1. UNCERTAINTIES

Purpose

No measurement produces an exact result; every meter has limited precision. This lab will introduce how physicists estimate and report the accuracy of measurements. Random error, systematic error, standard deviation, and standard deviation of the mean are defined through examples; These terms will be freely used in the remaining labs. Additionally the required format for spreadsheet hardcopy ('self-documented', 'final results format') is explained in text and video.

Introduction

This first-week lab is atypical: you need not complete the lab in the scheduled location at the scheduled time. You will not write up your results in your lab notebook. Rather the lab may be completed by yourself online at any location with internet access. (Do feel free to come to the scheduled room/time—your instructor's office will then be nearby and help will be immediately available— but generally students choose to work at home at a time convenient to themselves.) View the one-hour, 800 MB video:

`http://youtu.be/PxWS_HnsPh4` or
`http://www.physics.csbsju.edu/lab/105Lab1.mp4`

read this chapter and then complete the on-line lab report:

`http://www.physics.csbsju.edu/lab/105Lab1.html`

At your next lab period, turn in the required **spreadsheet exercise**. Given the large size of the video, you are encouraged to view or download it on-campus (but bring your own earbuds).

Theory: Measurement Errors

How much does a cat weigh?

When I put Tiger on a scale, the reading bounces around randomly: Tiger is not one to sit still. I'd guess that the average scale reading is more closely related to his weight than any particular reading, and indeed you'll learn this semester that the net impulse on an object determines the change in velocity of that object's center of mass (ΔV_{cm}). Since Tiger's center of mass isn't moving (much) we are assured that the time averaged scale reading is close to his weight. In particular, averaging over a time interval T, we exactly have:

$$\text{average scale reading} = \text{weight} + \frac{M\Delta V_{cm}}{T} \tag{1.1}$$

where M is Tiger's mass. So the longer we average those scale readings, the more accurately we'll know his weight, since the unknown (and likely small) quantity $M\Delta V_{cm}$ will be divided by an ever larger number, T. Of course if I knew the value of $M\Delta V_{cm}$, I could include it in the equation and gain accuracy, but aside from guessing that it's "small," I know nothing about it, not even its sign. The best I can do is exclude times (like when Tiger jumps on or off) when his center of mass is in noticeable motion.

Randomly fluctuating measurements like this are known as random uncertainties or random errors. In the context of this document, "uncertainty" and "error" mean exactly the same thing. Use the word "blunder" to describe what is sometimes called "human error": incorrect or accidental human actions that result in inaccurate measurements; for example not properly zeroing ("taring") the scale before putting Tiger on it.

At some point, it's nothing is gained by extending the averaging time because:

- The balance manufacturer only claimed a precision of ± 0.02 lbs, so there's little reason to reduce the random variation much below that level.
- If not level the scale will report only a component of weight. If off vertical by 3°, there will be in error of about -0.02 lbs. How carefully did I level the scale?
- A living cat does not have a weight. The mass enclosed by his skin changes as fluids (air, water, urine) and solids (food, feces) are exchanged. Because of this it's hard to imagine any *practical* significance to weight difference of order ± 0.1 lbs.

The first two items on this list could be called "systematic errors" or "calibration errors" or "biases". They result in incorrect measurements but do not signal their presence by a fluctuating answer. They can only be detected by testing with a standard, known mass. Since systematic errors are consistent, if we knew the value we could correct the reading, but as with random errors, at best we have some estimate of the magnitude of the error, but no idea of its precise value or sign.

Last item on this list is a "problem of definition:" the definition of the weight of a living being is ambiguous.

SO we can measure Tiger's weight and report guess-estimates for the random and systematic error, but the original questions was: How much does a cat weigh? A cat is a randomly selected cat, and of course it doesn't have a weight: different cats weigh different amounts. If we know the weight of the set of cats from which a cat was selected (or more likely: a collection of cats we judge to be similar), it is probable that a cat would be similar to the majority of cats. We need to report some typical weight and a range of variation that includes "most" cats. Notation: assume we have a set of N cat weights: $\{w_1, w_2, w_3, \ldots w_N\}$ (or equivalently: $\{w_i\}$ for i from 1 to N).

