PHYSICS FOR THE LIFE SCIENCES I

PHYSICS 105
LABORATORY MANUAL

It was a long spacious room, equipped with all kinds of scientific apparatus.

Spring 2021, Block A

Contents

Comic book scenes used in this manual are in the public domain and were obtained from the website http://Comicbookplus.com.

INTRODUCTION

> When you can measure what you are speaking about, and express it in numbers, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: It may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of science. *Lord Kelvin*

Purpose

Why does this course include a laboratory? In class you will be studying physical laws and you will probably become proficient at manipulating the symbols and speaking the vocabulary of physics. Yet it is possible to do this without a clear understanding of what those symbols and formulas represent. For example, in the lecture part of this course you will study the equation

$$\mathbf{F} = m\mathbf{a}, \tag{1.1}$$

where $\mathbf{F}$ represents the net force, m is a mass and $\mathbf{a}$ refers to the acceleration of the mass. And it is quite possible to recite these words and manipulate the symbols to solve textbook problems without understanding the reality of force, mass, acceleration, or the relationship among them. Physics deals with quantities that are *measurable*, and one of the best ways to find meaning in a physical law is to perform measurements of the relevant quantities. When you have learned how to measure acceleration, you may have a better idea of what acceleration is. So one purpose of the lab is to reinforce some concepts through the experience of making measurements. Another main purpose of the lab is to teach techniques in taking and analyzing data, and in reporting results. There will be emphasis on evaluating the amount of *uncertainty* that goes with measurements and results. Measurements are always imperfect and so the results are necessarily a bit uncertain, and it is clearly important to know the limits of their accuracy.

Lab Partners

It is often helpful to have more than two hands as you perform an experiment, so you will typically work with *one partner*. Each partner should be involved in the data-taking, the analysis, and the writing of the report. Of course the COVID-19 pandemic still requires 6-foot separations as much as possible.

Laboratory Materials

In a normal year you would record your results (using ink) in a laboratory notebook. While we believe that learning to keep a proper lab notebook is an important skill, COVID-19 makes dealing with such physical artifacts difficult; this year you will be submitting your lab reports electronically.

We do want to distinguish a lab notebook from a formal scientific report. A real lab notebook includes a record of what you did when, including mistakes. In a formal report the ducks are nicely in a row, and all mistakes and dead ends have been eliminated. Your electronic report this year should be in the style of a lab notebook.

Schedule

The experiments planned for the semester are synchronized as well as possible with the material covered in the Physics 105 lectures. A multiple choice Assessment Test will be taken near the end of the semester; it will count as a lab experiment (even though the Assessment Test is not a lab test). *This schedule may change during the semester. Your instructor will keep you updated.*

Class		Date	Labs
1	M	Jan 25	Uncertainties
2	T	Jan 26	
3	R	Jan 28	Projectile Motion
4	F	Jan 29	
5	M	Feb 01	Kinetic Friction
6	T	Feb 02	
7	R	Feb 04	Ballistic Pendulum
8	F	Feb 05	
9	M	Feb 08	Rotational Motion
10	T	Feb 09	
11	R	Feb 11	Archimedes' Principle
12	F	Feb 12	
13	M	Feb 15	Gas Behavior
14	T	Feb 16	
15	R	Feb 18	NO LAB

You should be enrolled in a Physics Lab section along with your lecture. It is your responsibility to attend your scheduled lab section. Switching lab sections may be impossible; seek accommodation from your lab instructor in advance of the switch with a plan on how to complete the lab. Generally the equipment for each experiment will be available only during the scheduled times.

Laboratory Apparatus

A list of apparatus is given with each experiment. Much of the apparatus is delicate and must be handled carefully. Please try to observe safe practice with the lab equipment.

In particular,

- Do not attempt to adjust or work with the apparatus until you have been instructed in its use.
- Check each item of equipment you receive, making sure that you have all the items required and that all are in working order.
- Report any problems to your instructor at once.
- Do not, under any circumstances, make electrical connections to batteries or electrical outlets until your circuit has been checked and approved by the laboratory instructor.

Disregarding these simple precautions is not only dangerous, it can cost you time. Moreover, electrical instruments, in particular, are generally very sensitive. Many can be damaged in a fraction of a second if they are connected incorrectly.

Laboratory Procedure

Since the laboratory is required, attendance will be taken. The time that you will have to perform each experiment is limited, so make the best use of time by reading the manual, answering pre-lab questions, and preparing your lab notebook before the beginning of each lab period. Pre-lab questions will be posted on Canvas and must be completed by the night before your regularly scheduled lab period.

Laboratory Reports

The reports should be submitted at the end of each lab period. If circumstances are extreme please see your lab instructor for possible extensions. It is likely that your score will be penalized under such circumstances.

Warning: failure to complete all labs may result in substantial grade reduction or failure of the course.

A lab report should be written in such a way that someone who *did not* do the experiment can read it, together with the instructions in the lab manual, and understand exactly what happened. It certainly does not need to be long or wordy to be good. Ordinarily, it should include the following items.

NAMES. The title of the experiment, your name, and your partner's name.

DATE. The date the experiment was performed and report was handed in.

PURPOSE. A brief statement (in your own words) of the objective or purpose of the experiment.

THEORY. A brief explanation defining which variable stands for which quantity in the formulas you use. You do not need to re-type the formulas from the manual but must at a minimum refer to the equation number and explain them as noted.

METHODS AND MATERIALS. You do not need to outline each step but should at least give a sentence to set up the basic procedure. An example might be "We used (equipment X) to measure (item Y) as in Chapter Z". You then only need to also report modifications to the lab manual procedure, or measurements that required judgment (for example, how you estimated uncertainty δx for a length measurement).

DATA. All the important numbers you encounter, including *units* and *uncertainties*, plus any other relevant observations should be recorded as the experiment progresses. Usually your data will be entered into Excel, this spreadsheet can then be copy/pasted into your report. Be sure that columns have names and units.

ANALYSIS/CALCULATIONS. Analysis and calculations will generally be performed in spreadsheets (Excel) or plotting/fitting program (*WAPP*$^+$), these can be copied and pasted into your report. Be sure to "self-document" spreadsheets to illustrate how calculations were performed. All graphs should have titles and axis labels (with units). From the *WAPP*$^+$ fit report, be sure to copy the chi-square value along with the fit model and fit parameters.

RESULTS/CONCLUSIONS. One objective of laboratory work is to clarify theory and indicate when and how well it applies to practical situations. Give comparisons between your results and the theoretical or accepted values. Does your experimental range of uncertainty overlap the accepted value? You are usually guided to discuss Results in lab manual questions. Your Results/Conclusions should end with a final Conclusion statement. Based on the Results, what does the experiment tell you? The can be as simple as yes or no, the experiment supported the theory to be studied. Be sure to connect the Conclusions to the Purpose you stated at the start of the lab.

QUICK REPORT. At the end of lab in a non-COVID semester, each group turns in a 3×5 "quick report" card. This year, you will instead record this information in a section of your electronic lab report. You will be told the information that belongs in this report, but generally it includes one or more of your numerical results from your final results (properly recorded: significant figures, units, uncertainty). Show this quick report to your instructor before you leave lab.

Grading

Pre-lab questions on Canvas will be graded on a five point scale. Reports will be graded by your TA on a 100-point scale. Your TA may provide some helpful criticism while grading, so please go over your labs carefully after they are graded. The TAs are expected to grade fairly and impartially, using a grading system approximately like the following. Please talk to your lab instructor if any problems arise.

A checklist appears at the end of each experiment in this manual which you may choose to use as a guide so you will not forget some important part of the experiment.

	Quality			
Component	**Inadequate**	**Minimal**	**Adequate**	**Superior**
Purpose 0 – 5 pts	Purpose omitted, or not related to lab 0 pts	Purpose stated but not clearly related to measurements 1 pt	States a purpose clearly related to some, but not all of the important measurements 2 – 3 pts	Correctly states a purpose related to all of the important measurements 4 – 5 pts
Theory 0 – 10 pts	Theory omitted or unrelated to lab 0 – 1 pt	Unclear 2 – 4 pts	Complete, but with a few mistakes 5 – 7 pts	Clearly stated with all relevant variable listed 8 – 10 pts
Methods 0 – 10 pts	Omitted, or incoherent 0 – 1 pt	Unclear, steps out of order 2 – 4 pts	Complete, but not well stated 5 – 7 pts	Clear, concise and complete 8 – 10 pts
Data 0 – 15 pts*	Data missing or incomplete 0 – 2 pt*	Data listed but not labeled correctly or without units 3 – 6 pts*	Data listed with some units and labels 7 – 11 pts*	Data well organized with clear labels and all units 12 – 15 pts*
Analysis/ Calculations 0 – 35 pts*	Calculations and graphs missing or not relevant 0 – 4 pts*	Calculations and graphs incomplete 5 – 13 pts*	Calculations and graphs nearly complete and correct 14 – 25 pts*	Neat and complete with clear example calculations 26 – 35 pts*
Results/ Conclusion 0 – 25 pts	Missing or does not relate to purpose 0 – 2 pts	Conclusion not supported by data or analysis 3 – 9 pts	Written conclusions with supporting evidence but incomplete 10 – 16 pts	Written conclusion, logical arguments, included uncertainties 17 – 25 pts

*Depending on the experiment, the breakdown of points between Data and Analysis/Calculations may vary.

Pre-Lab Questions 0 – 5 pts	Completed on Canvas by the night before lab.


YOU MEAN YOU CAN UNDER-STAND HIM NOW? GET HIM TO DO IT TO ME, TOO!
WILL YOU TREAT JOHNNY TOO, SO HE CAN UNDERSTAND YOU?
NO! IT WOULD BE DANGEROUS! IT CANNOT BE USED SAFELY ON ONE AS YOUNG AS HE!

1. UNCERTAINTIES

Purpose

No measurement produces an exact result; every meter has limited precision. This lab will introduce how physicists estimate and report the accuracy of measurements. Random error, systematic error, standard deviation, and standard deviation of the mean are defined through examples; these terms will be freely used in the remaining labs. Additionally the required format for spreadsheet hardcopy ('self-documented', 'final results format') is explained in text and video.

Introduction

This first-week lab is atypical: you need not complete the lab in the scheduled location at the scheduled time. You will not write up your results in your lab notebook. Rather the lab may be completed by yourself online at any location with internet access. (Generally students choose to work at home at a time convenient to themselves, but do feel free to come to the scheduled room/time—your instructor's office will then be nearby and help will be available.) View the one-hour, 800 MB video:

`http://youtu.be/PxWS_HnsPh4` or
`http://www.physics.csbsju.edu/lab/105Lab1.mp4` (for older browsers)
`http://www.physics.csbsju.edu/lab/105Lab1.webm` (for Chrome, Firefox, Edge...)

Given the large size of the video, you are encouraged to view or download it on-campus (but bring your own earbuds if you work in the lab room).

Read this chapter and then complete the on-line lab report:

`http://www.physics.csbsju.edu/lab/105Lab1.html`

Turn in a hardcopy of the spreadsheet exercise at the beginning of your next lab period.

Theory: Measurement Errors

How much does a cat weigh?

When Dr. Kirkman puts Tiger on a scale, the reading bounces around randomly: Tiger is not one to sit still. The average scale reading is more closely related to Tiger's weight than any particular reading, and indeed you will learn this semester that the net impulse on an object determines the change in velocity of that object's center of mass (ΔV_{cm}). Since Tiger's center of mass is not moving (much) we are assured that the time averaged scale reading is close to his weight. In particular, averaging over a time interval T, we exactly have:

$$\text{average scale reading} = \text{weight} + \frac{M\Delta V_{cm}}{T} \tag{1.1}$$

where M is Tiger's mass.

So the longer we average those scale readings, the more accurately we'll know his weight, since the unknown (and likely small) quantity $M\Delta V_{cm}$ will be divided by an ever larger number, T. Of course if the value of $M\Delta V_{cm}$ was known, we could include it in the equation and gain accuracy, but aside from guessing that it's "small," I know nothing about it, not even its sign. The best we can do is exclude times (like when Tiger jumps on or off) when his center of mass is in noticeable motion.

Randomly fluctuating measurements like this are known as random uncertainties or random errors. In the context of this document, "uncertainty" and "error" mean exactly the same thing. Use the word "blunder" to describe what is sometimes called "human error": incorrect or accidental human actions that result in inaccurate measurements; for example not properly zeroing ("taring") the scale before putting Tiger on it.

At some point, nothing is gained by extending the averaging time because:

- The balance manufacturer only claimed a precision of ± 0.02 lbs, so there's little reason to reduce the random variation much below that level.
- If not level, the scale will report only a component of weight. If off vertical by $3°$, there will be an error of about -0.02 lbs. How carefully did we level the scale?
- A living cat does not have a weight. The mass enclosed by his skin changes as fluids (air, water, urine) and solids (food, feces) are exchanged. Because of this it's hard to imagine any *practical* significance to weight difference of order ± 0.1 lbs.

The first two items on this list could be called "systematic errors" or "calibration errors" or "biases". They result in incorrect measurements but do not signal their presence by a fluctuating answer. They can only be detected by testing with a standard, known mass. Since systematic errors are consistent, if we knew the value we could correct the reading, but as with random errors, at best we have some estimate of the magnitude of the error, but no idea of its precise value or sign.

The last item on this list is a "problem of definition:" the definition of the weight of a living being is ambiguous.

SO we can measure Tiger's weight and report guess-estimates for the random and systematic error, but the original questions was: How much does a cat weigh? A cat is a randomly selected cat, and of course it does not have a weight: different cats weigh different amounts. If we know the weight of the set of cats from which a cat was selected (or more likely: a collection of cats we judge to be similar), it is probable that a cat would be similar to the majority of cats. We need to report some typical weight and a range of variation that includes "most" cats. Notation: assume we have a set of N cat weights: $\{w_1, w_2, w_3, \dots w_N\}$ (or equivalently: $\{w_i\}$ for i from 1 to N).

The two most common measures of "typical" are average, a.k.a., mean (add up all the cat weights and divide by the number of cats):

$$\text{average of the weights} = \overline{w} = \frac{1}{N} \sum_{i=1}^{N} w_i \tag{1.2}$$

and median (the middle weight: half the cats weigh more than the median, half weigh less).

There are several common measures of "range of variation". The most common is the standard deviation: subtract the average from each weight producing a list of deviations-from-average, some positive some negative; add up the square of these deviations-from-average, divide by $N-1$, and take the square root of the result:

$$\text{standard deviation of the weights} = \sigma_w = \sqrt{\frac{\displaystyle\sum_{i=1}^{N}(w_i - \overline{w})^2}{N-1}} \tag{1.3}$$

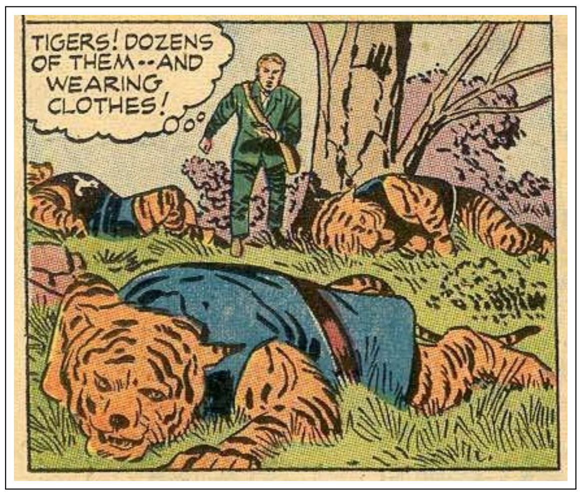
Remark: Calculators and spreadsheets have this function built in, so you should never have to use this formula to calculate standard deviation by hand. We present it to you so you have a general idea of what the standard deviation is: the square root of the average of the squared deviations-from-average. Re-read the previous sentence and make sure you understand what it is saying. The standard deviation is a measure of typical deviation-from-average.

If the distribution of cat weights is "normal" you should expect that about $\frac{2}{3}$ of the cat weights are in the interval: $(\overline{w} - \sigma_w, \overline{w} + \sigma_w)$. When plotting data, this likely interval is often marked with an "error bar" that displays the extremes of this interval as whiskers up and down from the average value. Unfortunately there is no consistent meaning assigned to an error bar, and at some point in your career you may encounter one of these alternative definitions for an error bar: 'equality' error bars that enclose half of cats in the range $(\overline{w} - 0.674\sigma_w, \overline{w} + 0.674\sigma_w)$; '95% confidence limits' enclosing 95% of cats in the range $(\overline{w} - 2\sigma_w, \overline{w} + 2\sigma_w)$; 'interquartile range' (the spread between the 25^{th} percentile and the 75^{th} percentile); or other error bar definitions. In this document, we will use the $\pm 1\sigma$ meaning, which means it's not hugely unlikely (i.e., $\frac{1}{3}$ of the time) for a datum to lie outside the error bar. You should recognize an unusual occurrence (5%) when a datum misses by 2 error bars.

Boxplots (a.k.a., box-and-whisker plots) are a standard, if less common, way to display the full range of variation which is encoded in the five-number summary: minimum, first quartile (25^{th} percentile), median, third quartile (75^{th} percentile), and maximum. A rectangle runs from first quartile to third quartile with a horizontal line indicating the median; 'whiskers' extend down to minimum and up to maximum. Unusual data points called outliers may also be specially marked.

How much does an average cat weigh?

If we actually weigh every cat in the world we could definitively answer this question, but usually we have weighed some subset of the world cat population. In the case of Tiger's weight, Newton's Laws prove that averaging over more weights improves the estimate of Tiger's weight. It seems evident that averaging over larger collections of cats would improve our estimate of the average cat's weight.



Indeed one can show that the likely inaccuracy of the average decreases with the inverse square root of the number of cats we weigh (and average). This **standard deviation of the mean** (SDOM) is given by:

$$\text{SDOM} = \sigma_{\overline{w}} = \frac{\sigma_w}{\sqrt{N}} \tag{1.4}$$

This formula is too good to be true: it claims we can have arbitrarily small uncertainties for the average value simply by collecting and averaging more data. However, this is only true when we are averaging out the fluctuations due to *random* errors (not systematic errors). Every time you use this formula you should ask yourself:

- Are you in fact seeking the uncertainty of the average weight of a group of cats? If you want to report the likely variation in the weight of cats in that group or the uncertainty in the weight of a randomly selected cat from that group, the standard deviation is what you should report.

- Since systematic errors are not reduced by averaging, is this reduction of the random errors useful, given that total error is what really matters?

It is important to recognize that the uncertainty in the weight of a (randomly selected) cat is much larger than the uncertainty in the weight of the average cat.

Average cat?

