured as

$$\boxed{\Delta S = \frac{Q}{T},}$$

and the second law says

$$\boxed{\Delta S \geq 0.}$$

So when heat Q flows from a hot object to a cold one, the entropy changes are

$$\Delta S_c = \frac{Q}{T_c} > 0, \text{ and } \Delta S_h = \frac{-Q}{T_h} < 0, \text{ so}$$

$$\Delta S_{\text{total}} = \Delta S_c + \Delta S_h = \frac{Q}{T_c} - \frac{Q}{T_h} = \frac{Q}{T_c T_h}(T_h - T_c) > 0.$$

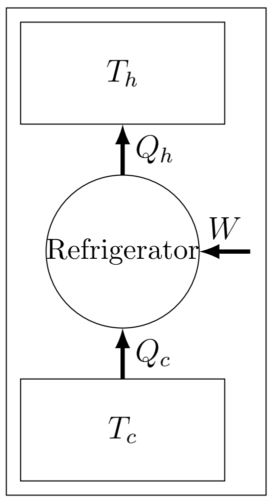
This is what we should expect to happen.

- The hot object loses entropy, because its molecules are slowed down in the transfer of heat.
- But the cold object gains entropy, and a larger amount than the hot object lost.
- The overall change is an increase in entropy if heat flows from hot to cold objects.
- This is what allows a heat engine to extract some of the transferred heat energy and turn it into work.
- The Carnot engine is an example of entropy remaining constant, because in that case

$$\text{Carnot engine: } \frac{Q_h}{T_h} = \frac{Q_c}{T_c} \Rightarrow \Delta S_{\text{C total}} = 0.$$

Does heat ever flow the other way? Yes, it can be arranged with a device that looks a lot like the heat engine, but with all the arrows reversed. A refrigerator is basically a heat engine in reverse:

- Some heat Q_c is taken from a cold reservoir at temperature T_c.
- Heat Q_h is delivered to the high temperature reservoir at T_h.
- But some work must be put into the machine to make this happen, because it lowers the entropy of the reservoirs.
- This work could be provided by a heat engine someplace else, that is raising the total entropy to provide the work W.



According to the first law, since energy is conserved, for a refrigerator

$$\boxed{W + Q_c = Q_h.}$$

And instead of efficiency, the measure of success is called the coefficient of performance, defined as the amount of Q_c moved per unit of work provided:

$$\boxed{COP_{\text{ref}} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c}.}$$

This quantity could be anything from 0 to infinity, theoretically.

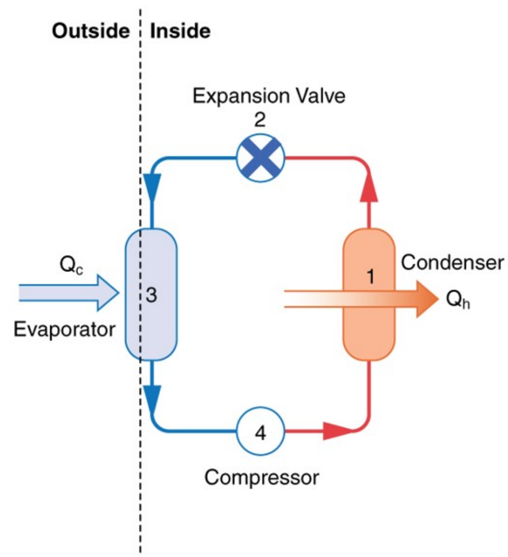
- A "perfect" refrigerator would be one that requires no work to cause heat to flow from cold to hot. It would have a $COP = \infty$, but it would also violate the second law of thermodynamics.
- A refrigerator doesn't work unless there is a source of energy available to do enough work.
- Keep the fridge plugged in, or it won't keep your groceries cold.

One more useful form of the refrigerator is called a heat pump. It is a refrigerator, but its purpose is to heat rather than to cool, so its coefficient of performance is defined as

$$\boxed{COP_{\text{hp}} = \frac{Q_h}{W} = \frac{Q_h}{Q_h - Q_c}.}$$

- This always turns out to be > 1,
- A "perfect" heat pump would have $COP_{hp} = \infty$, just like a refrigerator, but there is no way to get this reverse heat flow to happen without violating the second law of thermodynamics. Some work is needed to operate the machine.
- Heat pumps are very popular because they take available heat, such as heat from underground where it is warmer than the outside air in winter, and use it to heat a building.
- It requires some work W to move the heat to where it is wanted. But it is similar to finding that gasoline is free a few miles away, and driving your car there to get some. It's a good deal.

Figure 15.28 shows the idea of how a typical refrigerator or heat pump functions:



- A substance that is easily compressed into a liquid and expanded into a gas is circulated betwen the two temperature reservoirs.
- As a gas, it absorbs heat Q_c at temperature T_c.
- When compressed to a liquid, it gives off heat Q_h at temperature T_h.
- Then it is allowed to expand again by lowering the pressure, and the cycle repeats.
- The compressor, and a pump to keep the fluids moving, require work to keep them operating.