The two most common measures of "typical" are average, a.k.a., mean (add up all the cat weights and divide by the number of cats):

$$\text{average of the weights} = \overline{w} = \frac{1}{N}\sum_{i=1}^{N} w_i \tag{1.2}$$

and median (the middle weight: half the cats weigh more than the median, half weigh less).

There are several common measures of "range of variation". The most common is the standard deviation: subtract the average from each weight producing a list of deviations-from-average, some positive some negative; add up the square of these deviations-from-average, divide by $N-1$, and take the square root of the result:

$$\text{standard deviation of the weights} = \sigma_w = \sqrt{\frac{\displaystyle\sum_{i=1}^{N}(w_i-\overline{w})^2}{N-1}} \tag{1.3}$$

Remark: You should never actually use this formula: calculators and spreadsheets have this function built in. We present it to you so you have a general idea of what the standard deviation is: The square root of the average of the squared deviations-from-average. Re-read the previous sentence and make sure you understand what it is saying. The standard deviation is a measure of typical deviation-from-average.

If the distribution of cat weights is "normal" you should expect that about $\frac{2}{3}$ of the cat weights are in the interval: $(\overline{w}-\sigma_w, \overline{w}+\sigma_w)$. This likely interval is often marked with an "error bar" that displays the extremes of this interval as whiskers up and down from the average value. Leaving $\frac{1}{3}$ of cats outside of this normal range is sometimes viewed as insufficiently inclusive; error bars that encompass 95% of the cats (typically this is nearly the same as the interval: $(\overline{w}-2\sigma_w, \overline{w}+2\sigma_w)$) may be used and called 95% confidence limits. Some folks op for 'equality' error bars that enclose half of the cats, which may be approximately the smaller range: $(\overline{w}-0.674\sigma_w, \overline{w}+0.674\sigma_w)$; this is also called the interquartile range (the spread between the 25^{th} percentile and the 75^{th} percentile). Unfortunately there is no consistent meaning assigned to an error bar. In this this document, we will use the $\pm 1\sigma$ meaning, which means it's not hugely unlikely (i.e., $\frac{1}{3}$ of the time) for a datum to lie outside the error bar. You should recognize an unusual occurrence (5%) when a datum misses by 2 error bars.

Boxplots (a.k.a., box-and-whisker plots) are a standard, if less common, way to display the full range of variation which is encoded in the five-number summary: minimum, first quartile (25^{th} percentile), median, third quartile (75^{th} percentile), and maximum. A rectangle runs from first quartile to third quartile with a horizontal line indicating the median; 'whiskers' extend down to minimum and up to maximum. Unusual data points called outliers may also be specially marked.

Many have argued (the author included) that a better measure of variation would be the average absolute value of the deviations-from-average: while that is both a more robust and intuitive measure, the awkward mathematics of absolute value make it less commonly used by statisticians.

How much does an average cat weigh?

If we actually weigh every cat in the world we could definitively answer this question, but usually we have weighed some subset of the world cat population. In the case of Tiger's weight, Newton's Laws prove that averaging over more weights improves the estimate of Tiger's weight. It seems evident that averaging over larger collections of cats would improve our estimate of the average cat's weight. Indeed one can show that the likely inaccuracy of the average decreases with the inverse square root of the number of cats we weigh (and average). This **standard deviation of the mean** (SDOM) is given by:

$$\text{SDOM} = \sigma_{\overline{w}} = \frac{\sigma_w}{\sqrt{N}} \tag{1.4}$$

This formula is too good to be true: it claims we can have arbitrarily small uncertainties simply by collecting more data. Every time you use this formula you should ask yourself:

- Since systematic errors are not reduced by averaging, is this reduction of the random errors useful, given that total error is what really matters?
- Is the average really the quantity I want to know? In the case of Tiger's weight I could show a direct relationship between the average and the actual weight. However that is not always the case. For example, the path followed by the average initial speed ball is not the average path of balls. The weight of an average radius steel sphere is not average weight of steel spheres.