In 1947 the Cambridge statistician R.A.Fisher published weight data on 144 cats[1]. I've collected recent data on 25 Minnesota in-home cats. Which defines the average cat? Figure 1.1 displays the two distributions as a boxplot, as the mean with standard deviation error bars (STDEV), and as the mean with standard deviation of the mean error bars (SDOM). The boxplot shows that these samples overlap in part; STDEV shows no overlap between 1σ errors bars, but clearly 2σ errors bars (typically 95% inclusive) would again show a small overlap. SDOM shows distinctly different means. Clearly there is some systematic difference between well-fed MN home cats and cats used in post-war UK pharmaceutical research. Most Minnesotans would find it incredible that a $6\frac{1}{2}$ lb cat was "significantly above average". (More evidence that in Minnesota every cat is "above average".) Herein lies the danger (and benefit) of SDOM: large datasets can reduce 'errors' to practically meaningless levels (in the UK data, less than 0.1 lb). If the average is exactly the quantity we seek, we can measure it with extraordinary accuracy. If the average is not exactly equal to the quantity of interest, or if the sample we use to define average is not representative, we risk humiliating reversals from future work.

In summary: a measurement has no value if it lacks an estimate of its accuracy. In these labs you must always report both the measurement and its uncertainty (often the word "error" is used instead) that may be a mix of reading variation, hard-to-estimate, pernicious systematic errors and problem of definition errors. The deviation of a hypothetical future measurement can be estimated by the standard deviation of a set of previous measurements. The uncertainty in an average value can be estimated by SDOM of a set of previous measurements (if certain conditions are met).

Absolute and Percent Errors

If Tiger's weight is 14.9 ± 0.1 pounds, the '0.1 pounds' is called the absolute error. The absolute error is a direct estimate of the possible deviation between the measurement and the actual value, i.e., in this case that Tiger's weight is probably in the interval (14.8, 15.0) pounds. The absolute error always carries the same units/dimensions as the base measurement.

[1] The data was actually produced as a part of a 1946 study on the lethal dose of digitalis in cats.

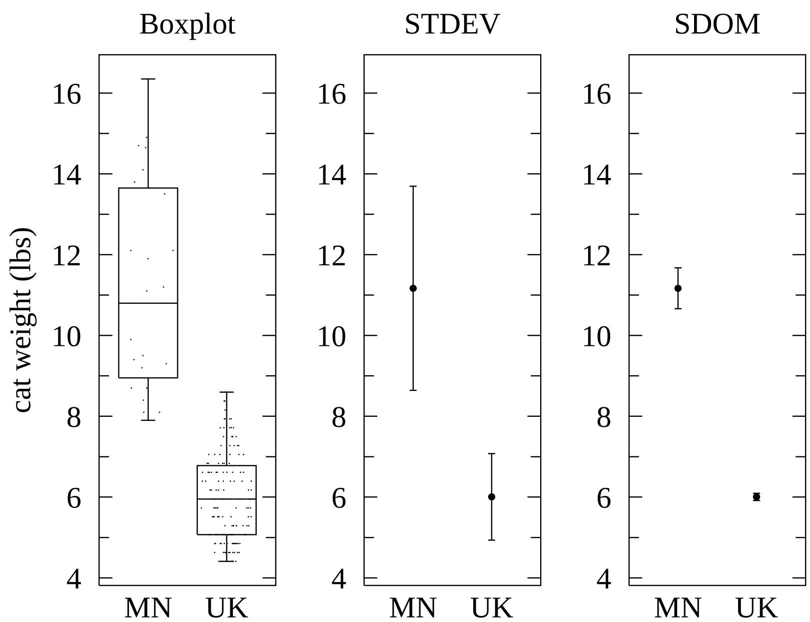


Figure 1.1: In search of the "average cat," we present two datasets on cat weight. UK reports the weight of 144 cats used in a pharmaceutical study reported by R.A.Fisher in 1947; MN reports the weight of 25 MN in-home cats collected by the author. The boxplot's rectangle shows the central 50% of cats with the median marked with a bisecting horizontal line and whiskers out to the group's maximum and minimum. STDEV displays the mean with 1σ error bars. SDOM displays the mean with error bars extending to the standard deviation of the mean. Clearly while there is some overlap between the distributions, the UK and MN means differ significantly. The difference between the datasets demonstrates that additional study is required to uncover the 'average cat'; it would be foolhardy to accept either mean$\pm$SDOM as the measure of the average cat. While SDOM provides an interesting measure of a group mean, that group's mean may not be exactly the quantity we seek.

Often it is useful to express the possible deviation as a percentage of the measurement. In this case, since:

$$\left(\frac{0.1}{14.9}\right) \times 100 = 0.6711 \approx 0.7 \tag{1.5}$$

we may report Tiger's weight as 14.9 pounds $\pm 0.7\%$. The percent error never carries units and (aside from the factor of 100) is the same thing as 'relative error' or 'fractional error'.

Judging Uncertainty

A significant fraction of your lab grade this semester will be based on your ability to judge the uncertainty of your measurements. The accuracy of most electronic instruments is recorded in the manufacturer's specifications; typically these are estimates of possible systematic error. (We will supply you with those estimates in lab.) Accuracies may be reported as 'absolute errors': the direct estimate of the likely standard deviation, or as 'percent errors': where the likely standard deviation can be calculated by taking the given percentage of the displayed value, or both.

For example, the manufacturer reports that the accuracy of the voltmeter that many of you will be using next semester is $\pm(0.1\% + 4 \text{ digits})$, where one 'digit' is the place value of the rightmost digit on the display. So if the display showed 6.238, the uncertainty would be $6.238 \times 0.1\% + .004 \approx 0.010$; if the display showed 71.49 the uncertainty would be $71.49 \times 0.1\% + .04 \approx .11$ instead.

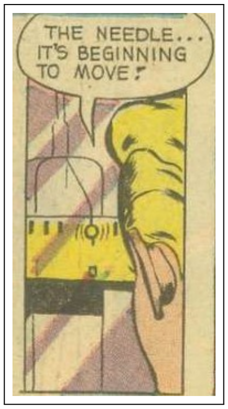
This semester many of your measurements will be made using rulers. Generally when using a ruler (or any device whose scale you read by eye) you can estimate the fractional bit between the marked gradations. Error estimates that are a third or a quarter (at most a half) of those marked divisions are then appropriate.

An important special case applies to counting events that have no preference over time or space. For example, the count of weeds growing in a uniform farm field, or the decay of uranium atoms which are just as likely to decay this year as in the year 4,000,000,000 B.C. Whole number counts of such events (weeds in an acre or decays during a year) will be randomly different for different acres or years, but there is a simple estimate for the standard deviation of such measurements based on just one measurement: the square root of the number of events. For example, if we measure 121 weeds in one acre, we estimate that $\frac{2}{3}$ of the acres will have between 121–11=110 and 121+11=132 weeds on them. (So $\frac{1}{3}$ of the acres will have fewer than 110 or more than 132 weeds.) This special case is known as Poisson statistics.

In this semester's labs we will not be expecting precise error estimates. Because of the limited lab time you will often be called on to judge the typical range-of-deviation by eye rather than using the standard deviation formula. You should develop the ability to judge such deviations (accurate to maybe a factor of two) just by eye. The main point this semester is that every measure has uncertainty, and this needs to be factored into all your further calculations with the data.

Types of Measuring Instruments

We can enumerate two broad categories of measuring instruments: analog and digital.



The ruler and anything else that has a continuous scale where the operator can "read between the lines" is referred to as analog. Triple beam balances and meters with pointers are examples of analog instruments.

The operator of an analog instrument must judge both the value and appropriate uncertainty. Quite often the uncertainty is estimated at plus or minus a fraction of the smallest scale division, but it always involves a judgment call by the operator.

Digital instruments are quite different. They are incremental rather than continuous and one can not "read between the lines." In typical use they provide a steady reading with no obvious random fluctuation. The manufacturer usually provides the uncertainty specifications (typically systematic errors) of digital instruments. For example, a digital balance may read 10.3 grams where the last digit, according to the balance manufacturer, has an uncertainty of two (0.2 gram in this case). For other models it might be something different, such as 0.1 gram. Usually the uncertainty involves the last digit, but sometimes it is expressed as a percentage of the total value or a combination of percentage plus the last digit.

Comparing Measurements with Uncertainty

It is unlikely that two measurements of the same quantity will exactly agree. However usually (i.e., approximately $\frac{2}{3}$ of the time) the intervals spanned by their error bars will overlap. If they miss by more than 2 error bars, most folks would call it a significant disagreement. Occasionally measurements can be compared to precise predictions of theory (for example, the ratio of a circle's measured circumference to diameter should be precisely π), in which case usually the theoretical prediction should lie within the error bars of a measurement.

Clearly one can wash over disagreements simply by enlarging the error estimates. However, the value of a measurement depends strongly on the error; technically the 'weight' of a measurement is proportional to the inverse square of the error. So a number with $2\times$ bigger error bars has $\frac{1}{4}$ the weight. Thus the worth of a measurement is judged by its error; measurements with small systematic and random errors are considered the best measurements.

If results show significant disagreement, then either (A) we're in a statistically unlikely (but not impossible) case or (B) mistakes have been made or (C) something interesting

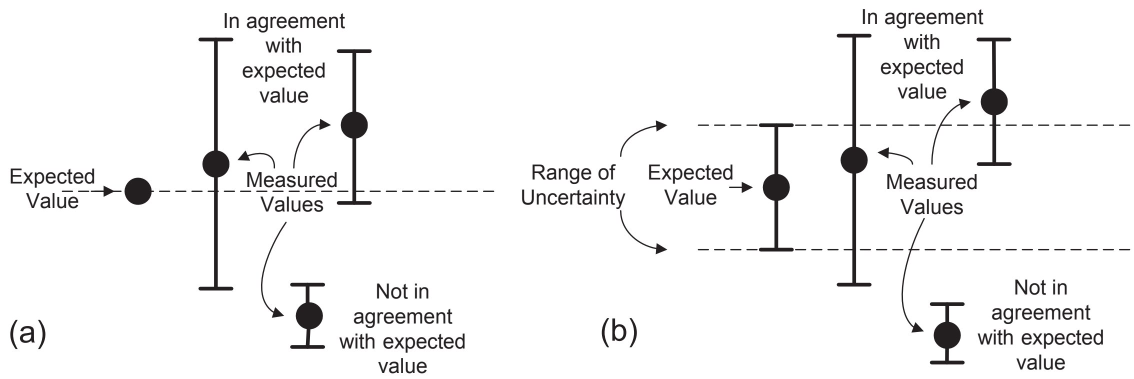


Figure 1.2: Examples of measured values with uncertainty, compared to an expected value (a) with negligible uncertainty, and (b) with the indicated range of uncertainty.

is going on which makes two correctly measured quantities disagree.

While we have not made a valid study of the frequency of these possibilities, my sense is that (B) is by far the most common situation, and that usually the mistake is undetected systematic errors in the measurement.

Reporting Measurements

You should record (in your lab book or Excel) every digit displayed by an instrument and every digit you estimate when reading a ruler. Do not round your raw data! Of course you must also report an uncertainty for those measurements. When it comes to reporting *final results* (in contrast to raw data or intermediate calculations) we have strict requirements as to exactly how you record those numerical results. Our uncertainty in the uncertainty itself is always large; hence round any uncertainties to one or two significant figures.

One significant figure[2] is always easier and almost always as accurate as two sigfigs. Recall: You may have estimated your range-of-deviation just by eye. Manufacturer's specification of errors almost always report just one sigfig (recall: $\pm(0.1\% + 4 \text{ digits})$) so one sigfig would be appropriate for an error calculated from those numbers. One can show that in 'normal' cases, formula-calculated standard deviations will have uncertainty greater than 10% unless the number of available data points, N, exceeds 50.

Once you have properly rounded your errors, display all the digits of your measurement until the place-value of the least (rightmost) significant digit in the measurement matches

[2]If you are unfamiliar with 'sigfigs', see page 70

the place-value of the least significant digit in the error. Some examples:

$3.14159 \pm .00354 \Longrightarrow 3.142 \pm .004$ or $3.1416 \pm .0035$
$2.71828 \pm .05472 \Longrightarrow 2.72 \pm .05$ or $2.718 \pm .055$
$321456 \pm 345 \Longrightarrow 321500 \pm 300$ or 321460 ± 350
$.1678 \pm 3.51 \Longrightarrow 0 \pm 4$ or 0.2 ± 3.5

Finally, remember to record the units of your measurement!

Spreadsheet Descriptive Statistics

In many cases uncertainties will be given by manufacturers specifications (typically systematic errors) or by-eye estimates of deviations-from-average, e.g., Tiger's weight. For repeated measurements, spreadsheets allow easy calculation of average, standard deviation and standard deviation of the mean.

To find the average, select an unused cell and enter the formula: `=AVERAGE(A1:A36)` where the list of cells you want averaged (here `A1:A36`—the first 36 cells in the first column which is called `A`) is enclosed by parenthesis. Typically you will get the list of cells by sweeping through them with left mouse button depressed or by clicking on the first cell and then the last cell in the list while also holding down the Shift key. After hitting Enter, your formula will be replaced by the appropriate value.

To find the standard deviation, use the formula: `=STDEV(A1:A36)`.

The standard deviation represents the uncertainty for any single measurement, e.g., a cat's weight.

If the mean is the value whose error you want to estimate, you need the SDOM (standard deviation of the mean). To find this, use the formula:
`=STDEV(A1:A36)/SQRT(COUNT(A1:A36))`, i.e.,

$$\text{SDOM} = \sigma_{\overline{x}} = \frac{\sigma_x}{\sqrt{N}} \tag{1.6}$$

where N=`COUNT(A1:A36)`. In this case we know there are 36 data points, so we might simplify by typing `=STDEV(A1:A36)/SQRT(36)` or even `=STDEV(A1:A36)/6`

The standard deviation of the mean represents the uncertainty in the mean, e.g., the average cat's weight.

Spreadsheet Self-Documentation

If you print out your spreadsheet, the formula used to calculate a cell will not be displayed; the grader will have no idea how the printed number was calculated. So in an adjacent cell you must display the formula used. This is easily done by copying and pasting the formula into an adjacent cell (which often results in an error or nonsense) and then editing out the '=' and hitting Enter. Excel will now display (and print if requested) the text of the formula used. Of course, you will also provide words ('headings') naming column and cell values (including units). Note: in documents and your lab notebook results (measurement/error/unit) are recorded in the form: '14.9±0.1 lbs'; do not do this in your spreadsheet! Text like '±' and 'lbs' make the numbers sharing that cell unavailable for further calculation. Instead put each number in its own cell and in an adjacent cell report the name of the quantity and units (the units are typically enclosed in parenthesis):

Tiger's weight (lbs)	14.9
Uncertainty in Tiger's weight (lbs)	0.1

Theory: Calculating with Uncertain Numbers

Measurements are usually followed by calculations which make use of the measurements. Perhaps you are calculating the area of a rectangle. Imagine that your distance measurement of 8.8 ± 0.3 m is to be multiplied by another uncertain distance measurement: 2.1 ± 0.2 m. One way of determining the uncertainty of the calculated result is to use a "high-low" approximation.

High-Low Method

The high-low method involves calculating the result three times: once without uncertainty—use your best values—and then two additional times: finding the highest possible result and the lowest possible result, as illustrated below and in Fig. 1.3. The answers to the three calculations are as follows:

$$\begin{array}{ll} \text{Best:} & 8.8\,\text{m} \times 2.1\,\text{m} = 18.48\,\text{m}^2 \\ \text{High:} & 9.1\,\text{m} \times 2.3\,\text{m} = 20.93\,\text{m}^2 \\ \text{Low:} & 8.5\,\text{m} \times 1.9\,\text{m} = 16.15\,\text{m}^2 \end{array}$$

The range of results could now be expressed as (18.48 + 2.45) m^2 to (18.48-2.33) m^2. This unsymmetric form is awkward, so the difference between the two extremes is split evenly $((20.93 - 16.15)/2)\,\text{m}^2 = 2.39\,\text{m}^2$, and the result expressed as $18.48 \pm 2.39\,\text{m}^2$.

Following our final value reporting rules, this is:

$$18 \pm 2 \text{ m}^2 \quad \text{or} \quad 18.5 \pm 2.4 \text{ m}^2$$

Again, there is no point in expressing uncertainties with more digits (for instance, 18.48 ± 2.39 m^2 or 18.48 ± 2 m^2): at least in these labs our error estimates never deserve 3-digit accuracy and there is no point in reported digits in the base number that are overwhelmed by the likely errors.

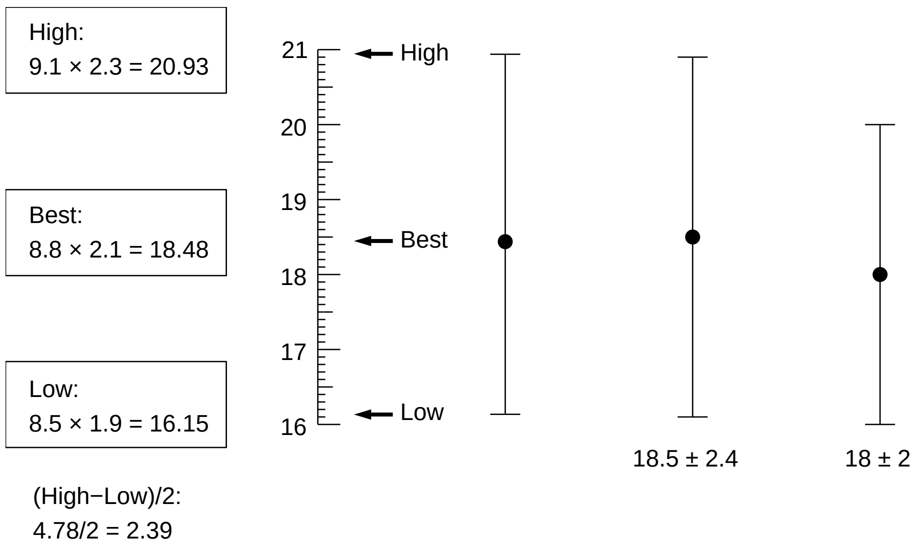


Figure 1.3: Example of calculating uncertainties by the high-low method.

It does not make any difference how complicated the calculations are when you use the high-low method. Always apply the uncertainties in such a manner as to maximize, and then minimize the result. In the case of division, maximize the numerator and minimize the denominator in order to maximize the result. Do the opposite to minimize the result.

One disadvantage to the high-low method is that several calculations are needed (the high, the low, their average and half their difference) to end up with a best estimate and its uncertainty. As a result it will be unclear which errors contribute most significantly to the final error; i.e., which measurements should be improved first.