It is important to recognize that the uncertainty in the weight of a cat is much larger than the uncertainty in the weight of the average cat.

Average cat?

In 1947 the Cambridge statistician R.A. Fisher published weight data on 144 cats[1]. I've collected recent data on 25 Minnesota in-home cats. Which defines the average cat? Figure 1.1 displays the two distributions as a boxplot, as the mean with standard deviation error bars (STDEV), and as the mean with standard deviation of the mean error bars (SDOM). The boxplot shows that these samples overlap in part; STDEV shows no overlap between 1σ errors bars, but clearly 2σ errors bars (typically 95% inclusive) would again show a small overlap. SDOM shows distinctly different means. Clearly there is some systematic difference between well-fed MN home cats and cats used in

[1] The data was actually produced as a part of a 1946 study on the lethal dose of digitalis in cats.

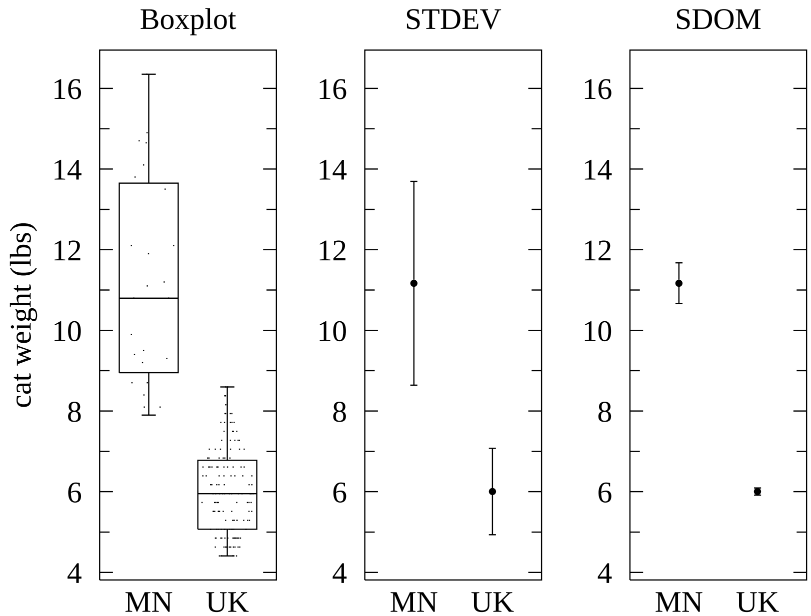

Figure 1.1: In search of the "average cat," I present two datasets on cat weight. UK reports the weight of 144 cats used in a pharmaceutical study reported by R.A. Fisher in 1947; MN reports the weight of 25 MN in-home cats collected by the author. The boxplot's rectangle shows the central 50% of cats with the median marked with a bisecting horizontal line and whiskers out the the group's maximum and minimum. STDEV displays the mean with 1σ error bars. SDOM displays the mean with error bars extending to the standard deviation of the mean. Clearly while there is some overlap between the distributions, the UK and MN means differ significantly. The difference between the datasets demonstrates that additional study is required to uncover the 'average cat'; it would be foolhardy to accept either mean±SDOM as the measure of the average cat. While SDOM provides an interesting measure of a group mean, that group's mean may not be exactly the quantity we seek.

post-war UK pharmaceutical research. Most Minnesotans would find it incredible that a $6\frac{1}{2}$ lb cat was "significantly above average". (More evidence that in Minnesota every cat is "above average".) Here in lies the danger (and benefit) of SDOM: large datasets can reduce 'errors' to practically meaningless levels (in the UK data, less than 0.1 lb). If the average is exactly the quantity we seek, we can measure it with extraordinary accuracy. If the average is not exactly equal to the quantity of interest, or if the sample we use to define average is not representative, we risk humiliating reversals from future work.