Fractional Uncertainty Method

There is a simpler approach we can sometimes use, known as the fractional uncertainty method, in which the uncertainty in a result is calculated directly from the measured values and their uncertainties. The fractional uncertainty formulas (see Appendix A) are often fast and easy calculations. For instance, in the case of multiplication or division, fractional uncertainties are additive. Going back to our two distance measurements,

$$\frac{\text{uncertainty in area}}{\text{area}} = \frac{\text{uncertainty in distance 1}}{\text{distance 1}} + \frac{\text{uncertainty in distance 2}}{\text{distance 2}},$$

or more briefly,

$$\frac{\delta A}{A} = \frac{\delta D_1}{D_1} + \frac{\delta D_2}{D_2},$$

where A represents the area, D_1 and D_2 are the measured distances, and their uncertainties are indicated using the Greek letter δ ("delta"). We already know how to calculate the area $A = D_1 \cdot D_2$, and the uncertainty in the area can be found with one more calculation:

$$\delta A = A\left(\frac{\delta D_1}{D_1} + \frac{\delta D_2}{D_2}\right).$$

Using our measurements,

$$\delta A = 18.48 \text{ m}^2 \times \left(\frac{0.3 \text{ m}}{8.8 \text{ m}} + \frac{0.2 \text{ m}}{2.1 \text{ m}}\right) = \pm 2.39 \text{ m}^2$$

In this case the fractional uncertainty method and the high-low method produced exactly the same result; more generally they will differ, but never significantly. One advantage of the fractional uncertainty method is we can determine which terms are contributing the most to the final error. In this case $0.3/8.8 = .034 < .2/2.1 = .095$, so the smaller absolute error (0.2) actually contributes more to final error.

In future labs, we'll tell you which method works easier/better for the particular experiment in question. In general, the high-low method will be used for complicated, multi-step calculations while the fractional uncertainty approach will be used with fairly simple calculations. The rules for using fractional uncertainties are in Appendix A.

For both the high-low method and the fractional uncertainty method the worst-case scenario is assumed, which may be unrealistic. If the actual deviations were random (or uncorrelated), as is often the case with measurement errors, one would expect some partial cancellation of the effects of uncertainty. Why, for example, would both distance measurements always be high or always be low? Why could not one be a bit high while the other is a bit low? If a calculation involves many steps, should not there be at least some cancellation of random errors? These questions suggest that a statistical approach to estimating uncertainty is needed.

Procedure

View the one hour, 800 MB, on-line video:

`http://youtu.be/PxWS_HnsPh4` or
`http://www.physics.csbsju.edu/lab/105Lab1.mp4` (older browsers)
`http://www.physics.csbsju.edu/lab/105Lab1.webm` (Chrome, Firefox, Edge…)

and then complete the on-line lab report:

`http://www.physics.csbsju.edu/lab/105Lab1.html`

Part I of the on-line lab report consists of multiple choice questions related to this chapter of the Lab Manual. Part II seeks definitions (in your own words) and examples of random and systematic error. This material is covered in the video and this chapter. Part III requires calculations in a spreadsheet. The required calculations and format are covered in the video. The numerical results of these calculations (with at least 4 sigfigs) are reported in the on-line form. A pdf of the completed spreadsheet ('self-documented' and in proper 'final results' format) must be submitted to Canvas before the start of the next lab period. Note: this is an individual project.

Critique of Lab

As a service to us and future students we would appreciate it if you would also include a short critique of the lab—simply jot down your comments in the Excel pdf. Please comment on such things as the clarity of the Lab Manual and video, relevance of experiment, and if there is anything you particularly liked or disliked about the lab.

Quick Report

Generally, as you leave lab, each group should turn in a 3×5 "quick report" card. Typically it includes one or two numerical results (properly recorded: significant figures, units, uncertainty) from your Conclusion. However, for this lab, there is no quick report card required.

Checklist

At the end of each chapter in this Lab Manual you should find a 'checklist': a brief listing of the required components for the lab. The checklist is necessarily terse, but it should provide reminders for critical elements of your lab report.

CHECKLIST

Part I: multiple choice questions from material in this chapter	
Part II: short answers, in your own words, not plagiarized from Google	
Part III: numerical results reported online with at least 4 sigfigs	
PDF of Excel spreadsheet: self documented, final results format	
Lab Critique jotted in Excel pdf	

2. PROJECTILE MOTION

Pre-Lab Videos

WAPP$^+$ video: `http://youtu.be/_PNC_Vn7llI` or
`http://www.physics.csbsju.edu/lab/COVID_WAPPonline.mp4` for older browsers
`http://www.physics.csbsju.edu/lab/COVID_WAPPonline.webm` for Chrome, Firefox, Edge. . .

process video: `http://youtu.be/jgTtuN-Ss0c` or
`http://www.physics.csbsju.edu/lab/COVID_process.mp4` for older browsers
`http://www.physics.csbsju.edu/lab/COVID_process.webm` for Chrome, Firefox, Edge. . .

Read `https://www.csbsju.edu/instructional-technology/technology-training/microsoft-onedrive` for information on your CSBSJU OneDrive used in this and future labs.

Purpose

In this experiment you will examine the ballistic motion of a freely-falling object and determine whether its path follows the expected two dimensional motion discussed in lecture (with constant horizontal velocity and constant vertical acceleration).

Apparatus

1. Projectile launcher and ball bearing
2. Paper and carbon paper
3. Board on vertical stand
4. Meter stick
5. Tape
6. a computer with spreadsheet (e.g., Excel) and web browser for accessing *WAPP*$^+$

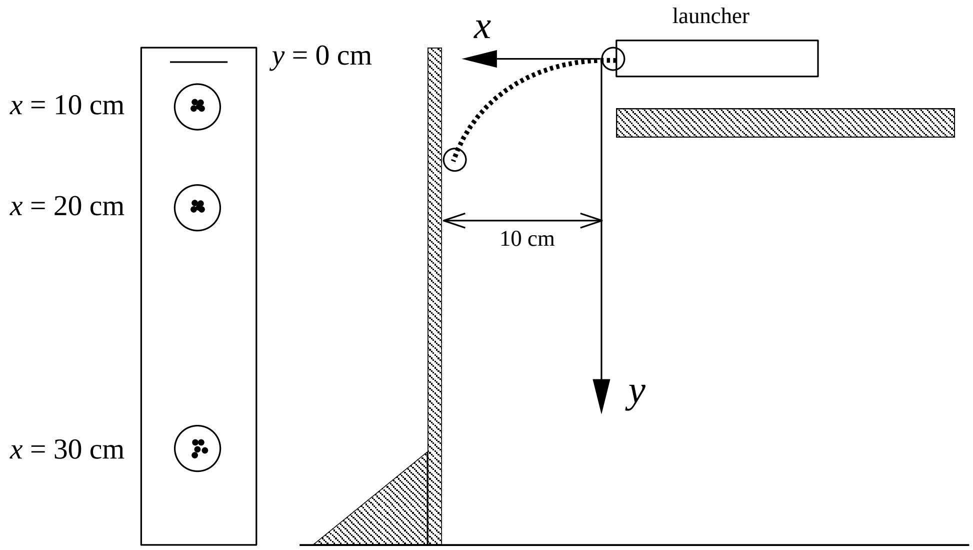


Figure 2.1: Projectile motion apparatus.

Introduction

Galileo was greatly concerned with motion of all kinds, particularly projectile motion. In his work Two New Sciences, he wrote:

On the Motion of Projectiles

PROPOSITION I. THEOREM I.

> When a projectile is carried in motion compounded from equable horizontal and from naturally accelerated downward [motions], it describes a semiparabolic line in its movement.

The apparatus for this experiment is simple but can yield reasonably good data if careful measurements are made. Essentially, you will be launching ball bearings as projectiles, and measuring their travel in the vertical and horizontal directions. A spring-loaded launcher will shoot the ball horizontally off a table. Carbon paper marks the ball's point of impact on a vertical board. The horizontal distance x it travels is controlled by moving the board further away, and the vertical distance y it travels will be measured. The relationship between these distances determines the shape of the ball's flight path.

Theory

If the projectile has an initial velocity v_{0x} in the x direction and is in free-fall in the y direction (starting from $v_{0y} = 0$), then the equations of motion for the projectile are

$$x = v_{0x}t \tag{2.1}$$
$$y = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}gt^2 \tag{2.2}$$

where t is the time since launch. Solving Eq. (2.2) for t we obtain $\pm\sqrt{\frac{2y}{g}}$; the negative solution is not relevant here, so the time of flight is

$$t = \sqrt{\frac{2y}{g}}. \tag{2.3}$$

Substituting t into Eq. (2.1) we can write the horizontal velocity

$$v_{0x} = \frac{x}{t} = x\sqrt{\frac{g}{2y}} \tag{2.4}$$

and the relationship of y to x can be re-written in the form

$$y = \frac{g}{2v_{0x}^2}x^2 \tag{2.5}$$

This is the equation of a parabola with its vertex at the origin. You will be testing whether the horizontal velocity does in fact remain constant.

Procedure

1. Examine the apparatus to find out how it works. The ball is to be launched horizontally, and hit the board at some point. A level is available for you to make the board vertical, and the launcher horizontal.

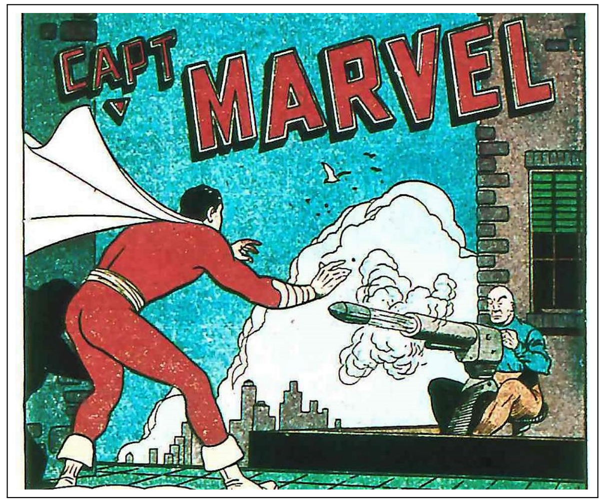
2. Load the ball into the launcher and push it in with a stick or pencil (you will not want your finger stuck in there) until it “clicks” one time.

3. Attach white paper to the board and then a layer of carbon paper so the carbon is facing the white paper and will leave a mark where the ball hits. Position the board at the end of the launcher and fire the ball to get a mark corresponding to $y = 0$ from which to measure other y values.

4. Move the board 10 cm away from the end of the launcher; this means your first horizontal distance is $x = 10$ cm.

   - Try to give a reasonable estimate of the uncertainty δx too.

   Now fire the ball three times, and it will leave three carbon marks on the paper. You can peek under the carbon to be sure there are three identifiable marks.



5. Continue these measurements for horizontal distances of approximately 20, 30, 40, . . . , 100 cm, then remove the carbon paper and place the marked paper on a table for measurement.

6. Make an Excel spreadsheet of your x and y data with their uncertainties. For each trial, estimate (by eye) and mark the average vertical (y) location of the ball hits and accurately measure the corresponding fall distance y (relative to the $y = 0$ position). Take half the distance between the outermost points as an estimate of δy.

x(cm)	δx(cm)	y(cm)	δy (cm)	v_{0x} (cm/s)
10				
20				
...				

7. Use Eq. (2.4) to calculate the v_{0x} values in each row.

   - As a spreadsheet function it will look something like: `=A2*SQRT(980/2/C2)` (the 980 assumes units of cm/s^2).
   - **Q1:** Do your values of v_{0x} show any systematic trend with x, or do they appear to be randomly varying?
   - The values of v_{0x} that result from smaller values of x and y are probably the most uncertain and may differ significantly from the others.
   - It is expected that most of the v_{0x} values will turn out fairly constant; if they don't, consult your lab instructor for help.

8. Use the spreadsheet to calculate the average value of v_{0x} using the formula `=AVERAGE(E2:E11)` (or whatever cells are appropriate). Then compute the standard deviation of the mean using `=STDEV(E2:E11)/SQRT(10)`. This would give you a reasonable estimate of v_{0x} and its uncertainty. Remember to self-document your spreadsheet!

9. Now analyze the experimental data using *WAPP*$^+$.

   (a) Your data probably has significant uncertainties in both x and y, so use this default setting for errors on the opening *WAPP*$^+$screen.

   (b) Copy and paste your columns of data into *WAPP*$^+$.

   (c) Identify which column is which: probably (X, X Errors, Y, Y Errors, Ignore).

   (d) Select a power-law fit: $y = Ax^B$ in accordance with Eq. (2.5).

   - The first thing to check is whether $y(x)$ is a parabola, which would have $B = 2$.
   - When you have fitted this function, check whether $B \pm \delta B$ is near 2, and whether $\overline{\chi}^2$ is reasonably good (not too far from 1).

   Record these results, or ask your instructor for help if things don't look right.

   (e) If the data appear to fit a parabola, then set $B = 2$ on *WAPP*$^+$and do a fixed power-law fit.

   - Again, examine the Fit Report results to be sure that $\overline{\chi}^2$ is reasonably good (not too far from 1).
   - Print the useful area of the Fit Report screen.
   - **Q2:** Does your fit support the scientific validity of Eq. (2.5)?

   (f) Make and print a nice graph of the $y(x)$ data with the fitted curve.

   (g) Now the value of A will be of interest (when B was held fixed) because, according to Eq. (2.5), A should correspond to $g/(2v_{0x}^2)$. Hence an estimate of v_{0x} can be obtained:

   $$v_{0x} = \sqrt{\frac{g}{2A}} \tag{2.6}$$

   And because A is uncertain, this calculated v_{0x} will have a corresponding uncertainty:

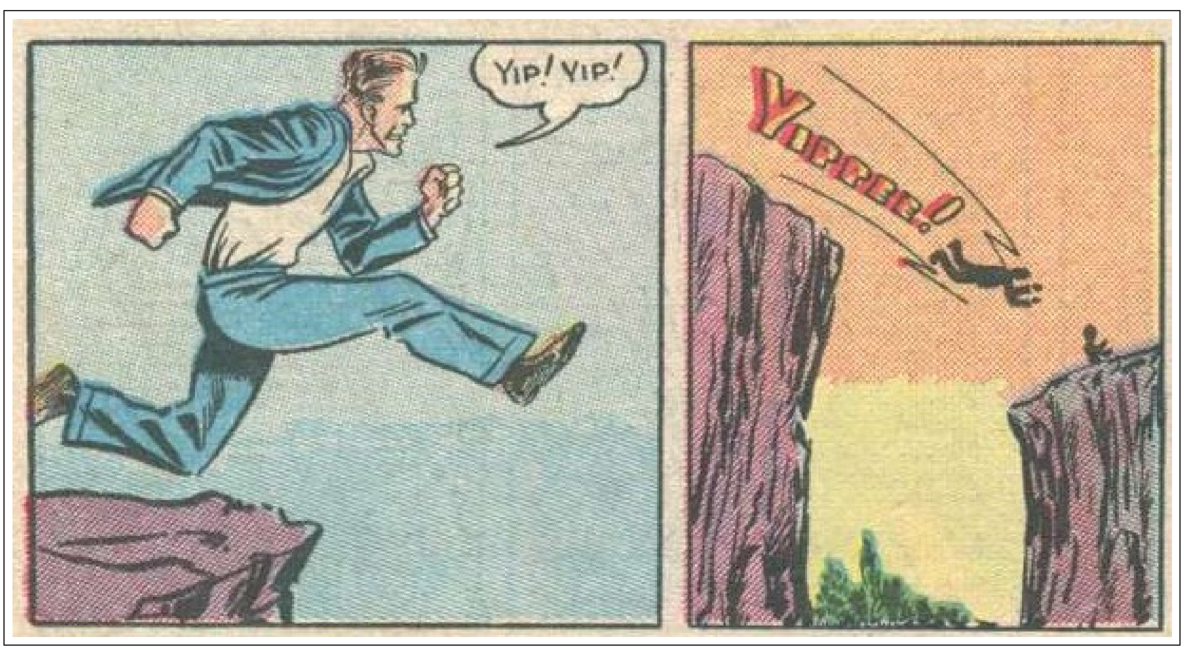
   $$\delta v_{0x} = v_{0x}\frac{1}{2}\frac{\delta A}{A}. \tag{2.7}$$

   So calculate your estimate of $v_{0x} \pm \delta v_{0x}$ based on your *WAPP*$^+$ results; record the result with proper units and sigfigs. (Self-document or show a sample calculation.) **Q3:** Does it agree fairly well with the value calculated from your table of data (in item 8 above)? It should be reasonably close, but because *WAPP*$^+$ has taken into account the uncertainties of your data points, this result should be a more accurate estimate.

10. Record your "Quick Report" with the following items: The *WAPP*$^+$$B$ parameter from your first fit, the *WAPP*$^+$$A$ parameter from your second fit, and v_{0x} calculated from the *WAPP*$^+$fit. Be sure to include proper significant figures, units and uncertainties on all values. Show your instructor.

CHECKLIST

Names, date, purpose	
Theory, with relevant quantities defined	
Brief Methods & Materials or Procedure outline	
Table of x, y data with uncertainties	
Excel calculations of v_{0x} (self-documented)	
Q1: Comments on constancy of v_{0x}	
WAPP$^+$ fit report of $y(x)$	
Comparison of B with expected value 2	
WAPP$^+$ fit report of $y(x)$ (fixed B)	
Q2: Is the $y(x)$ fit reasonable?	
Graph of $y(x)$ (fixed B)	
$v_{0x} \pm \delta v_{0x}$ calculated from $A \pm \delta A$ (proper sigfigs and units)	
Q3: Comparison of v_{0x} from Excel and *WAPP*$^+$	
Conclusions	
"Quick Report" check	



3. FRICTION

Purpose

This experiment allows you to study some of the basic factors that influence the amount of friction between two surfaces. You will also gain experience using a computer to not only analyze data, but collect it as well. In this case with the program *Logger Pro*, which we will continue to use in future labs.