In summary: a measurement has no value if it lacks an estimate of its accuracy. In these labs you must always report both the measurement and its uncertainty (="error") that may be a mix of reading variation, hard-to-estimate, pernicious systematic errors and problem of definition errors. The deviation of a hypothetical future measurement can be estimated by the standard deviation of set of previous measurements. The uncertainty in a average value can be estimated by SDOM of a set of previous measurements (if certain conditions are met).

Absolute and Percent Errors

If Tiger's weight is 14.9 ± 0.1 pounds, the '0.1 pounds' is called the absolute error. The absolute error is a direct estimate of the possible deviation between the measurement and the actual value, i.e., in this case that Tigers weight is probably in the interval (14.8, 15.0) pounds. The absolute error always carries the same units/dimensions as the base measurement.

Often it is useful to express the possible deviation as a percentage of the measurement. In this case, since:

$$\left(\frac{0.1}{14.9}\right) \times 100 = 0.6711 \approx 0.7 \tag{1.5}$$

we may report Tiger's weight as 14.9 pounds $\pm 0.7\%$. The percent error never carries units and (aside from the factor of 100) is the same thing as 'relative error' or 'fractional error'.

Judging Uncertainty

As significant fraction of your lab grade this semester will be based on your ability to judge the uncertainty of your measurements. The accuracy of most electronic instruments is recorded in the manufacturer's specifications; typically these are estimates of possible systematic error. (We will supply you with those estimates in lab.) Accuracies may be reported as 'absolute errors': the direct estimate of the likely standard deviation, or as 'percent errors': where the likely standard deviation can be calculated by taking the given percentage of the displayed value, or both.

For example the manufacturer reports that accuracy of the voltmeter you will be using next semester is $\pm(0.1\% + 4\text{digits})$, where one 'digit' is the place value of the rightmost digit on the display. So if the display showed 6.238, the uncertainty would be $6.238 \times 0.1\% + .004$; if the display showed 71.49 the uncertainty would be $71.49 \times 0.1\% + .04$.

This semester many of your measurements will be made using rulers. Generally when using a ruler (or any device whose scale you read by eye) you can estimate the fractional bit between the marked gradations. Error estimates that are a third or a quarter (at most a half) of those marked divisions are then appropriate.

An important special case applies to counting events that have no preference over time or space. For example, the count of weeds growing in a uniform farm field, or the decay of a Uranium atoms which are just as likely to decay this year as in the year 4,000,000,000 B.C.. Whole number counts of such events (weeds in an acre or decays during a year) will be randomly different for different acres or years, but there is a simple estimate for the standard deviation of such measurements based on just one measurement: the square root of the number of events. For example, if I measure 121 weeds in one acre, I estimate that $\frac{2}{3}$ of the acres will have between 121–11=110 and 121+11=132 weeds on them. (So $\frac{1}{3}$ of the acres will have fewer than 110 or more than 132 weeds.) This special case is known as Poisson statistics.

In this semester's labs we will not be expecting precise error estimates. Because of the limited lab time you will often be called on to judge the typical range-of-deviation by eye rather than using the standard deviation formula. You should develop the ability to judge such deviations (accurate to maybe a factor of two) just by eye. The main point this semester is that every measure has uncertainty, and this needs to be factored into all your further calculations with the data.

Types of Measuring Instruments

We can enumerate two broad categories of measuring instruments: analog and digital.

The ruler and anything else that has a continuous scale where the operator can "read between the lines" is referred to as analog. Triple beam balances and meters with pointers are examples of analog instruments. The operator of an analog instrument must judge both the value and appropriate uncertainty. Quite often the uncertainty is estimated at plus or minus a fraction of the smallest scale division, but it always involves a judgment call by the operator.