Apparatus

1. a computer with spreadsheet (e.g., Excel)
2. Vernier interface box and *Logger Pro*
3. Smart Pulley (the pulley with wire attached), and string
4. Table clamp
5. Mass and hanger set, washers
6. Blocks with hooks and bolt, friction board
7. Digital balance, $\delta M = 0.02$ g

Introduction

The force of friction opposes the slipping of surfaces that are in contact, and the amount of friction depends on the nature of the surfaces. The force of friction can be determined by measuring the acceleration of a body when all the other forces are known. You will be doing this for a wooden block as it accelerates across the table top, pulled forward by a hanging mass, and with friction opposing the motion of the block. Figure 3.1 shows the experimental setup. The string connecting the hanging mass to the wooden block turns a pulley as the block moves. The computer keeps track of the pulley's rotations and quickly calculates the block's acceleration. You will then use the acceleration data to compute the coefficient of kinetic friction μ_k of the two sliding surfaces. To investigate whether μ_k depends on factors other than the two surfaces, you can change the sliding block's speed by varying its mass (M) and the falling mass (m), and the area of contact between the surfaces can be changed by turning the block on edge. It is also fairly easy

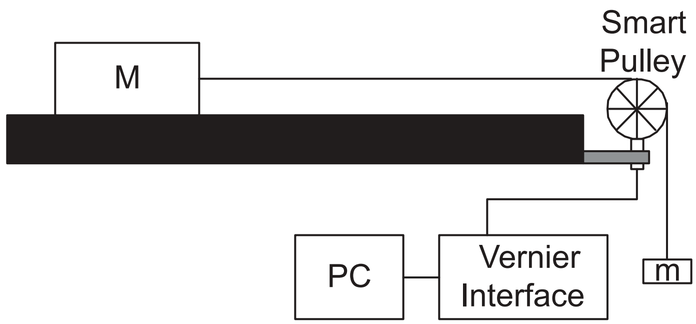


Figure 3.1: Kinetic Friction Apparatus

to measure a coefficient of static friction μ_s using the same apparatus.

Theory

a. Kinetic friction

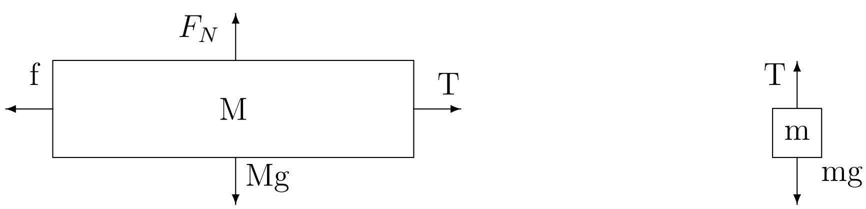


Figure 3.2: Free-body diagrams.

Figure 3.2 is a free-body diagram of the sliding block, showing the force T exerted by the string, the weight Mg of the block, the normal force F_N of the table acting on the block, and the frictional force $f = \mu_k F_N$.

The vertical forces must cancel, so $F_N = Mg$. Newton's second law gives

$$T - \mu_k Mg = Ma \tag{3.1}$$

The hanging mass m is also shown in the free-body diagram, with tension T upward and the weight mg downward. Since it accelerates downward,

$$mg - T = ma, \tag{3.2}$$

hence $T = m(g - a)$. Substituting for T in Eq. (3.1) we can solve for μ_k:

$$\mu_k = \frac{m}{M} - \frac{a}{g}\left(1 + \frac{m}{M}\right). \tag{3.3}$$

From this equation we can calculate μ_k for each measured a if m, M and g are known.

b. Static friction

What if the hanging mass m is not large enough to get the other mass M to slide? In that case, the friction is said to be static, the acceleration $a = 0$ in Eqs. (3.1)–(3.2), and Eq. (3.3) would give a simple expression for the static friction coefficient: $\mu_s = m/M$. Note, however, despite the simplicity of the formula, that this is not a constant — it has some largest experimental value which you could find by increasing m or decreasing M until motion begins. We should expect that the largest $\mu_s > \mu_k$; it (usually) takes more force to start an object sliding than to keep it sliding.

Procedure

1. Set up the experiment as shown in Fig. 3.1, checking to see that the string pulls horizontally on the block; raise or lower the pulley as needed. The LabQuest interface will have a smart pulley plugged into Dig 1 (Digital Input 1).

2. The smart pulley signals the computer every time one of the spokes interrupts a light beam; if it is properly connected you should see its red light blink when this happens. The computer measures time intervals between pulley signals to calculate distance, speed, and acceleration of the string that turns the pulley.

3. Start the *Logger Pro* program by clicking on the desktop icon.[3] Under the file menu, open **kin_fr**. For certain digital sensors (like the one used in this experiment) a confirmation window will open that asks you to connect the sensor[4]. Click the connect button. A toolbar along with a data table and an acceleration vs. time graph will appear. Among the toolbar icons, you may find the Autoscale and Zoom icons particularly useful.

4. Now try it. Place enough mass on the hanger such that the block will slide on the table without needing any initial push.

[3] If you see the message "No Device Connected" in the upper left corner of the screen, the interface must be unplugged at the computer end of the cable for a few seconds and then re-connected. You should then see this message disappear.

[4] If this message fails to appear, follow the suggestion above: un-plug and re-plug the USB cable and exit and re-enter Logger Pro.

5. Pull the block (M) back until the hanging mass (m) is raised to the pulley. Damp any pendulum motion in m. Hold the block at rest until you have clicked on the green Collect button, then release the block (the space bar also works to start collecting). Hit Stop when the block stops. Data runs can be stored (**Store Latest Run** under the Experiment menu) or discarded (**Clear Latest Run**). Storing is helpful when you would like to review runs later, but can be messy since all runs appear on the same graph. For this experiment, you don't need to save the runs.

6. Since there are often oddities at the beginning or end of a run, you can select an **acceptable portion** of the data by holding down the left mouse button while dragging the cursor to enclose the desired portion of the graph.

   Clicking on **Statistics** under the Analyze menu will now analyze the selected portion of the data. A box will appear containing the mean acceleration for the run (among other things). Record the selected mean each time you make a run. To reduce clutter, you can delete the statistics box after recording the mean acceleration.

   When you make subsequent data runs, new curves will be plotted in different colors along with older stored curves. When a new region of interest is selected and you click on **Statistics**, you will be asked to choose the run for which you wish to calculate statistics.

7. Make a table for calculating μ_k from Eq. (3.3), based on different sets of M and m, with the different resulting a values. An Excel spreadsheet might be set up something like the following to guide you through the data-taking and calculations: As outlined in the table, you should do 20 trials:

   - Start with the wood block and some additional masses (total mass M_1), with the block lying on its large face, and using 5 different hanging masses m. (**make sure the string is level: adjust the pulley height as required**)
   - Next, remove the additional masses and just test the block (mass M_2) using your 5 m values (use the same m values as before).
   - Third, turn the block (mass M_2) onto a side with a smaller area for 5 more runs (**raise the pulley height to keep the string horizontal, as required in the theory**).
   - Finally, replace the extra masses (total mass M_1) with the block lying on the small face.

   Doing all of these different arrangements will help you decide whether mass, surface area, or velocity affects the kinetic friction coefficient.

M (g)	m (grams)	$\overline{a}$ (m/s^2)	μ_k		
M_1	m_1	...	...	Extra masses	
M_1	m_2	...	...	Large Side	
M_1	m_3	...	...	Average	SDOM
M_1	m_4	...	...	$\overline{\mu_k}$	$\sigma_{\overline{\mu_k}}$
M_1	m_5	...	...		

M (g)	m (grams)	$\overline{a}$ (m/s^2)	μ_k		
M_2	m_1	...	...	No extra masses	
M_2	m_2	...	...	Large Side	
M_2	m_3	...	...	Average	SDOM
M_2	m_4	...	...	$\overline{\mu_k}$	$\sigma_{\overline{\mu_k}}$
M_2	m_5	...	...		

M (g)	m (grams)	$\overline{a}$ (m/s^2)	μ_k		
M_2	m_1	...	...	No extra masses	
M_2	m_2	...	...	Small Side	
M_2	m_3	...	...	Average	SDOM
M_2	m_4	...	...	$\overline{\mu_k}$	$\sigma_{\overline{\mu_k}}$
M_2	m_5	...	...		

M (g)	m (grams)	$\overline{a}$ (m/s^2)	μ_k		
M_1	m_1	...	...	Extra masses	
M_1	m_2	...	...	Small Side	
M_1	m_3	...	...	Average	SDOM
M_1	m_4	...	...	$\overline{\mu_k}$	$\sigma_{\overline{\mu_k}}$
M_1	m_5	...	...		

8. To calculate the μ_k values, enter into Excel the formula that corresponds to Eq. (3.3).

   - In your formulas refer to the cells that contain M, m, a rather than typing in the actual mass or accelaration values. Once you get the formula correct in a cell, you can simply copy it to all the other μ_k cells.
   - The μ_k values should come out reasonably close to each other. Find the mean $\overline{\mu_k}$ and the standard deviation of the mean (SDOM) $\sigma_{\overline{\mu_k}}$ for each of the four groups of data.
     – You can calculate the mean of a list of numbers by placing in a cell the formula `=AVERAGE( )`; inside the parentheses goes the list of cells containing the numbers to be averaged, such as **D2:D6** or whatever (you can also do this by dragging the mouse to highlight these cells).
     – The standard deviation of the mean of these values, $\sigma_{\overline{\mu_k}}$, would be obtained by placing in some cell the formula `=STDEV(D2:D6)/SQRT(5)`.
   - If you have set up your spreadsheet as above, the mean and SDOM cells can also be simply copied to each of the following blocks.

- Remember to self-document (once) all of these formulas!
- As this will be your Final Results table, have Excel display the proper number of sigfigs for each $\overline{\mu_k}$ and uncertainty $\sigma_{\overline{\mu_k}}$.
- Include your spreadsheet results in your lab report document.

9. Finally, get an estimate of the STATIC coefficient of friction μ_s using the materials provided. According to the free body diagram Fig. 3.2, $f = \mu_s Mg = mg$ if M is nearly at the point where it starts to slide (but $a = 0$). You could find the largest m for which M remains at rest, and use it to compute μ_s. However changing m usually bumps the system, so we suggest carefully reducing M until motion begins. There is usually a fairly large inconsistency/uncertainty in this minimum M, so try it a few times. Use the variation (**explain your method**) to estimate an uncertainty in μ_s.

10. In your lab report summarize your results and comment briefly on them:
    - Did the coefficient of friction appear to vary with surface area?
    - Did it vary with the mass of the block (compare results for M_1 and M_2)?
    - Did it vary with the speed of the block (compare results for some M when pulled by large m and small m)?
    - Is it a reasonable assumption that the coefficient of kinetic friction is constant?
    - Is μ_s higher or lower than μ_k?

11. Record your "Quick Report" listing your four results for $\overline{\mu_k}$ and your estimate of μ_s, all with uncertainties and the proper number of significant figures. Show your instructor.

CHECKLIST

Names, date, purpose	
Theory, with relevant quantities defined	
Brief Methods & Materials or Procedure outline	
Table of M, m, a, μ_k with units	
Calculation of average and SDOM of μ_k (self document, sigfigs!)	
Measurement of static coefficient of friction μ_s	
Answers to the questions in #10	
Conclusions	
"Quick Report" check	

4. BALLISTIC PENDULUM

Purpose

Conservation of momentum and conservation of mechanical energy are two great laws that apply in many situations. In this lab you calculate situations where they do and do not apply.

Apparatus

1. Projectile launcher, steel ball
2. Ball catcher suspended from a supporting rod
3. Plastic rail with aluminum foil marker
4. Tape measure and scale (somewhere in the lab)
5. Plumb line
6. Meter stick
7. a computer with spreadsheet (e.g., Excel)

Introduction

If there are no external forces, the total momentum of colliding bodies is unchanged. In the study of collisions, two extremes are generally defined. A *perfectly elastic* collision is one in which the total kinetic energy of the system does not change. Collisions of microscopic objects such as atoms, for example, often fit into this category. Other collisions are inelastic. A *perfectly inelastic* collision is one in which the colliding objects stick together. In this experiment you'll be using a device known as a ballistic pendulum to study inelastic collisions and to measure the speed of a projectile.

If all the forces present are conservative forces, the total mechanical energy (potential energy plus kinetic energy) is unchanged. Since gravity is a conservative force and since friction is minimal, in the motion of a pendulum energy is nearly conserved.

In this experiment you'll be shooting a steel ball horizontally from a launcher, as illustrated in Fig. 4.1. The ball will be caught by a container suspended as a pendulum on strings. By measuring the distance the pendulum swings you determine the ball's initial speed v. With this information you should also be able to determine how far the launcher can shoot the ball.

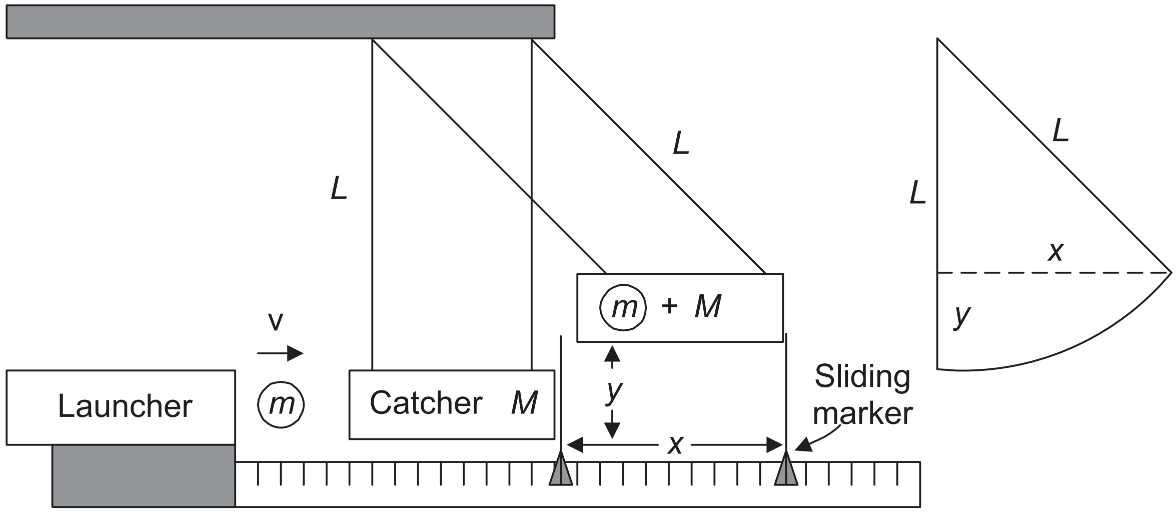


Figure 4.1: Ballistic pendulum experiment.

Theory

When the ball collides with the catcher, the forces they exert on each other are opposite in direction but equal in magnitude, by Newton's third law. As a result, the momentum $\mathbf{p_i}$ before the collision equals $\mathbf{p_f}$, the momentum after the collision. In equation form,

$$\mathbf{p_i} = m\mathbf{v} = \mathbf{p_f} = (m + M)\mathbf{V}, \tag{4.1}$$

so the ball's initial speed v is related to the speed V of the ball and catcher after the collision by

$$v = \frac{m + M}{m} V, \tag{4.2}$$

where m is the ball's mass and M is the catcher mass. In principle you could measure the masses and V to determine v. But there is another way. Since the impact of the ball with the catcher is inelastic the kinetic energy of the system is not conserved <u>in</u> the collision. But <u>after</u> the collision, the swinging motion of the catcher is nearly frictionless, so mechanical energy is nearly conserved. The pendulum's kinetic energy changes to gravitational potential energy as it swings, rising a distance y so that

$$\frac{1}{2}(m + M)V^2 = (m + M)gy. \tag{4.3}$$

Solving for V in this expression gives

$$V = \sqrt{2gy} \tag{4.4}$$

Combining Eqs. (4.4) and (4.2), the initial speed of the ball is related to the height y:

$$v = \left(1 + \frac{M}{m}\right)\sqrt{2gy}. \tag{4.5}$$

Instead of measuring the small distance y directly, it is simpler to measure the horizontal distance x the pendulum swings. From Fig. 4.1, the Pythagorean theorem can be used to show that

$$L^2 = (L - y)^2 + x^2, \tag{4.6}$$

and algebra can be used to write y in terms of x and L as

$$y = L \pm \sqrt{L^2 - x^2}. \tag{4.7}$$

The minus sign gives the solution which is relevant here. (Can you explain what the other one means?) So by measuring x, the initial speed v of the ball can be calculated.

Mechanical energy will be lost in *any* non-elastic collision. In the *perfectly* inelastic case, writing the ratio of kinetic energies before $(\frac{1}{2}mv^2)$ and after $\left[\frac{1}{2}(m+M)V^2\right]$ the collision, and then using Eq. (4.2), it is easy to obtain the ratio

$$\frac{E_{before}}{E_{after}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}(m+M)V^2} = \frac{m+M}{m}. \tag{4.8}$$

Your measured quantities, x, L, m, and M all have some uncertainty about them, leading to uncertainty in the calculated v. The formulas connecting v with these variables would make the error analysis very complicated. But a major source of uncertainty is the behavior of the launcher. You will find variations in x partly because v in fact varies from shot to shot. The best way to handle this is simply to fire the ball repeatedly, and use statistical analysis to find the best estimate for v and its uncertainty; this will be outlined in the Procedure section.

Procedure

1. Be sure that the launcher on your table is mounted so that it shoots horizontally. Position it ~ 1 cm from the catcher. The launcher and catcher should be aligned along the same direction. The catcher should be at the same level as the launcher, and both should be level. The geometry of the entire structure can be changed by loosening and re-tightening the appropriate screws. Check with your lab instructor if you do not see how to do this.

2. Use the digital balance to measure the ball mass m. The catcher mass M should be labeled on it. Use the tape measure to measure the string length L. Record these values (with estimated error) in your notebook.

3. Notice that there are three settings for the launch speed — it is necessary to push the ball in until the <u>third click</u>, so that it can shoot past the end of the ruler later in the experiment. Push the ball into the launcher with a pen or pencil and listen for three clicks. Take some practice shots to make sure everything works correctly. Make sure the ball is cleanly caught and the catcher swings upward

without rotating. Make sure the aluminum foil marker slides with hardly any resistance on the rail and marks the furthest excursion of the catcher. Stop the catcher from hitting the launcher when it swings back (or the resulting bounces will move the marker). If there are problems check with your lab instructor.

4. Load the launcher. Check that the catcher is level, etc. Now carefully find and record the zero position x_0 of the catcher by pushing the aluminum foil marker against the catcher. Take an initial measurement of x by firing. The new position, x_f, of the marker minus the original position, x_0, indicates the distance x the catcher moved. For subsequent measurements, start with the marker about 1 cm short of the expected final position — this will reduce the amount of pushing required to mark the furthest extent of the swing.