Digital instruments are quite different. They are incremental rather than continuous and one can't "read between the lines." In typical use they provide a steady reading with no obvious random fluctuation. The manufacturer usually provides the uncertainty specifications (typically systematic errors) of digital instruments. For example, a digital balance may read 10.3 grams where the last digit, according to the balance manufacturer, has an uncertainty of two (0.2 gram in this case). For other models it might be something different, such as 0.1 gram. Usually the uncertainty involves the last digit, but sometimes it is expressed as a percentage of the total value or a combination of percentage plus the last digit.

Comparing Measurements with Uncertainty

It is unlikely that two measurements of the same quantity will exactly agree. However usually (i.e., approximately $\frac{2}{3}$ of the time) the intervals spanned by their error bars will overlap. If they miss by more than 2 error bars, most folks would call it a significant disagreement. Occasionally measurements can be compared to precise predictions of theory (for example, the ratio of measured circumference to diameter should be precisely π), in which case usually the theoretical prediction should lie within the error bars of a measurement.

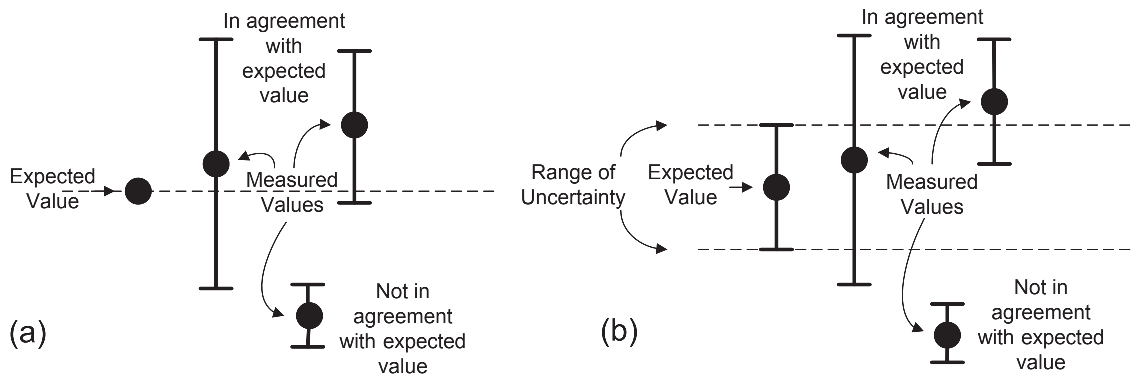

Figure 1.2: Examples of measured values with uncertainty, compared to an expected value (a) with negligible uncertainty, and (b) with the indicated range of uncertainty.

Clearly one can wash over disagreements simply by enlarging the error estimates. However the value of a measurement depends strongly on the error; technically the ‘weight’ of a measurement is proportional to the inverse square of the error. So a number with $2\times$ bigger error bars has $\frac{1}{4}$ the weight. Thus the worth of a measurement is judged by its error; measurements with small systematic and random errors are considered the best measurements.

If results show significant disagreement, then either (A) we’re in a statistically unlikely (but not impossible) case or (B) mistakes have been made or (C) something interesting is going on which makes two correctly measured quantities disagree.

While I haven’t made a valid study of the frequency of these possibilities, my sense is that (B) is by far the most common situation, and that usually the mistake is undetected systematic errors in the measurement.