5. Start recording the distances x that catcher moves based on the final position of the aluminum foil marker. Carefully put the marker back about 1 cm each time. Make a list of the x values. Record x for 5 good (smooth, no-twist) shots. In making your Excel spreadsheet table, recall the functions: `SQRT()` (square root), `AVERAGE()` (mean or average), and `STDEV()` (standard deviation, σ). The Standard Deviation of the Mean is $\sigma/\sqrt{N}$ where here $N = 5$. A sample data table could look like the following example:

Trial	x(cm)	y(cm)	v(cm/s)
1			
2			
...			
5			
		(self-doc)	(self doc)
Average speed			$\overline{v}$
Standard Deviation			σ_v
Standard Deviation of Mean			$\sigma_{\overline{v}}$

6. Enter the formula to calculate y based on Eq. (4.7); enter the formula to calculate v based on Eq. (4.5).

   Note: you can directly enter the required numerical values for m, M, L, and g in these formulas, or you could put those values in fixed cell locations and then refer to those unchanging locations using an odd notation using dollar signs: e.g. **A15** (a cell fixed at **A15**) or whatever. Remember to pay attention to consistent use of units (e.g., is $g = 9.8$ or $g = 980$; it depends on what unit you are using for length). Get the formula right once, then copy it to the other cells in a column. Remember to self-document those calculations!

7. Now you should have five measurements of v. What is the best estimate of v? The best we can do is to calculate the average, or mean, of all the speeds. So on your spreadsheet, add a formula to the cell at the bottom of the v data to compute the average, $\overline{v}$, of all five measurements, something like `=AVERAGE(D2:D6)`. What uncertainty should be attached to this mean? It turns out that the best estimate

for *uncertainty in the mean* is the so-called "standard deviation of the mean," $\sigma_{\overline{v}}$. Use the spreadsheet to do this too, by placing in some cell a formula similar to `=STDEV(D2:D6)/SQRT(5)`. Self-document please!

8. Include the spreadsheet with calculated mean and SDOM of v in your report document. Note: since this will be your Final Results Table, $\overline{v}$, σ_v, and $\sigma_{\overline{v}}$ should be in 'final results format' with units and the proper number of sigfigs.

9. A good way to verify these measurements of v is to predict the range R, the distance to the place the ball hits the floor, using the equations of projectile motion. Review the physics that yielded Eq. (2.5) which relates the vertical and horizontal distances traveled by a projectile. See that in this case the range will be

$$R = v\sqrt{\frac{2h}{g}}, \tag{4.9}$$

where h is the initial height of the ball above the floor. Position the launcher at the edge of the table, measure h and calculate the expected R as well as an uncertainty range δR using the high-low method. **DO NOT REMOVE THE PLASTIC RAIL FROM THE LAUNCHER**.

10. Measure and mark on the floor the spot where you expect the ball to land, as well as your uncertainty range. (A plumb line may be helpful for determining where to measure from on the floor.) Will you hit that mark on your first try (i.e., no 'practice shots')? Notify your lab instructor when you are ready to test your prediction.

11. In your lab report, compare the kinetic energy before the collision ($KE_{\text{before}} = \frac{1}{2}mv^2$) with the kinetic energy immediately after the collision. According to Eq. (4.3) you can calculate the post-collision kinetic energy from the potential energy at the top of the swing: $KE_{\text{after}} = PE_{\text{max}} = (m + M)gy$. Do this using

some pair of v and y values in your table of data. **Q1:** Do the energies differ? What accounts for the difference? Check whether the ratio $KE_{\text{before}}/KE_{\text{after}}$ turns out approximately to equal $(M + m)/m$ as predicted by Eq. (4.8).

12. Record your "Quick Report" with your value for v (units, significant figures, uncertainty), and whether you hit the mark on your first try. Show your instructor.

CHECKLIST

Names, date, purpose	
Theory, with relevant quantities defined	
Brief Methods & Materials or Procedure outline	
Measurements (with errors) of m, M, L	
Spreadsheet calculations of v, $\overline{v}$, $\sigma_{\overline{v}}$ (units, sigfigs!)	
Range calculation and uncertainty, and success of prediction	
Q1: Energy comparison.	
Conclusions	
"Quick Report" check	

5. ROTATIONAL MOTION

Purpose

In this lab, you will examine the moment of inertia of objects. This will be done in two ways: by calculations based on geometry (denoted by $\mathcal{I}$) and by measurements of changing rotational speed when a torque is applied to the object (denoted by I). Your results for both methods should agree within uncertainty.

Warning: $\mathcal{I}$ and I mean different things in this lab. Confusion will result if you fail to distinguish them!

Apparatus

1. a computer with spreadsheet (e.g., Excel)
2. Vernier rotation apparatus and interface box. Masses labeled on the components have an uncertainty of $\delta M = 2$ g
3. Compressed air source with pressure control valve
4. Digital caliper ($\delta D = \pm 0.02$ mm) and digital balance ($\delta m = 0.1$ g)

Introduction

The well-known quantities used to describe linear motion have analogous quantities related to rotation, some of which are listed in the table below.

LINEAR MOTION	ROTATIONAL MOTION
Displacement x	Angular displacement θ
Velocity $v = \Delta x/\Delta t$	Angular velocity $\omega = \Delta\theta/\Delta t$
Acceleration $a = \Delta v/\Delta t$	Angular acceleration $\alpha = \Delta\omega/\Delta t$
Force F	Torque τ
mass m	Moment of inertia I

In the 19^{th} century physicists were careful to distinguish two potentially different types of mass: inertial mass (which could be measured by finding the acceleration caused by a force: $m = F/a$) and a gravitational mass (which was defined by the force of gravity on the object, i.e., the way most every balance measures mass).

Experimentally[5] the two seemed to be identical. In 1908 Einstein postulated his Equivalence Principle which states these that two masses are exactly identical, so as a result today we just talk about the mass.

Analogously we can distinguish two different types of moment of inertia. We will denote with I the moment of inertia measured by finding the angular acceleration caused by a torque: $I = \tau/\alpha$. We will denote with $\mathcal{I}$ the moment of inertia calculated from the mass and shape of the object. In this lab you will see if these two things seem to be identical.

In this experiment you will be applying a torque τ to a steel disk to get it to rotate. Friction is nearly eliminated by a cushion of air supporting the moving disk, and the resulting angular acceleration α is measured by a computer attached to the apparatus. By relating τ and α you find the moment of inertia of the rotating disk, here called I. This will be compared with a value $\mathcal{I}$(disk) calculated from the mass and radius of the disk. If all goes well, the two values should agree. Next, a rectangular metal bar of mass M, length L and width w will be rotated; both I (based on angular acceleration) and $\mathcal{I}$(bar) (based on shape) will be determined. Again they should agree.

The apparatus is illustrated in Fig. 5.1. A mass hanging on a string attached to a pulley on the steel disk provides the torque. The disk circumference is marked with stripes which the computer detects optically, allowing the angular speed to be calculated. Measurements of the mass and radius of the disk will be used to get an independent estimate of the moment of inertia for comparison. The process will be repeated with a metal bar.

Theory

According to Newton's second law, a net force F acting on a mass m gives it an acceleration a, where

$$F = ma \tag{5.1}$$

The analogous form of this law for rotational motion says that a net torque τ acting on a body gives it an angular acceleration α such that

$$\tau = I\alpha, \tag{5.2}$$

where I is the moment of inertia for the body. So if τ and α can be measured, I can be calculated from these measurements.

[5] According to Wikipedia: the earliest experiments were done by Isaac Newton and improved upon by Friedrich Wilhelm Bessel. Roland von Eötvös famously carried out extremely accurate experiments starting around 1885.

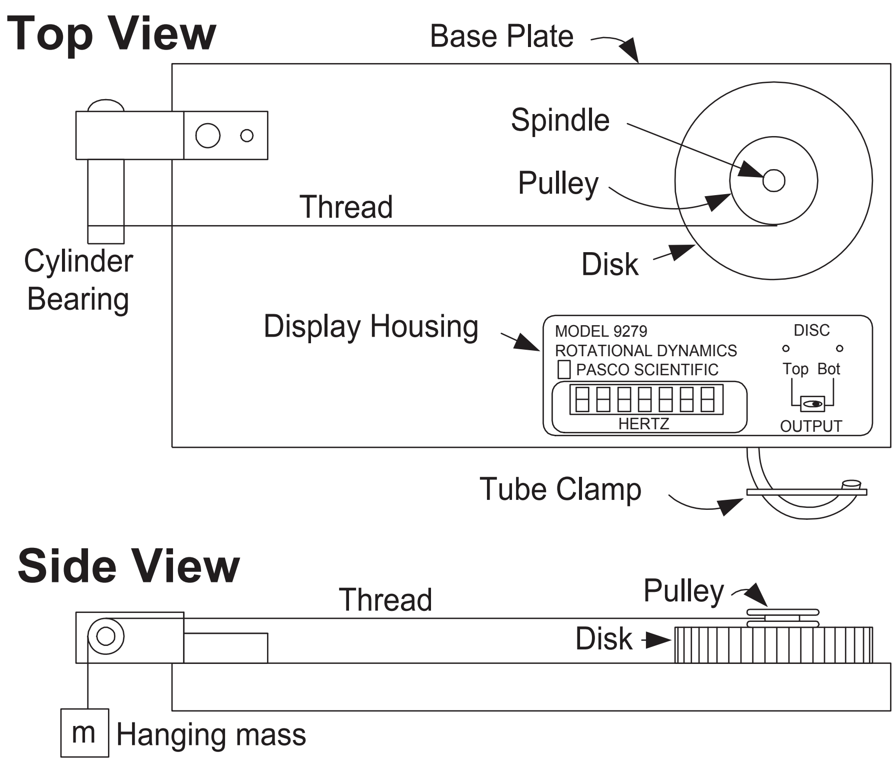


Figure 5.1: Rotational motion apparatus

With a mass m hanging on the string in Fig. 5.1, the tension T in the string satisfies:

$$mg - T = ma = mr\alpha, \tag{5.3}$$

since the acceleration a and the angular acceleration of the pulley are related by $a = r\alpha$, where r is the inner radius of the pulley where the string winds. Thus $T = mg - mr\alpha$. The torque on the rotating disk is then

$$\tau = Tr = mgr - m\alpha r^2 \tag{5.4}$$

So if you measure m, r and α you can get I from Eq. (5.2):

$$I = \frac{\tau}{\alpha} = \frac{mgr}{\alpha} - mr^2. \tag{5.5}$$

Alternatively a moment of inertia $\mathcal{I}$ can be calculated from the mass and shape of the object. Tables of $\mathcal{I}$ for simple shapes are found in most physics texts, with diagrams such as in Fig. 5.2. For a solid disk with the rotation axis through its center, the moment of inertia depends on the mass M and radius R of the disk, as follows:

$$\mathcal{I}(\text{disk}) = \frac{1}{2}MR^2. \tag{5.6}$$

For a rectangular bar rotated around an axis through its center as shown the moment of inertia is

$$\mathcal{I}(\text{bar}) = \frac{1}{12}M(L^2 + w^2), \tag{5.7}$$

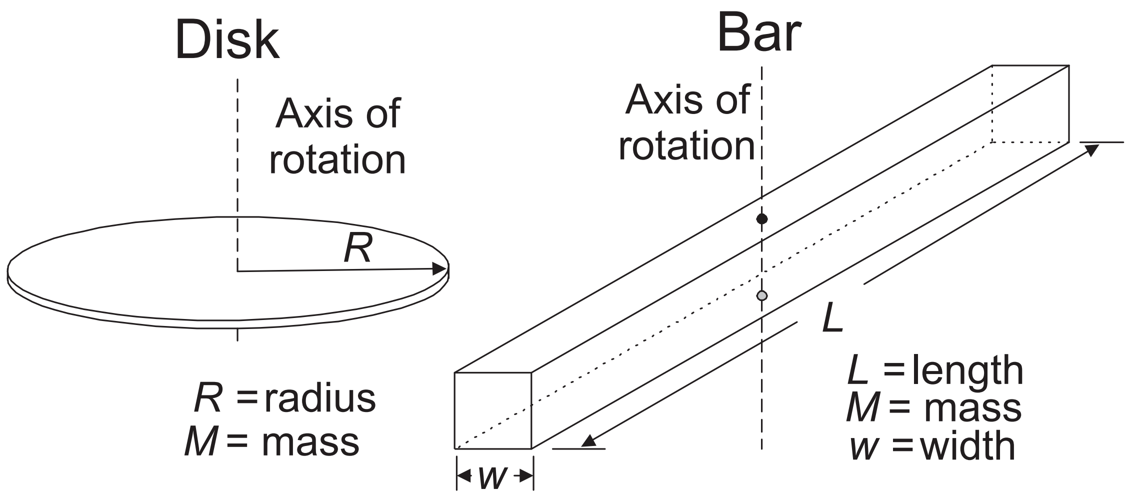


Figure 5.2: A solid disk and a solid bar rotated about axes through their centers.

where M is the mass of the bar, and L and w are its length and width.

The uncertainty of your $\mathcal{I}$(disk) and $\mathcal{I}$(bar) values depends on the uncertainties of your mass and shape measurements. Use the high-low method to determine these uncertainties. For the disk, calculate the highest value and the lowest value of $\mathcal{I}$(disk) from Eq. (5.6). For the bar, calculate the highest and lowest values of $\mathcal{I}$(bar) using Eq. (5.7). Half the difference between the high and low values will be your estimate for uncertainty.

The uncertainty in I depends strongly on the uncertainty in α, which is not simple to evaluate; the best way to estimate δI will be to take the standard deviation of the mean of several I measurements.

Including appropriate uncertainties with your measured values, your $\mathcal{I}$(disk) and I(disk) ought to agree. Similarly, when the bar is used, $\mathcal{I}$(bar) should agree with the measured inertia I(bar).

Procedure

1. The apparatus is delicate, and must be treated properly. The steel disk is designed to rotate on a thin cushion of compressed air that flows underneath it and along the spindle. *DO NOT remove the disk from the spindle, and DO NOT rotate the disk unless the air is turned on at a minimum pressure of 9 psi (some units require 12 psi).*

2. Use the bubble level to make sure the disk is level; if it needs leveling you can adjust it yourself or ask your lab instructor for help.

3. Next, measure and record the radius r of the pulley where the string winds. You may find it easier to measure the diameter with calipers and then divide by two.

You will need the radius to calculate the applied torques.

4. The pulley should attach to the top disk with the black thumbscrew. The string is anchored with a washer inside the pulley and it should come out the slot on the rim of the pulley. It then passes over the cylindrical bearing, and a mass m of about 25 grams hangs on the end of it. Weigh your m!

5. Turn on the air supply and adjust the supply so that the air pressure allows the top disk to turn freely (typically set to around 10 psi, and keep monitoring the air pressure throughout the experiment). The bottom disk should not freely rotate.

6. Make sure the ~ 25 g mass m on the string can drop without interference. If the string rubs on the table, move the apparatus to correct the problem. Check the air pressure again. By *gently* turning the top disk, wind the thread around the pulley until the top of the 25 g mass is level with the bottom of the cylindrical bearing bracket. Hold the top disk stationary for a moment, damp any pendulum motion of the hanging mass and then release the top disk. The falling mass m should slowly accelerate the disk. The thread should unwind from the pulley and, before the mass hits the ground, the thread should begin winding back on the pulley, slowing the rotation. If the mass reaches the ground, shorten the thread. Make a couple of test runs for practice.

7. The LabQuest interface will have the rotation apparatus plugged into Dig 1. Start the *Logger Pro* program by clicking on the desktop icon.[6] Under the file menu,

[6]If you see the message "No Device Connected" in the upper left corner of the screen, the interface must be unplugged at the computer end of the cable for a few seconds and then re-connected. You should then see this message disappear.

open **rot_dyn**. For certain digital sensors (like the one used in this experiment) a confirmation window will open that asks you to connect the sensor. Click the connect button. A toolbar along with a data table and an velocity vs. time graph will appear.

8. Begin a new data collection run by first winding up the thread and holding the disk steady. Tell the computer to begin taking data and *then* release the disk; let it proceed for at least six full cycles.

9. On the resulting velocity vs. time graph, click and drag the mouse to select an upward linear portion of your data for analysis (this corresponds to times when the hanging mass is descending).

   Clicking on **Linear Fit** under the Analyze menu will now analyze the selected portion of the data. A box will appear containing the slope (angular acceleration) for the selected portion of the run (among other things). Record the selected slopes each time you make a run.[7]

10. Make a data table in your spreadsheet similar to the one below. Record six angular accelerations for the disk and calculate τ from Eq. (5.4) and I from Eq. (5.2) for each measured α. **Be sure to also record your units for each column. The units must be consistent to compare results.**

α (s^{-2})	$\tau = mgr - m\alpha r^2$ ($\mathrm{g\cdot cm^2/s^2}$)	$I = \tau/\alpha$ ($\mathrm{g\cdot cm^2}$)
...	...	...
...	...	...
...	...	...
...	...	...
...	...	...
	(self doc)	(self doc)
	Mean	...
	Standard Deviation of Mean	...

11. Enter the formulas for τ and I and copy them through the 6 trials.

12. Now you should have 6 measurements of the moment of inertia I for the disk. Calculate the mean and standard deviation of the mean. Self document all these spread sheet calculations!

13. Record mass M and radius R of the top steel disk.

[7] There are two unusual sources of error in this apparatus. (1) The reflective strip on the rotating disk has alternate black and white bars that trigger the photogate. A larger white bar exists at the location the strip ends, this causes a calculation error that appears as a downward spike on the velocity graph. Although the spikes are visible, they contribute only a small error to the calculated slopes. (2) The software only allows two significant figures to be used for the separation between bars on the reflective strip, and this may show up as a systematic error that could affect your comparison between experimental and theoretical moments of inertia.

14. Calculate the moment of inertia $\mathcal{I}$(disk) from the mass and shape of the disk using Eq. (5.6). Calculate its uncertainty using the high-low method as shown below.[8]

    A spreadsheet is the easiest way to make these calculations. Make a little table:

Mass	Radius	$\mathcal{I} = \frac{1}{2}\, MR^2$
M_0	R_0	$\mathcal{I}_0$
M_{max}	R_{max}	$\mathcal{I}_{max}$
M_{min}	R_{min}	$\mathcal{I}_{min}$
$\delta\mathcal{I} = (\mathcal{I}_{max} - \mathcal{I}_{min})/2$		$\delta\mathcal{I}$

    The formula to calculate the best estimate of $\mathcal{I}$ (labeled above $\mathcal{I}_0$) using the cells containing the best estimates for M and R (M_0, R_0), can be copied to find $\mathcal{I}_{max}$ if the appropriate cells are filled with the high values of M and R ($M_{max} = M_0 + \delta M$, $R_{max} = R_0 + \delta R$). Similarly the low value $\mathcal{I}_{min}$ can be immediately calculated, and then $\delta\mathcal{I}$ can be found. Self documenting the spreadsheet saves you from showing a sample calculation!

15. Repeat steps 8–12 after attaching the bar to the top of the pulley with the red thumbscrew. Note that the Excel table used for the steel disk can be simply copied & pasted and then new values of α entered. The result is the moment of inertia for the combined system of disk+bar which we denote as $I(\text{disk} + \text{bar})$.

16. To determine I for the bar alone, subtract I(disk) (found in part 11) from I(bar+disk):

$$I(\text{bar}) = I(\text{disk} + \text{bar}) - I(\text{disk}) \tag{5.8}$$

    Find the uncertainty in this result by adding (not subtracting!) uncertainties in the two Is:

$$\delta I(\text{bar}) = \delta I(\text{disk} + \text{bar}) + \delta I(\text{disk}) \tag{5.9}$$

17. Calculate the moment of inertia $\mathcal{I}$(bar) from the mass and shape of the bar, and also calculate the uncertainties using the high-low method much as in #14. Make an Excel table with rows containing the best, maximum and minimum values and columns of M, w and L and a formula for Eq. (5.7).[9]

    The inertia $\mathcal{I}$(bar) from measurements of the bar shape should agree (within uncertainties) with the result I(bar) that you got using angular acceleration.

18. Your "Quick Report" should be a final results table (or list) in your report document that properly records (units, errors, sigfigs) your four moments of inertia.

object	I units	$\mathcal{I}$ units
disk	$\pm$	$\pm$
bar	$\pm$	$\pm$

[8] Alternately, for this case you can use $\delta\mathcal{I} = \mathcal{I}(\frac{\delta M}{M} + 2\frac{\delta R}{R})$. The analagous expression for the next part is not so simple.

[9] Alternately, you can use $\delta\mathcal{I}(bar) = \mathcal{I}(\frac{\delta M}{M} + 2\frac{L\delta L + w\delta w}{L^2+w^2})$.

19. Comment on your results. Do the two ways of determining the moment of inertia agree (within error)? If they differ, think about factors that were neglected and whether they may be significant. Our theory neglected friction which is probably actually present. If friction were actually eliminated, would the value of α increase of decrease? Given that effect on α, would the resulting value of I increase of decrease? Therefore would a correction for the effect of friction on what you observed increase or decrease the torque-based I values? Our theory neglected the moment of inertia of the small pulley. What would happen to I(disk) and I(bar) if the moment of inertia of the small pulley were correctly subtracted rather than simply neglected?

CHECKLIST

Names, date, purpose	
Theory, with relevant quantities defined	
Brief Methods & Materials or Procedure outline	
Mass M and radius R of disk (errors!)	
Mass M and dimensions L and w of bar	
Radius r of pulley recorded	
Hanging mass m recorded	
Table of α using disk; 6 trials	
Calculation of mean and SDOM for I(disk)	
Calculation of $\mathcal{I}$(disk) with high/low uncertainty	
Table of α for disk+bar; 6 trials	
Calculation of mean and SDOM for $I(\text{disk} + \text{bar})$	
Calculation of I(bar) and δI(bar)	
Calculation of $\mathcal{I}$(bar) with high/low uncertainty	
Final Results Table (proper units, errors, sigfigs)	
Answers to questions in part 19	
Conclusions	
“Quick Report” check	

6. ARCHIMEDES' PRINCIPLE

Purpose

In this experiment you will investigate Archimedes' Principle by measuring and comparing the buoyant forces and displaced fluid weights for different objects in fluids.

Apparatus

1. Unmarked cylinder containing water
2. Digital balance (±2 g)
3. Digital height gauge (±.003 cm)
4. Digital calipers (±.002 cm)
5. Objects that float and objects that sink
6. a computer with spreadsheet (e.g., Excel)

Introduction

An object submerged (partially or fully) in a fluid experiences an upward buoyant force that counteracts part or all of the force of gravity on the object. For instance, supporting a large rock under water is easier than in air. Many objects are found to float in water, such as ice cubes, life jackets, and huge ships made mostly of steel. Some time ago, Archimedes (287–212 B.C.) deduced the following principle: *the buoyant force on an object is equal to the weight of the fluid it displaces.*

The experiment consists of measuring the buoyant force on objects that sink and objects that float in water, and comparing this force with the weight of the displaced water. We know the density of water, $\rho_{\mathrm{w}} = 1.0\ \mathrm{g/cm^3}$, so we can determine the weight of the displaced water by finding its volume. The buoyant force is measured indirectly via Newton's Third Law: if the fluid is pushing the object up, the object must be pushing the fluid down, and that downward push can be measured by placing the fluid (and its container) on a scale.

Theory

A. Sinking Objects

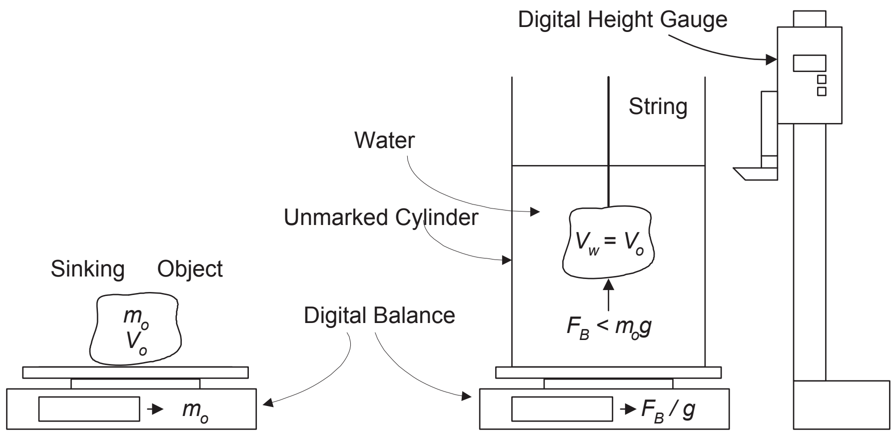


Figure 6.1: Testing Archimedes' principle for a sinking object.

For the first part of the experiment, we test Archimedes' principle for objects that *sink* in water, by measuring the buoyant force F_{B} and the volume V_{w} of water displaced. The uncertainties are probably going to be significant in this method, so you will need to pay attention to them. First, we use the digital balance to measure the mass, m_{obj}, of the object. Then, as illustrated in Figure 6.1, the object is suspended in water on a string. It may feel noticeably lighter than when in air, because the water exerts a buoyant force upward on the object. So according to Newton's Third Law, the object exerts an equal downward force on the water. This is conveniently measured with the digital balance; we use the TARE button to set the balance to read zero before the object is suspended, then the buoyant force will be given by the scale (actually, the scale gives the force divided by $g = 980\ \mathrm{cm/s^2}$, as if it were a mass instead of a force). So your measurements of buoyant forces will be:

$$F_{\mathrm{B}} = g \cdot \text{scale reading} \tag{6.1}$$

with an uncertainty of

$$\delta F_{\mathrm{B}} = g \cdot 2 \tag{6.2}$$

because the uncertainty of the scale is ± 2 grams.

Because the object sinks, the object's volume, V_{obj}, equals the the volume of water it displaces V_{w}, which can be calculated from the change in the water level of the cylinder, denoted by h:

$$V_{\mathrm{w}} = \pi r^2 h, \tag{6.3}$$

where r is the inside radius of the cylinder. The uncertainty in V_{w} can be approximated by

$$\delta V_{\mathrm{w}} = V_w \left(2\frac{\delta r}{r} + \frac{\delta h}{h} \right). \tag{6.4}$$

The weight of the displaced water is:

$$W_{\mathrm{w}} = V_{\mathrm{w}}\rho_{\mathrm{w}}g \tag{6.5}$$

where $\rho_{\mathrm{w}} = 1$ gram/cm^3 is the density of water. The uncertainty in the water weight is simply

$$\delta W_{\mathrm{w}} = \delta V_{\mathrm{w}} \cdot \rho_{\mathrm{w}}g \tag{6.6}$$

where δV_{w} is your estimate of the uncertainty in the volume [see Eq. (6.4)].

Now, according to Archimedes, the buoyant force F_{B} should equal the water weight W_{w}, and you can compare your measurements of F_{B} and W_{w} to see if they agree within their uncertainties.

From your data you can also compute the sinking object's density,

$$\rho = \frac{m_{\mathrm{obj}}}{V_{\mathrm{obj}}} \tag{6.7}$$

because the volume V_{obj} of the sinking object is the same as the volume of water V_{w} it displaces. The uncertainty associated with this density would be

$$\delta\rho = \rho\left(\frac{\delta m_{\mathrm{obj}}}{m_{\mathrm{obj}}} + \frac{\delta V_{\mathrm{obj}}}{V_{\mathrm{obj}}}\right). \tag{6.8}$$

B. Floating Objects

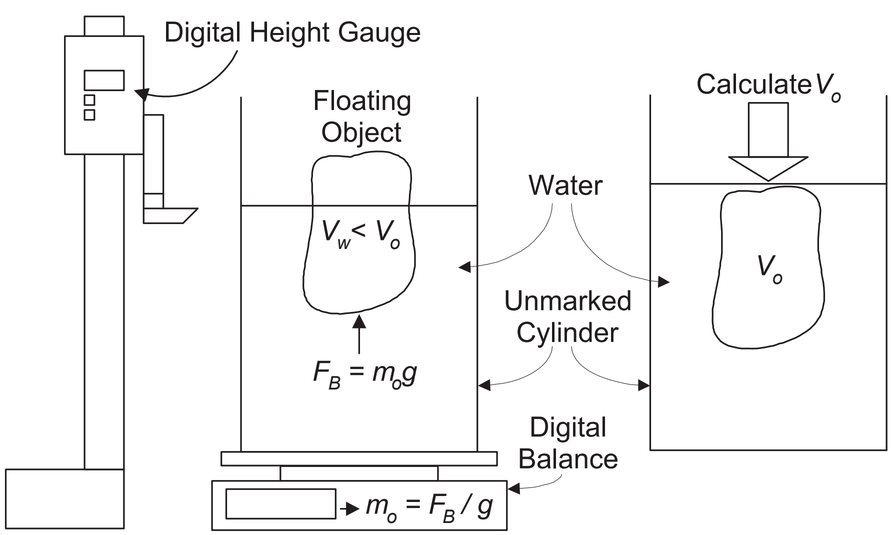


Figure 6.2: Testing Archimedes' principle for a floating object.

For the second part of the experiment, test Archimedes' principle using objects that *float* in water, as illustrated in Fig. 6.2. The buoyant force F_B in this case equals the weight of the object, and you will get it from the scale reading as before:

$$F_B = g \cdot \text{scale reading} \tag{6.9}$$

with an uncertainty of

$$\delta F_B = g \cdot 2 \tag{6.10}$$

because the uncertainty of the scale is ± 2 grams.

But a floating object displaces a water volume V_w which is something less than the object's volume, V_{obj}. As before, the weight of the displaced water is

$$W_w = V_w \rho_w g \tag{6.11}$$

where $\rho_w = 1$ gram/cm^3 is the density of water. And the uncertainty in the water weight is again simply

$$\delta W_w = \delta V_w \cdot \rho_w g \tag{6.12}$$

where δV_w is your estimate of the uncertainty in the volume [see Eq. (6.4)].

According to Archimedes, the buoyant force again should be equal to the weight of the displaced fluid.

After measuring the volume V_w displaced by the object when it floats, you should also measure the total volume V_{obj} of the object by pushing it down so it is completely submerged. Then you will be able to calculate the density of the floating object:

$$\rho = m_{obj}/V_{obj} \tag{6.13}$$

as before, and the uncertainty in density will again be

$$\delta\rho = \rho \left(\frac{\delta m_{obj}}{m_{obj}} + \frac{\delta V_{obj}}{V_{obj}} \right). \tag{6.14}$$

Procedure

1. Find two different objects that sink in water. As shown in Fig. (6.1), submerge the objects in the water using the string to suspend the object in the bulk of the water, but not in contact with the bottom or sides of the container. Record the scale reading and water level increase, h. Calculate buoyant forces F_B, the water volumes V_w displaced when the objects are submerged, and the corresponding water weights W_w. A table like the following will help. Note that Eqs. (6.1–6.6) are relevant here; you need to convert each equation to an Excel-cell formula which can then be copied to the second row. A convenient force unit is the dyne: 1 dyne (dyn) = 1 g·cm/s^2, so a mass of 1 gram has a weight of 980 dyn. Recall the odd Excel formula for π: `PI()`. Remember to self-document.

Object	Scale Reading (g)	F_B (dyn)	δF_B (dyn)	h (cm)	V_w (cm^3)	δV_w (cm^3)	W_w (dyn)	δW_w (dyn)	Overlap? Yes/No

2. Archimedes' principle says that two seemingly unrelated quantities: F_B and W_w are the same. Of course they are unlikely to be exactly the same; we can say that the relationship is confirmed if the range of possible values for F_B, i.e., the interval $(F_B - \delta F_B, F_B + \delta F_B)$, overlaps the range of possible values for W_w, i.e., the interval $(W_w - \delta W_w, W_w + \delta W_w)$. Is Archimedes' principle confirmed?

3. Measure the masses m_{obj} of the objects. **Q1:** How do the measured masses compare to the scale readings from step #1 for the same objects suspended in water? Explain. Is the volume of water V_w displaced by the objects as measured in #1 more than, less than, or equal to the volume of the object V_{obj}? Explain.

4. Figure out the densities of the masses (with proper sig figs) using a table similar to the one below. Eqs. (6.7–6.8) are relevant here.

	Mass			Volume		Density	
Object	m_{obj} (g)	δm_{obj} (g)	h (cm)	V_{obj} (cm^3)	δV_{obj} (cm^3)	ρ (g/cm^3)	$\delta\rho$ (g/cm^3)

5. Find two objects that float in water and, as shown in Fig. (6.2), float them. For the floating objects, it is okay if they touch the sides of the cylinder. Record the scale reading and the height <u>as they float</u>. Calculate the buoyant forces F_B, displaced water volumes V_w, etc. as before. This will fill in the final two rows in the table you started in #1.

6. Does Archimedes' principle appear to be correct for the floating objects?

7. Measure the masses m_{obj} of the objects. Measure the volumes of the objects V_{obj} by pushing them under the surface. Enter your results into the final two rows of the table you started in #4. **Q2:** How do the measured masses compare to the scale readings from step #5 for the same objects floating in water? Explain. Is the volume of water V_w displaced by the objects as measured in #5 more than, less than, or equal to the volume of the object V_{obj}? Explain.

   Figure out the densities of the objects (with proper sig figs).

8. How do the densities of the floating and sinking objects compare to the density of water?

9. Use what you have learned to measure the density ρ_ℓ of an unknown liquid, and its uncertainty $\delta\rho_\ell$. Your instructor will show you the equipment. Explain your procedure.

10. Just like in this lab, suppose you have a container of water sitting on a scale, and the scale reports the weight of the water and container. Now if you lower a rock, supported on a string, into the water. What effect will this have on the scale? Is the change in the scale reading: the weight of the rock, the tension in the string, the buoyant force, …? Explain briefly in your notebook why the scale reading changes (after all the rock is not in direct contact with the container) and what the change in scale reading (in grams) represents.

11. Record your "Quick Report" listing (units, significant figures, uncertainties) the densities of one floating object, one sinking object, and the unknown liquid.

CHECKLIST

Names, date, purpose	
Theory, with relevant quantities defined	
Brief Methods & Materials or Procedure outline	
Table for F_B and W_w comparisons (self-documented)	
Table for densities (self-documented, proper sig figs/uncertainties for densities)	
Comments on results for Archimedes' principle	
Q1, Q2	
Comments on densities of floating and sinking objects	
Measurement of density ρ_ℓ with uncertainty $\delta\rho_\ell$ of unknown liquid	
Discussion of step 10	
Conclusions	
"Quick Report" check	

7. GAS BEHAVIOR

Purpose

The purpose of this experiment is to give you some experience with a simple thermodynamic system—a container of air. You will be studying the relationship of pressure and volume for a gas sample, and plotting the PV diagram of a simple heat engine.

Apparatus

1. a computer with spreadsheet (e.g., Excel) and web browser for accessing *WAPP*$^+$
2. Vernier interface and *Logger Pro*
3. Pneumatic cylinder
4. Heat engine apparatus: piston, air chamber, tubing; the piston diameter $= 3.25 \pm .01$ cm
5. Pressure Sensor
6. Volume sensing pulley system
7. Water baths: hot and cold water; ice
8. A 100 g mass with a hook
9. Thermometer

Introduction

This lab has two parts: (I) to verify Boyle's law relating gas volume and pressure and (II) to acquaint you with a simple heat engine. Both parts make use of the ideal gas law and pressure vs. volume (PV) diagrams.

The ideal gas law relates the gas pressure P, volume V, and absolute temperature T:

$$PV = nRT \tag{7.1}$$

where n is the number of moles of the gas, T is the temperature in kelvins, and R is the universal gas constant, 8.31 J/mol·K. No real gas is "ideal" but under the conditions of this experiment (temperatures and pressures not too low or high), Eq. (7.1) should be an excellent approximation.

Boyle's Law

In Eq. (7.1) we can group together constant terms. Boyle's Law says that if n and T are constant, then V is proportional to $1/P$; that is,

$$V = \frac{nRT}{P}. \tag{7.2}$$

To test Boyle's law, we need a gas container with no leaks, so n will remain constant. A pneumatic cylinder as shown in Fig. 7.1 will be used. A rod extending into the cylinder allows you to change the gas volume and pressure, and sensors attached to a computer will record these measurements. It is also assumed that temperature remains constant, so the process is *isothermal*; for this to be approximately true the changes in V and P must be done slowly, so the cylinder and the gas inside it can stay in equilibrium with the room's temperature.