Reporting Measurements

You should record (in your lab book or Excel) every digit displayed by an instrument and every digit you estimate when reading a ruler. Don’t round your raw data! Of course you must also report an uncertainty for those measurements. When it comes to reporting *final results* (in contrast to raw data or intermediate calculations) we have strict requirements as to exactly how you record those numerical results. Our uncertainty in the uncertainty is always large; hence round any uncertainties to one or two significant figures (“sigfigs”[2]). One sigfig is always easier and almost always as accurate as two sigfigs. (Recall: You may have estimated your range-of-deviation just by eye. Manufacturer’s specification of errors almost always report just one sigfig (recall: $\pm(0.1\% + 4\text{digits})$) so one sigfig would be appropriate for an error calculated from those numbers. One can show that in ‘normal’ cases, formula-calculated standard deviations will have un-

[2]If you are unfamiliar with sigfigs, see page 94, or consult the textbook, or Wiki.

certainty greater than 10% unless the number of available data points, $N > 50$.) Once you have properly rounded your errors, display all the digits of your measurement until the place-value of the least (rightmost) significant digit in the measurement matches the place-value of the least significant digit in the error. Some examples:

$$3.14159 \pm .00354 \Longrightarrow 3.142 \pm .004 \text{ or } 3.1416 \pm .0035$$
$$2.71828 \pm .05472 \Longrightarrow 2.72 \pm .05 \text{ or } 2.718 \pm .055$$
$$321456 \pm 345 \Longrightarrow 321500 \pm 300 \text{ or } 321460 \pm 350$$
$$.1678 \pm 3.51 \Longrightarrow 0 \pm 4 \text{ or } 0.2 \pm 3.5$$

Finally, remember to record the units of your measurement!

Spreadsheet Descriptive Statistics

In many cases uncertainties will be given by manufacturers specifications (typically systematic errors) or by-eye estimates of deviations-from-average, e.g., Tiger's weight. For repeated measurements, spreadsheets allow easy calculation of average, standard deviation and standard deviation of the mean.

To find the average, select an used cell and enter the formula: `=AVERAGE(A1:A36)` where the list of cells you want averaged (here `A1:A36`—the first 36 cells in the first column which is called `A`) is enclosed by parenthesis. Typically you will get the list of cells by sweeping through them with left mouse button depressed or by clicking on the first cell and then the last cell in the list while also holding down the Shift key. After hitting Enter, your formula will be replaced by the appropriate value.

To find the standard deviation, use the formula: `=STDEV(A1:A36)`.

The standard deviation represents the uncertainty for any single measurement, e.g., a cat's weight.

If the mean is the value whose error you want to estimate, you need the SDOM (standard deviation of the mean). To find this, use the formula:
`=STDEV(A1:A36)/SQRT(COUNT(A1:A36))`, i.e.,

$$\text{SDOM} = \sigma_{\overline{x}} = \frac{\sigma_x}{\sqrt{N}} \tag{1.6}$$

where N=`COUNT(A1:A36)`. In this case we know there are 36 data points, so we might simplify by typing `=STDEV(A1:A36)/SQRT(36)` or even `=STDEV(A1:A36)/6`

The standard deviation of the mean represents the uncertainty in the mean, e.g., the average cat's weight.

Spreadsheet Self-Documentation

If you print out your spreadsheet, the formula used to calculate a cell will not be displayed; the grader will have no idea how the printed number was calculated. So in an adjacent cell you must display the formula used. This is easily done by copying and pasting the formula into an adjacent cell (which often results in an error or nonsense) and then editing out the '=' and hitting Enter. Excel will now display (and print if requested) the text of the formula used. Of course, you will also provide words ('headings') naming column and cell values (including units). Note: in documents and your lab notebook results (measurement/error/unit) are recorded in the form: '14.9±0.1 lbs'; don't do this in your spreadsheet! Text like '±' and 'lbs' make the numbers sharing that cell unavailable for further calculation. Instead put each number in its own cell and in an adjacent cell report the name of the quantity and units (the units are typically enclosed in parenthesis):

14.9	Tiger's weight (lbs)
0.1	Error in Tiger's weight (lbs)

Theory: Calculating with Uncertain Numbers

Measurements are usually followed by calculations, which make use of the measurements. Perhaps you are calculating the area of a rectangle. Imagine that your distance measurement of 8.8 ± 0.3 units is to be multiplied by another uncertain distance measurement: 2.1 ± 0.2 units. One way of determining the uncertainty of the calculated result is to use a "high-low" approximation.

High-Low Method

The high-low method involves calculating the result three times: once without uncertainty — use your best values — and then two additional times: finding the highest possible result and the lowest possible result, as illustrated below and in Fig. 1.3. The answers to the three calculations are as follows,

$$\begin{array}{lll} \text{Best:} & 8.8 \times 2.1 & =18.48 \\ \text{High:} & 9.1 \times 2.3 & =20.93 \\ \text{Low:} & 8.5 \times 1.9 & =16.15 \end{array}$$

The range of results could now be expressed as $18.48 + 2.45$ to $18.48 - 2.33$ units squared. This unsymmetric form is awkward, so the difference between the two extremes is split evenly ($20.93 - 16.15 = 4.78$, and $4.78/2 = 2.39$), and the result expressed as $18.48 \pm$

2.39. Following our final value reporting rules this is:

$$18 \pm 2 \quad \text{or} \quad 18.5 \pm 2.4 \text{ units squared.}$$

Again, here is no point in expressing uncertainties with more digits (for instance, 18.48 ± 2.39 or 18.48 ± 2): at least in these labs our error estimates never deserve 3-digit accuracy and there is no point in reported digits in the base number that are overwhelmed by the likely errors.

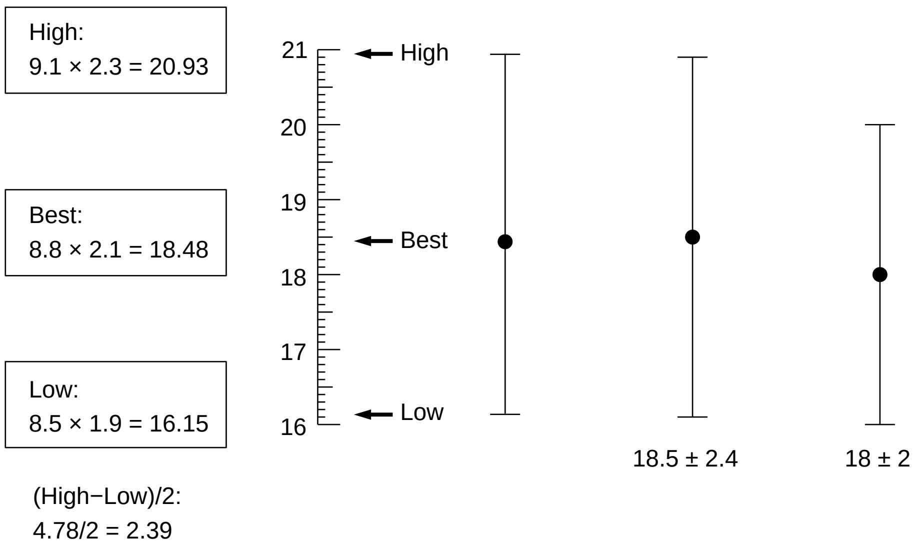

Figure 1.3: Example of calculating uncertainties by the high-low method.

It doesn't make any difference how complicated the calculations are when you use the high-low method. Always apply the uncertainties in such a manner as to maximize, and then minimize the result. In the case of division, maximize the numerator and minimize the denominator in order to maximize the result. Do the opposite to minimize the result.

One disadvantage to the high-low method is that several calculations are needed (the high, the low, their average and half their difference) to end up with a best estimate and its uncertainty. As a result it will be unclear which errors contribute most significantly to the final error; i.e., which measurements should be improved first.