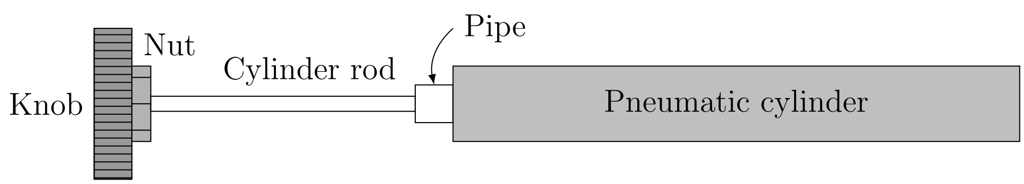
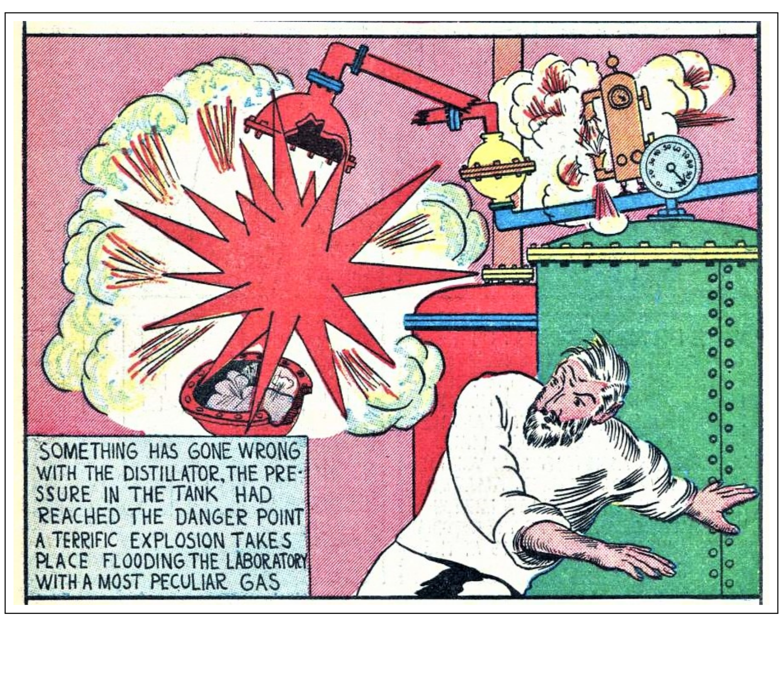


Figure 7.1: Boyle's Law apparatus. The gas is contained within the pneumatic cylinder. The cylinder rod is used to change the gas volume. A chain limits the possible expansion. The initial volume of the gas varies slightly between devices, a fact we must allow for in our analysis.



Procedure

1. The LabQuest interface will have the rotary motion sensor plugged into Dig 1 and the gas pressure sensor plugged into CH 1.

2. Start the *Logger Pro* program by clicking on the desktop icon.[10] Under the file menu, open **gas1**. A pressure vs. volume graph will appear.

3. The rotary motion sensor and metal pneumatic cylinder are mounted on a vertical rod. Do not alter the position of the cylinder. It may be necessary to move the rotary motion sensor inward or outward to make it align with the cylinder string.

4. With the string attached to the cylinder rod draped over the *large* pulley of the rotary motion sensor, the rotation of the sensor measures the linear displacement of the piston. The initial position of the piston results in an initial volume of about 28.5 mL, but in fact this initial volume varies a bit between devices. This unknown, systematic, device-dependent volume must be determined as part of the analysis. The random uncertainty in volume is about 0.2 mL.

5. To connect the pressure sensor you will need to rotate the sensor approximately one-half turn to make a good connection. The pressure sensor measures absolute pressure (including atmospheric pressure) with an uncertainty of about 0.2 kPa.

6. With the string attached to the handle draped over the *large* pulley of the rotary motion sensor, click Collect and slowly pull the handle up until the chain limits further expansion. Logger Pro allows 10 seconds for this expansion. If you finish early either hit Stop or hold the rod at its maximum expansion until the 10 seconds expires. If you find the volume decreasing when it should be increasing, change the string direction on the rotary motion pulley and re-do the measurement.

7. Start *WAPP*$^+$ selecting formulas (in this case constants) for both x and y errors. Do Bulk.

8. Click and drag to select a portion of the graph with regular behavior. (Avoid the stick-slip at the start of the gas expansion and any odd behavior at the end of the run.)

9. Copy the P and V columns and paste them into the "Block Copy & Paste" section of *WAPP*$^+$. Pressure should be in the first column; it will be x data. Volume data (the second column) should appear on vertical (y) axis. Enter the constant x and y errors in the appropriate box.

10. Select the "Inverse x" function: $y = A + B/x$. The A value represents the unknown device-dependent initial volume (it should be small); the B value should be nRT.

[10] If you see the message "No Device Connected" in the upper left corner of the screen, the interface must be unplugged at the computer end of the cable for a few seconds and then re-connected. You will then see this message disappear.

11. According to Eq. (7.2), the B parameter ought to be equal to nRT. Since R is well-known ($8314\ \frac{\text{mL}\cdot\text{kPa}}{\text{mol}\cdot\text{K}}$) and T can be measured in the room, you will calculate n from B. The uncertainty in n will be related to the uncertainty in the B parameter according to $\delta n = n\frac{\delta B}{B}$. Is n a reasonable value? (As a rough guess, you may recall that a mole of gas at standard temperature and pressure fills a volume of 22.4 L. Set up a ratio with your cylinder's volume to make a rough guess for the fraction of a mole in the cylinder).

12. Make any relevant comments on the scientific validity of Boyle's Law based on your analysis.

13. When writing your report, give some thought as to how a small amount of air leakage and how a slight temperature increase, caused by pressing down on the piston, might affect your measurements. **Q1:** Would these errors be considered random or systematic?

Heat Engine

Gases are useful for converting thermal energy into mechanical energy because they undergo much larger volume changes than solids or liquids. For a gas confined at low temperature in a container with a piston, warming the gas will raise the piston and a mass placed on top of it, thereby converting thermal energy to mechanical energy. An *engine* operates in a *cycle*; the mass must be removed and the gas cooled to return it to its starting point for a complete cycle.

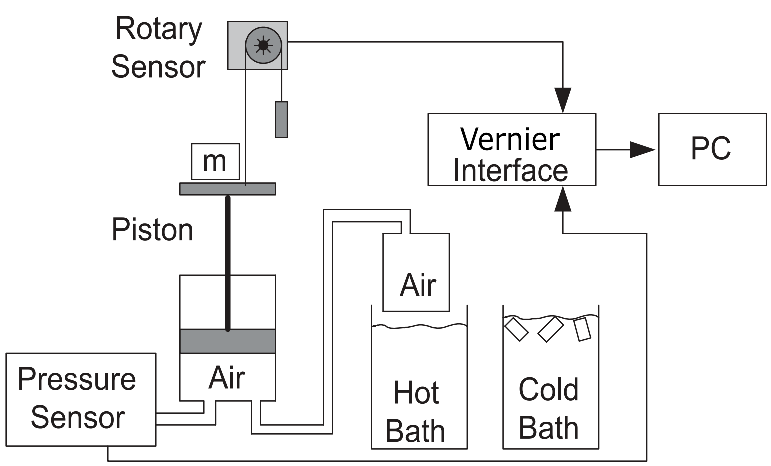


Figure 7.2: Heat engine apparatus. The mass m moves up or down when the air chamber is moved to the hot or cold water bath.

As illustrated in Fig. 7.2, the heat engine in today's lab consists of a graduated cylinder with a graphite piston, an aluminum air chamber on the end of a plastic tube, and hot and cold water baths for immersing the air chamber. Sensors attached to the computer allow you to monitor the pressure and volume of the gas. In this way it is similar to the

Boyle's law apparatus, but the volume and pressure changes will be much smaller, and the temperature is obviously not held constant.

The engine operates in a cycle consisting of four stages: (1) a 100-g mass is placed on the piston, (2) the gas is warmed to lift the mass, (3) the mass is removed, and (4) the gas is cooled, completing the cycle. P and V will be monitored around the cycle.

Figure 7.3 illustrates this simple engine cycle and a corresponding PV diagram.

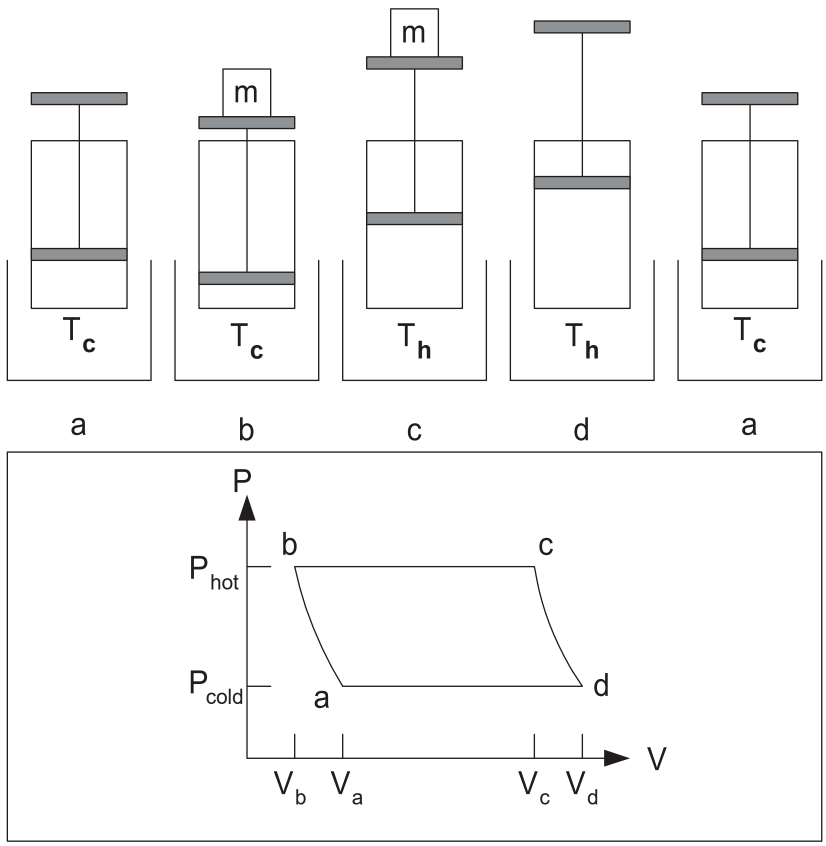


Figure 7.3: Engine cycle and PV diagram.

We can assume that the $a \to b$ segment (where a mass is added compressing the gas), and the $c \to d$ segment (where the mass is removed), happen quickly enough to be *adiabatic*, so no heat is transferred.[11] The other segments, $b \to c$ and $d \to a$, are assumed to be *isobaric*, or constant pressure processes.

According to the first law of thermodynamics, the heat Q added to the gas appears in the form of work done <u>by</u> the gas, W, and a change in the internal energy of the gas, U:

$$Q = \Delta U + W. \tag{7.3}$$

[11] You may wonder why this isn't an isothermal process, since the piston illustration shows the cylinder in contact with a single temperature during the $a \to b$ and $c \to d$ segments of the cycle. The compression from the mass is fast, but thermal equilibrium takes a few seconds to be achieved—for this reason, we consider the process as adiabatic. In reality, these segments are probably somewhere between adiabatic and isothermal processes.

The area of a PV cycle gives the net work done.

Procedure

1. Get two water baths ready in pails: a cold one with some ice water, and a hot one with hot water from the tap. The precise temperature of neither is critical; obviously the ice bath will be close to 0°C.

2. Undrape the string from the Boyle's Law pneumatic cylinder and replace it by draping the string from the heat engine over the same large pulley of the rotary motion sensor. It may be necessary to move the rotary motion sensor inward or outward for alignment.

3. Disconnect the plastic tube from the pneumatic cylinder where it connects to the pressure sensor. You will need a starting volume in the heat engine cylinder that can both increase and decrease. Position the piston somewhere near the center of the cylinder and lock it in place with the thumbscrew. Re-connect the plastic tube to the pressure sensor, then release the thumbscrew.

4. Under the file menu on *Logger Pro*, open **gas2**. The initial volume will be designated zero and can swing positively and negatively. This is OK since we are only looking for changes in the volume (not the actual volume).

5. Try a test run to assure that the P and V sensors work, and to practice the routine for making a nice cycle. Here's the routine:

   (a) Start with the air chamber in the cold water bath.

   (b) Click on the Collect button of the *Logger Pro* program. Each run will end after 50 seconds, which should be enough time to finish the cycle.

   (c) Gently (but quickly) place the 100 g mass on the piston. Watch the PV graph. P should increase, while V decreases, like adiabat $a \to b$ of Fig. 7.3.

   (d) Immediately, move the air chamber into the hot water bath. V should rise as the temperature increases, but P should remain constant, like the $b \to c$ isobar of Fig. 7.3. Leave the air chamber in the hot bath until the volume increase stalls.

   (e) Remove the 100 g mass from the piston. P should decrease, while V increases, like the $c \to d$ adiabat of Fig. 7.3.

   (f) Immediately move the air chamber into the cold bath, and leave it there for now. P should remain constant and V should decrease, like the $d \to a$ isobar of Fig. 7.3. Everything working OK? If not, get help. Click on the Stop button to end the trial run.

   (g) Your PV graph should return to approximately the starting position (making one complete engine cycle). To improve it, try to do it quicker, especially moving the air chamber from one temperature bath to the other immediately after moving the 100 g mass, to reduce heat transfers between the air chamber and the temperature baths at the b and d corners of the cycle.

6. Try the cycle a few times until you get a good PV diagram. (Be watching for evidence of gas leaks. Does the PV curve return to its starting place? Does P drop when it is expected to remain constant?) Include an image of the cycle in your report document.

7. Here are some things to determine from your data:

   (a) Are the $b \to c$ and $d \to a$ segments nearly horizontal?

   (b) Is the pressure difference $\Delta P = P_{bc} - P_{ad}$ between the upper and lower isobars consistent with the amount expected from adding a 100-g mass to the piston? Recall that $P =$ Force/Area. (Convert your basic measurements to MKS units to make this comparison; recall that Pa $=$ N/m^2.)

   (c) The area inside a PV cycle should correspond to the net work done by the gas in lifting the mass.

      i. Estimate the area inside your cycle in milliJoules (kPa·mL=mJ). The cycle is roughly a parallelogram where the area is equal to the product of the base and the height:

      $$W = \Delta V(\text{either base}) \times \Delta P. \tag{7.4}$$

      Note: the pressure and volume at the cursor's location is continuously displayed at the bottom left corner of the graph.

      ii. Calculate the work done in lifting the mass: $mg\Delta y$. The vertical displacement of the piston Δy can be determined by dividing ΔV by piston area A. Recall that if you use MKS units the result will be in joules and that mL=$10^-6\ rmm^3$.

      iii. Compare the results of (i) and (ii).

8. For your "Quick Report" list your measured n (with correct units, significant figures, and uncertainty) and both calculations of the work done in your PV engine cycle.

9. Following the suggestions given in the Introduction to the Laboratory Manual, make a brief critique of this lab.

CHECKLIST

Names, date, purpose	
Sketch of the apparatus	
Theory, with relevant equations and quantities defined	
Brief Methods & Materials or Procedure outline	
Record room temperature	
WAPP$^+$ fit report for V vs. P	
WAPP$^+$ graph of V vs. P	
WAPP$^+$ B parameter and the calculation of n (sigfigs!)	
Q1: Random or systematic errors?	
PV cycle graph	
comparison: ΔP and Force/Area	
comparison: PV cycle area and $mg\Delta y$	
Turn in "Quick Report"	


IF THIS WORKS WE'LL BUILD VATS TWENTY TIMES THIS SIZE!
ACCORD-ING TO OUR FIG-URES SIX HOURS STEAMING AT 260 DEGREES SHOULD DO IT!

APPENDIX A — UNCERTAINTIES

Introduction

All *measured* quantities have some uncertainty associated with them. It follows that anything you *calculate* using this measurement will also have some uncertainty associated with it. Any time you report a quantity, for example a quantity called Y, you should also report the amount of uncertainty δY it has, in the form $Y \pm \delta Y$. This appendix will assist you in determining the uncertainties for measured and calculated quantities.

Measurements

Quantity Y measured once

The amount of uncertainty in a measurement usually depends on how the measurement was made. For instance, if you measure the temperature in a room using a mercury-column Celsius thermometer that is marked in 1° increments, you should be able report the temperature to the nearest degree. You might report that the temperature is $T =$ 25°C, and mean that the temperature is in the range 24.5°C $\leq T \leq$ 25.5°C; if this is the case, you should report it as $T = 25.0 \pm 0.5$ °C. It is common when reporting length data to take the uncertainty as 1/2 the smallest unit on the measuring device. Often you can do a bit better than this. You might, for example, be able to estimate where the mercury column ends between two degree markings, and report a temperature of, say, 25.3°C. If you mean that the temperature is certainly between 25.25°C and 25.35°C, then say so by reporting $T = 25.30 \pm 0.05$ °C. However, if you only mean that it's certainly between 25.1 and 25.5 degrees, you should report $T = 25.3 \pm 0.2$ °C. The idea is to report a measurement as accurately as possible, but to also state clearly its range of possible values.

It can be very useful to put the uncertainty into a fractional or percentage form. As we shall see, the percent uncertainty in a calculated quantity is often related in a simple way to the percent uncertainty of the measured quantity. For example, if $x = 37.2\pm$ 0.2 cm, it could also be written as $x = 37.2$ cm $\pm 0.3\%$ since $0.2/37.2 = 0.00269$.

In summary, when only a single measurement of quantity Y is performed, express it as $Y \pm \delta Y$, meaning you're reasonably sure the value lies in the interval $(Y - \delta Y, Y + \delta Y)$. Note the following terms:

$$\text{Absolute uncertainty in } Y = \delta Y \tag{A.1}$$

$$\text{Fractional uncertainty in } Y = \frac{\delta Y}{Y} \tag{A.2}$$

$$\text{Percentage uncertainty in } Y = \frac{\delta Y}{Y} \times 100 \tag{A.3}$$

Quantity Y measured N times

Sometimes repeated measurements of a quantity give different results. This may be because the quantity actually varies; for example, your weight may vary from day to day. On the other hand, it may be purely a consequence of how the measurement was made; for instance, you may be measuring the time for a pendulum to swing once, and get a different result each time, even though the pendulum always behaves the same. In either case, repetition can tell us something useful. We can take the average, or mean, $\overline{Y}$ as the best overall value of the quantity Y. We can also find how much the measurement typically varies from the average. This typical variation is called the standard deviation, σ_Y. So, for example, it is likely that the next pendulum swing will fall in the range $(\overline{Y} - \sigma_Y, \overline{Y} + \sigma_Y)$. The best estimate of the uncertainty in $\overline{Y}$ (called the standard deviation of the mean), is $\sigma_{\overline{Y}} = \sigma_Y/\sqrt{N}$. The idea is that the mean of many measurements is less uncertain than the measurements themselves. If we have N measurements of quantity Y, call them $(Y_1, Y_2, \ldots, Y_N)$, and they are all taken to be equally trustworthy, then:

$$\begin{aligned}
\text{Mean of } Y &= \overline{Y} = \tfrac{Y_1+Y_2+\ldots+Y_N}{N} \\
\text{Standard deviation of } Y &= \sigma_Y = \sqrt{\tfrac{(Y_1-\overline{Y})^2+(Y_2-\overline{Y})^2+\ldots+(Y_N-\overline{Y})^2}{N-1}} \\
\text{Standard deviation of } \overline{Y} &= \sigma_{\overline{Y}} = \sigma_Y/\sqrt{N}
\end{aligned}$$

Standard deviations can be tedious to calculate if N is large; many hand calculators have a button for computing σ_Y; on spreadsheets try `STDEV(...)`, e.g., `STDEV(B3:B10)`.