Fractional Uncertainty Method

There is a simpler approach we can sometimes use, known as the fractional uncertainty method, in which the uncertainty in a result is calculated directly from the measured values and their uncertainties. The fractional uncertainty formulas (see Appendix A) are often fast and easy calculations. For instance, in the case of multiplication or division, fractional uncertainties are additive. Going back to our two distance measurements,

$$\frac{\text{uncertainty in area}}{\text{area}} = \frac{\text{uncertainty in distance 1}}{\text{distance 1}} + \frac{\text{uncertainty in distance 2}}{\text{distance 2}},$$

or more briefly,

$$\frac{\delta A}{A} = \frac{\delta D_1}{D_1} + \frac{\delta D_2}{D_2},$$

where A represents the area, D_1 and D_2 are the measured distances, and their uncertainties are indicated using the Greek letter δ ("delta"). We already know how to calculate the area $A = D_1 \cdot D_2$, and the uncertainty in the area can be found with one more calculation:

$$\delta A = A \left(\frac{\delta D_1}{D_1} + \frac{\delta D_2}{D_2} \right).$$

Using our measurements,

$$\delta A = 18.48 \times \left(\frac{0.3}{8.8} + \frac{0.2}{2.1} \right) = \pm 2.39$$

In this case the fractional uncertainty method and the high-low method produced exactly the same result; more generally they will differ, but never significantly. One advantage of the fractional uncertainty method is we can determine which terms are contributing the most to the final error. In this case $0.3/8.8 = .034 < .2/2.1 = .095$, so the the smaller absolute error (0.2) actually contributes more to final error.

In future labs, we'll tell you which method works easier/better for the particular experiment in question. In general, the high-low method will be used for complicated, multi-step calculations while the fractional uncertainty approach will be used with fairly simple calculations. The rules for using fractional uncertainties are in Appendix A.

For both the high-low method and the fractional uncertainty method the worst-case scenario is assumed, which may be unrealistic. If the actual deviations were random (or uncorrelated), as is often the case with measurement errors, one would expect some partial cancellation of the effects of uncertainty. Why, for example, would both distance measurements always be high or always be low? Why couldn't one be a bit high while the other is a bit low? If a calculation involves many steps, shouldn't there be at least some cancellation of random errors? These questions suggest that a statistical approach to estimating uncertainty is needed.

Procedure

View the one hour, 800 MB, on-line video:

`http://youtu.be/PxWS_HnsPh4` or
`http://www.physics.csbsju.edu/lab/105Lab1.mp4`

and then complete the on-line lab report:

`http://www.physics.csbsju.edu/lab/105Lab1.html`

Part I of the on-line lab report consists of multiple choice questions related to this chapter of the Lab Manual. Part II seeks definitions (in your own words) and examples of random and systematic error. This material is covered in the video and this chapter. Part III requires calculations in a spreadsheet. The required calculations and format are covered in the video. The numerical results of these calculates (with at least 4 sigfigs) are reported in the on-line form. Hardcopy of the completed spreadsheet ('self-documented' and in proper 'final results' format) must be turned in at the start of Lab 2.

Critique of Lab

As a service to us and future students we would appreciate it if you would also include a short critique of the lab—simply jot down your comments on the Excel hardcopy. Please comment on such things as the clarity of the Lab Manual and video, relevance of experiment, and if there is anything you particularly liked or disliked about the lab.

Quick Report

Generally, as you leave lab, each group should turn in a 3×5 "quick report" card. Typically it includes one or two numerical results (properly recorded: significant figures, units, uncertainty) from your Conclusion. There is no quick report card for this lab.

Checklist

At the end of each chapter in this Lab Manual. you should find a 'checklist': a brief listing of the required components for the lab. The checklist is necessarily terse, but it should provide reminders for critical elements of your lab report.

CHECKLIST

Part I: multiple choice questions from this Manual	
Part II: short answers, in your won words, not plagiarized from Google	
Part III: numerical results with at least 4 sigfigs	
Hardcopy of Excel spreadsheet: self documents, final results format	
Lab Critique jotted on Excel hardcopy	