Quantity Y measured N times with different uncertainties

Suppose we somehow get several measurements of some quantity Y, and the measurements have different uncertainties associated with them. There is a way to get a "best" estimate of Y by taking all the measurements into consideration, but counting most heavily on the more precise ones; it's called the *weighted average*. Furthermore, we can attach an uncertainty to it that is less than the smallest of the individual uncertainties. If the measurements of Y are $(Y_1, Y_2, \ldots, Y_N)$ and their uncertainties are $(\delta_1, \delta_2, \ldots, \delta_N)$, we calculate weighting factors $W_1, W_2, \ldots, W_N$ given by:

$$W_i = \frac{1}{\delta_i^2}$$

and use them to compute

$$Y_{best} = \frac{W_1 Y_1 + W_2 Y_2 + \ldots + W_N Y_N}{W_1 + W_2 + \ldots + W_N}$$

Notice that this is like an average of the Y values, but with each one weighted by a factor that increases as its uncertainty decreases. So the ones that are most reliable influence the average the most. The weighted average has a "best" uncertainty associated with it, given by

$$\delta Y_{best} = \frac{1}{\sqrt{W_1 + W_2 + \ldots + W_N}}$$

Calculations

This important section gives formulas for finding uncertainties in calculations. Suppose quantities $A, B, C, \ldots$, are measured and used to compute quantity Y. The uncertainty δY in the value of Y depends on the uncertainties $(\delta A, \delta B, \delta C, \ldots)$ in the measurements and the way the measurements are used in the calculation. A little calculus gives approximate formulas to use for a "worst-case" estimate. Some statistical analysis gives similar but slightly better formulas to use when the uncertainties are all independent and random.

When the measurements $A, B, C, \ldots$ have estimated uncertainties $\delta A, \delta B, \delta C, \ldots$, the uncertainty δY depends on the formula for Y in the following ways.

$$
\begin{aligned}
\text{Addition and Subtraction: If} \quad Y &= A \pm B \pm C \pm \ldots, \\
\text{then} \quad \delta Y &\leq \delta A + \delta B + \delta C + \ldots \\
\text{Multiplication and Division: If} \quad Y &= \tfrac{AB}{D}, \\
\text{then} \quad \tfrac{\delta Y}{Y} &\leq \tfrac{\delta A}{A} + \tfrac{\delta B}{B} + \tfrac{\delta D}{D} \\
\text{Exponentiation: If} \quad Y &= A^a B^b D^d, \\
\text{then} \quad \tfrac{\delta Y}{Y} &\leq a\tfrac{\delta A}{A} + b\tfrac{\delta B}{B} + d\tfrac{\delta D}{D} \\
\text{Logarithms: If} \quad Y &= \ln A, \\
\text{then} \quad \delta Y &= \tfrac{\delta A}{A}
\end{aligned}
$$

General remarks on calculated uncertainties

Note how fractional uncertainties like $\delta A/A$ appear in formulas for fractional uncertainty in Y. When measured quantities are multiplied, divided, or raised to a power in calculating Y, the easiest way to find δY is to use fractional or percentage uncertainties.

In exponent formulas, the exponents do not have to be integers. For example, the period T of a pendulum depends on the square root of its length L; the formula is $T = (2\pi/\sqrt{g})L^{1/2}$, so measuring $L \pm \delta L$ gives $\delta T/T = \delta L/2L$. The significance of a measured quantity in determining δY obviously increases with higher exponents. As another example, the volume of a cube is $V = L^3$, where L is the length of a side. Measuring $L \pm 10\%$, we obtain $V \pm 30\%$. To get V within 10%, we must measure L within about 3%.

Significant Digits

The rules for significant digits are important in determining where to truncate values when using a computer or calculator. *As a rule, the last significant figure in any stated number should be of the same order of magnitude (in the same decimal position) as the uncertainty.* In the physics lab, you should use the following rules in addition to reporting uncertainties:

1. *Nonzero digits.* Nonzero digits always count as significant figures.
2. *Zeros.* There are three classes of zeros:
   (a) **Leading zeros** are zeros that *precede* all the nonzero digits. These do not count as significant figures. In the number 0.0025, the three zeros simply in-

dicate the position of the decimal point. This number has only two significant figures.

(b) **Captive zeros** are zeros *between* nonzero digits. These always count as significant figures. The number 1.008 has four significant figures.

(c) **Trailing zeros** are zeros at the *right end* of the number. They are significant only if the number contains a decimal point. The number 100 has only one significant figure, whereas the number 1.00×10^2 has three significant figures. The number one hundred written as 100. also has three significant figures.

3. *Exact numbers.* Often calculations involve numbers that were not obtained using measuring devices but were determined by counting: 10 experiments, 3 apples, 8 molecules. Such numbers are called *exact numbers.* They can be assumed to have an infinite number of significant figures. Other examples of exact numbers are the 2 in $2\pi r$ (the circumference of a circle) and the 4 and 3 in $4/3\pi r^3$ (the volume of a sphere).

Exact numbers also can arise from definitions. For example, one inch is defined as *exactly* 2.54 centimeters. Thus in the statement 1 in = 2.54 cm, neither the 2.54 nor the 1 limits the number of significant figures when used in a calculation.

For calculations in the lab it is a good idea to carry at least one extra significant figure until the final result is obtained, then round to the proper number of digits as the last step. The rule of thumb is to drop all but one extra significant figure when multiplying/dividing (do this first). Lastly, for addition and subtraction, line up decimal places and add. In your final result round to the left-most decimal place in the added (subtracted) numbers. For example, to add $1.05 + 3.797 + 6.71236$ one would line up the decimals:

$$\begin{array}{rl} & 6.71236 \\ & 3.797 \\ + & 1.05 \\ \hline & 11.55936 \end{array} = 11.56 \text{ because 1.05 has the left-most decimal place}$$

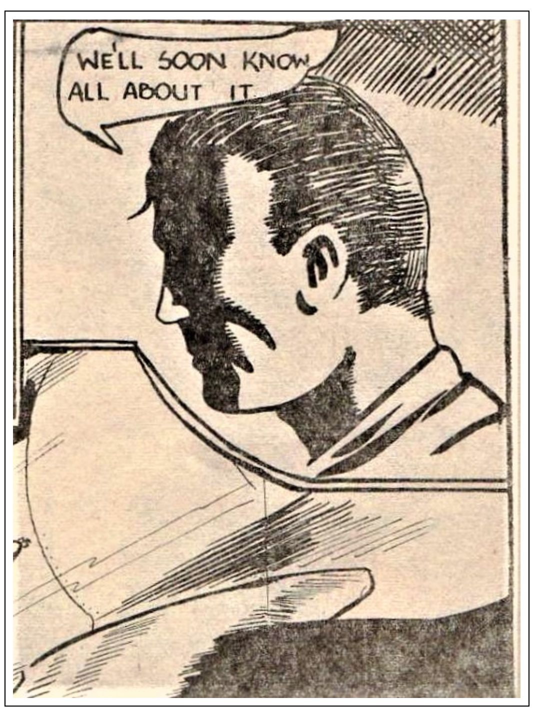
At some point you will likely find a number reported *without* a value for its uncertainty. In this case the author has assumed some value of uncertainty that you can only guess. A reasonable guess would be $\pm$ one unit of the last decimal place (e.g., 15.5 would be assumed to be 15.5 ± 0.1).

Reporting Measurements

You should record (in your lab book or Excel) every digit displayed by an instrument and every digit you estimate when reading a ruler. Do not round your raw data! Of course you must also report an uncertainty for those measurements. When it comes to reporting *final results* (in contrast to raw data or intermediate calculations) there are strict requirements as to exactly how you record those numerical results. Our uncertainty in the uncertainty is always large; hence round any uncertainties to one or two significant figures. One sigfig is always easier and almost always as accurate as two sigfigs. (Recall: You may have estimated your range-of-deviation just by eye. Manufacturer's specification of errors almost always report just one sigfig so one sigfig would be appropriate for an error calculated from those numbers. One can show that in 'normal' cases, formula-calculated standard deviations will have uncertainty greater than 10% unless the number of available data points, $N > 50$.) Once you have properly rounded your errors, display all the digits of your measurement until the place-value of the least (rightmost) significant digit in the measurement matches the place-value of the least significant digit in the error. Some examples:

$3.14159 \pm .00354 \Longrightarrow 3.142 \pm .004$ or $3.1416 \pm .0035$
$2.71828 \pm .05472 \Longrightarrow 2.72 \pm .05$ or $2.718 \pm .055$
$321456 \pm 345 \Longrightarrow 321500 \pm 300$ or 321460 ± 350
$.1678 \pm 3.51 \Longrightarrow 0 \pm 4$ or 0.2 ± 3.5

Finally, remember to record the units of your measurement!



APPENDIX B — DATA HANDLING

Tables

Experimental data are usually recorded first in a tabular form, as shown below. The table should have labeled columns for the controlled quantity and the resultant quantities. Each column label should indicate the units for that quantity. A representative estimate of the uncertainty should be made and recorded for the first entry and any other entries where there is a significantly different uncertainty. Leave room for more columns if other quantities will be calculated from the direct measurements. A sample calculation should also be shown at the bottom of the table for those quantities which are calculated.

Point #	L (cm)	$\pm\delta L$ (cm)	T (sec)	$\pm\delta T$ (sec)	Other Quantities (units)
1	1.50	0.05	0.25	0.01	...
2	1.60	"	0.40	0.02	...
...	...	...	...	...	...

Graphs

It is much easier to recognize a pattern from a graph than a table, so we will use graphs extensively (in this laboratory you will be using the program *WAPP*$^+$ to create graphs as well as fit curves to your data; more information on *WAPP*$^+$ is available in the next appendix). A well-constructed graph should provide detail beyond simply the plotted points. Notice the following features on the graph in Fig. B.1:

- Labels for each axis, with name and units.
- A descriptive title (not just a restatement of the axis labels).
- Axis ranges chosen so data points fill most of the plot area.
- Uncertainty for each data point using error bar symbols. If the uncertainty is smaller than the data point symbol, be sure to state this in your notebook on or near the graph.
- A fit curve through the data when appropriate. The fit information should be included in your notebook near the graph.

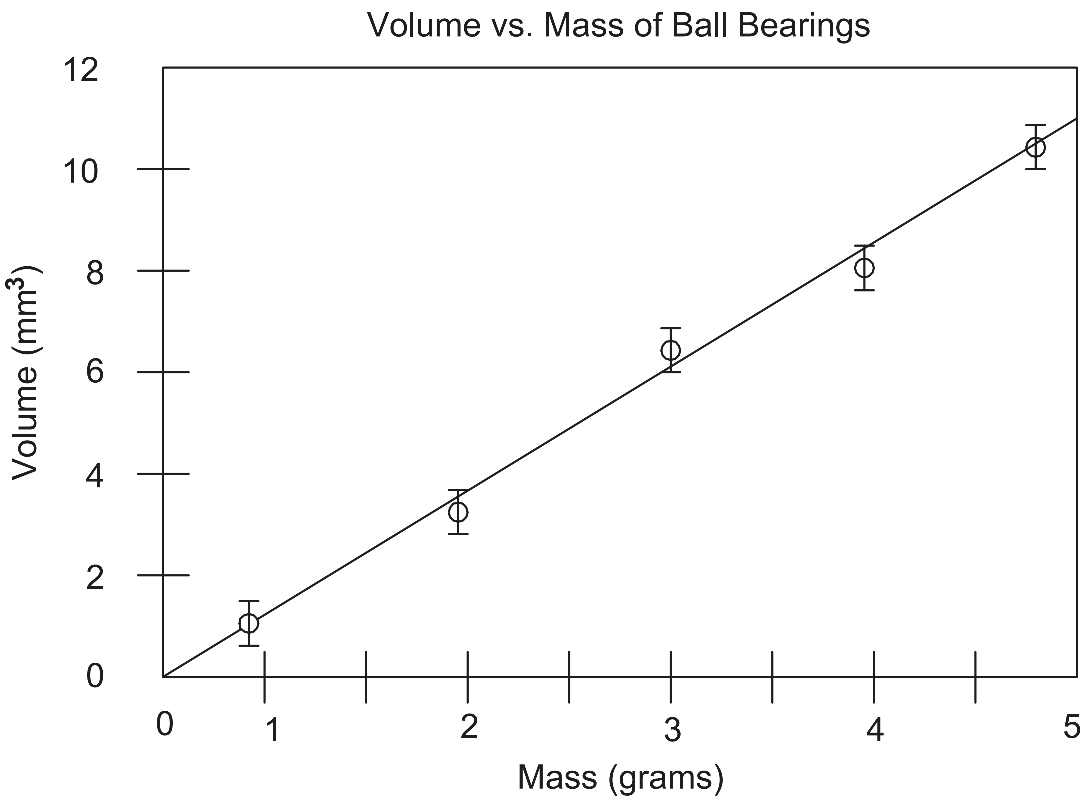


Figure B.1: A sample graph.

APPENDIX C — *WAPP*$^+$

Using *WAPP*$^+$

1. Find *WAPP*$^+$: *WAPP*$^+$ is a web-based program that can be found at

   `http://www.physics.csbsju.edu/stats/WAPP2.html`.

   However it's often faster just to Google "`wapp+`". You should bookmark this page for fast future reference. The videos:

   `http://youtu.be/9sJWetyzPdM` or locally:
   `http://www.physics.csbsju.edu/lab/105_WAPPonline.mp4` for Edge
   `http://www.physics.csbsju.edu/lab/105_WAPPonline.webm` for Chrome, Firefox, ...

   provide an introduction to this program.

2. Start Excel

3. Enter data into Excel: Enter your data values and formula-based calculations in spreadsheet columns. For example, generically calling a controlled variable x, with uncertainties δx, you may be measuring a related variable y with uncertainty δy, and then perhaps computing a third quantity z that depends on the variables somehow, and has an uncertainty δz that depends on the other measurements and their uncertainties. Use the spreadsheet columns to record data and perform calculations:

x	δx	y	δy	z	δz

4. *WAPP*$^+$ y and x error: The default setting for *WAPP*$^+$ is "Enter y error for each data point" and "Enter x error for each data point".

   Other options are available. For instance, if the uncertainties are uniform for δx and/or δy, or readily expressed with a formula, you can avoid transferring those columns and instead enter just once the uniform or formula-based error. (A box will appear on the following page so you can enter those values/formulas.) Not uncommonly the error in one coordinate is so small it can be neglected; you may also make that selection now. Make your desired selection and then click the **Do Bulk** button. A "Data Entry" page should appear.

5. Copy the numbers from Excel: Either by using:

   (a) Block Mode:
   Highlight an array of up to six columns of numbers (no column headings, just numbers) in Excel. Copy those values using your favorite method (e.g., right click, control-C, or clicking the appropriate button on the top ribbon). Paste those numbers into the large *WAPP*$^+$ data entry box that has a background image of spreadsheet cells, using your favorite method (right click, control-V, or clicking Paste from the Edit menu). There is no reason for the columns to line up with either the cell-image or the column headings. As far as the browser is concerned this is just a big lump of numbers. The numbers you paste in will almost certainly not align with anything. *WAPP*$^+$ is only concerned that there is some sort of separator between numbers (space, comma, tab, new-line, etc.); it is not concerned about the column alignment visible to you.

   (b) Column Mode:
   If appropriate, you can use the "Column Copy & Paste" region and, in separate copy & paste actions, move the appropriate columns of data into the appropriate *WAPP*$^+$ columns.

6. Tell *WAPP*$^+$ which column is which: In Block Mode, you must tell *WAPP*$^+$ the identity of each column you pasted in, so it can perform the data analysis you wish to do. In Column Mode, the headings were already in place when you entered the data.

7. Tell *WAPP*$^+$ which function should fit the data: Select from the choices:

   - Linear: $y = A + Bx$ (which should be the default).
   - Power: $y = Ax^B$
   - Exponential: $y = A\exp(Bx)$
   - Natural Log: $y = B\log(x/A)$
   - Inverse x: $y = A + B/x$
   - Inverse y: $1/y = A + Bx$
   - Inverse x & y: $1/y = A + B/x$
   - Arrhenius: $y = A\exp(B/x)$
   - Inverse-Natural Log: $1/y = B\log(x/A)$
   - Quadratic: $y = A + Bx + Cx^2$ (only available if there is no x uncertainty)
   - Resonance: $1/y^2 = A + Bx^2 + Cx^4$ (only available if there is no x uncertainty)

   If you look at the bottom of the *WAPP*$^+$ data entry page you will find these words:

Occasionally the value of B is set by theory to a known value. If that situation applies to this data, enter here that fixed value of B. (Usually you will leave this blank.)

$B=$ $\boxed{}$

8. Compute fit: click the $\boxed{\text{Submit Data}}$ button. The resulting page includes the 'fit report' that gives the best values for parameters A and B and the uncertainties for each.

   There is an important quantity called "reduced chi-squared" ($\overline{\chi}^2$) which tells how well the computed curve fits the data:

   - if $\overline{\chi}^2$ is approximately 1 then the curve is a good fit within the error bars;
   - if $\overline{\chi}^2$ is much larger than 1, the curve misses the error bars by too much to be considered reliable;
   - if $\overline{\chi}^2$ is much less than 1, the curve misses the points by distances that are unexpectedly small compared to the error bars — your error bars are probably larger than necessary. *WAPP*$^+$ may give you a warning about this, but do not feel bad, just think about whether the error bars could have been smaller, and fix them if necessary; otherwise go on.

   So if the $\overline{\chi}^2$ value is far from 1 either direction, you ought to check either your data or error estimates. By looking at the plot of the curve you may see a problem you should correct.

9. Request a plot: For a linear (normal) plot select "X Scale Options" Linear and "Y Scale Options" Linear (which should be the default). Power law data should produce a straight line if plotted on Log-Log paper. For a Log-Log plot select "X Scale Options" Log and "Y Scale Options" Log. For the final version of a plot, enter appropriate x axis labels, y axis labels, and plot title. Click $\boxed{\text{Make Plot}}$ button. On the following page click PDF File. If everything looks OK, make a final pdf plot.

10. Print the fit report: Hit the browser's back arrow a couple of times to return to the fit report page. The bottom part of the fit report page involves plotting the data and need not be printed. Rather than print the entire page, you can save some trees by highlighting (left-click sweep) the report area, down to and including the line Data Reference, and then print just the selected area.